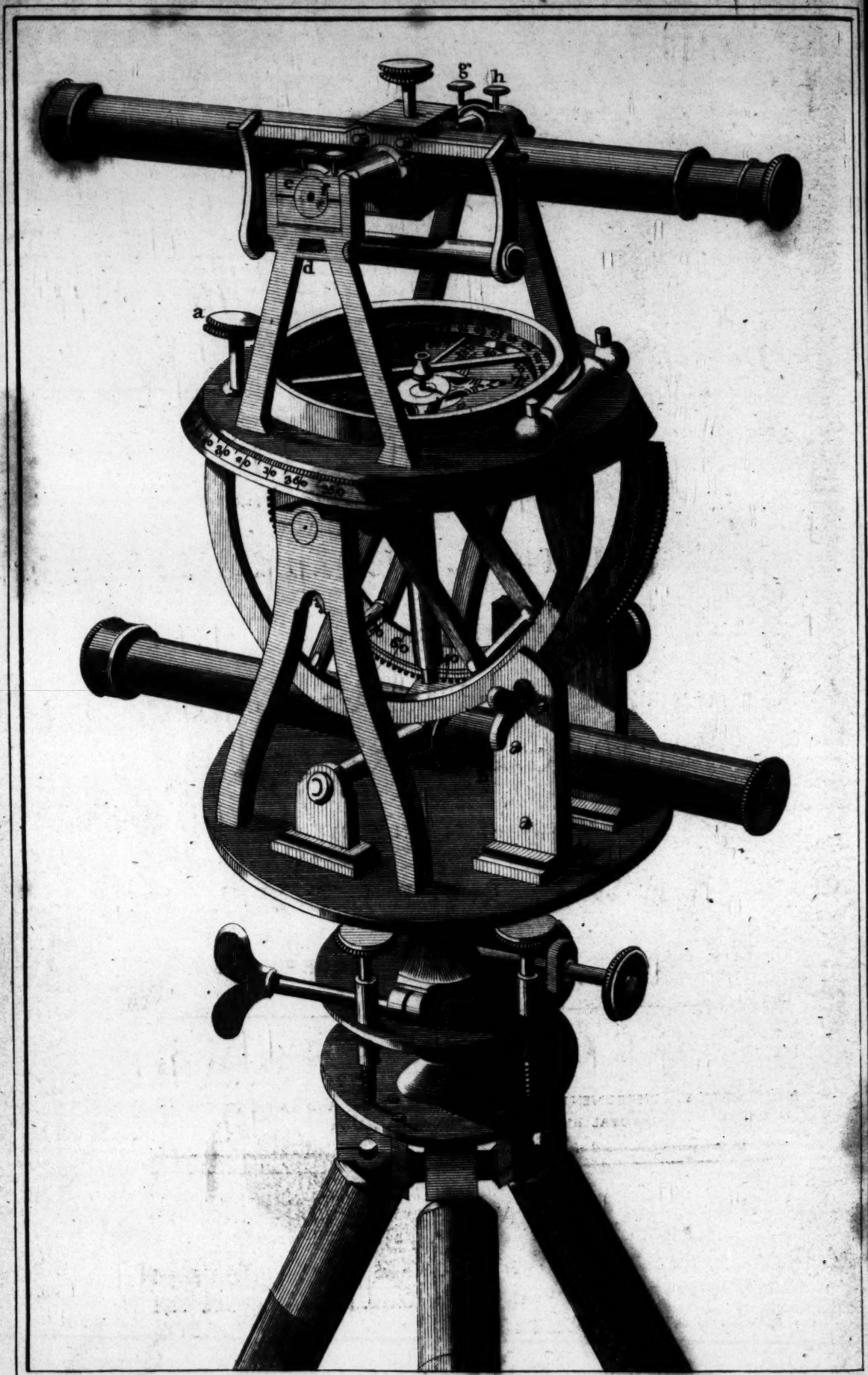


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GEOMETRICAL
AND
GRAPHICAL
ESSAYS,

CONTAINING

A DESCRIPTION OF THE
MATHEMATICAL INSTRUMENTS

USED IN

Geometry, Civil and Military Surveying,
LEVELLING AND PERSPECTIVE;

WITH MANY

NEW PROBLEMS,

ILLUSTRATIVE OF EACH BRANCH.

By GEORGE ADAMS,

MATHEMATICAL - INSTRUMENT MAKER TO HIS MAJESTY, AND OPTICIAN TO HIS
ROYAL HIGHNESS THE PRINCE OF WALES.

L O N D O N:

Printed for the AUTHOR, by R. HINDMARSH, Printer to His Royal
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M. DCC. XCI.

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GRAPHICAL

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By GEORGE ADAMS.

MATHEMATICAL INSTRUMENTS KEPT TO THE MEASURE AND WEIGHTS OF THE
ROYAL SOCIETY THE YEAR 1711.

Printed for the Author, by J. BARNARD, at the Sign of the Crown, in St. Paul's Church-yard, 1711.
By the Author, at the Sign of the Crown, in St. Paul's Church-yard, 1711.

M. DCC. XI.

TO
THE MOST NOBLE
CHARLES, DUKE OF RICHMOND,
AND LENNOX,
Master General of the Ordnance, &c.

THESE ESSAYS

ARE,

WITH GREAT RESPECT,

JUSTLY INSCRIBED,

BY HIS GRACE'S

Most Obedient,

Humble Servant,

GEORGE ADAMS.

TO

THE MOST NOBLE

CHARLES, DUKE OF RICHMOND,

AND LENOX,

Master General of the Ordnance, &c.

THESE ESSAYS

WITH GREAT RESPECT,

JUSTLY INSCRIBED,

BY HIS GRACE:

Mark Obolensky

Humble Servant,

GEORGE ADAMS

P R E F A C E.

THOSE who have had much occasion to use the mathematical instruments constructed to facilitate the arts of drawing, surveying, &c. have long complained that a treatise was wanting to explain their use, describe their adjustments, and give such an idea of their construction, as might enable them to select those that are best adapted to their respective purposes.

This complaint has been the more general, as there are few active stations in life whose professors are not often obliged to have recourse to mathematical instruments. To the civil, the military, and the naval architect their use must be familiar; and they are of equal, if not of more importance to the engineer and the surveyor; they are the means by which the abstract parts of the mathematics are rendered useful in life, they connect theory with practice, and reduce speculation to use.

Monsieur Bion's treatise on the construction of mathematical instruments, which was translated into English by Mr. Stone, and published in 1723, is the only regular treatise * we have upon this subject: the numerous improvements that have been made in instruments since that time, have rendered this work but of little use. It has been my endea-

a

vour

* I do not speak of Mr. Robertson's work, as it is confined wholly to the instruments contained in a case of drawing instruments.

vour by the following essays to do away this complaint; and I have spared no pains to render them intelligible, and make them useful. Though the materials of which they are composed lie in common, yet it is presumed, that essential improvements will be found in almost every part.

These essays begin by defining the necessary terms, and stating a few of those first principles on which the whole of the work is founded: they then proceed to describe the mathematical drawing instruments; among these the reader will find an account of an improved pair of triangular compasses, a small pair of beam compasses with a micrometer screw, four new parallel rules, and other articles not hitherto described: these are followed by a large collection of useful geometrical problems; I flatter myself, that the practitioner will find many that are new, and which are well adapted to lessen labour, and promote accuracy. In describing the manner of dividing large quadrants, I have first given the methods used by instrument-makers, previous to the publication of that of Mr. Bird, subjoining his mode thereto, and endeavouring to render it more plain to the artist by a different arrangement. This is succeeded by geometrical and mechanical methods of describing circles of every possible magnitude, for the greater part of which I am indebted to *Joseph Priestley*, Esq. of Bradford, Yorkshire, whose merit has been already noticed by an abler pen than mine.* From this I proceed to give a short view of elliptic and other compasses, and a description of *Suard's geometric pen*, an instrument not known in this country, and whose curious properties will exercise the ingenuity of mechanics and mathematicians.

Trigonometry

* Priestley's Perspective.

Trigonometry is the next subject; but, as this work was not designed to teach the elements of this art, I have contented myself with stating the general principles, and giving the cannons for calculation, subjoining some useful and curious problems, which, though absolutely necessary in many cases that occur in county and maritime surveying, have been neglected by every practical writer on this subject, except M. Mackenzie,* and B. Donn.† Some will also be found, that are even unnoticed by the above-mentioned authors.

Our next article treats of surveying, and it is presumed the reader will find it a complete, though concise system thereof. The several instruments now in use, and the methods of adjusting them, are described in order; and I think it will appear evident, from a view of those of the best construction, that large estates may be surveyed and plotted with greater accuracy than heretofore.

The great improvements that have been made within these few years in the art of dividing, have rendered observers more accurate and more attentive to the necessary adjustments of their instruments, which are not now considered as perfect, unless they are so constructed, that the person who uses them can either correct or allow for the errors to which they are liable. Among the various improvements which the instruments of science have received from *Mr. Ramsden*, we are to reckon those of the theodolite here described; the surveyor will find also the description of a small quadrant that should be constantly used with the chain, improvements in the circumferenter, plain-table, protractor, &c. In treating of surveying, I thought

a 2

to

* Treatise on Maritime Surveying.

† Donn's Geometrician.

to have met with no difficulty ; having had however no opportunity of practice myself, I had recourse to books ; a multiplicity have been written upon this subject, but they are for the most part imperfect, irregular, and obscure. I have endeavoured (with what success must be left to the reader's judgment) to remove their obscurities, to rectify their errors, and supply their deficiencies : but whatever opinion he may form of my endeavours, I can venture to say, he will be highly gratified with the valuable communications of *Mr. Gale*,* and *Mr. Milne*, here inserted, and which I think will contribute more to the improvement of the art of surveying, than any thing it has received since it's original invention.

The reader will, I hope, excuse me, if I stop a moment to give him some account of Mr. Gale's improvements ; they consist, first, in a new method of plotting, which is performed by scales of equal parts, (without a protractor,) from the northings and southings, eastings and westings, taken out of the table which forms the appendix to this work ;† this method is much more accurate than that in common use, because any small inaccuracy, that might happen in laying down one line, is *naturally corrected* in the next ; whereas in the common method of plotting by scale and protractor, any inaccuracy in a former line is *naturally communicated* to all the succeeding lines. The next improvement consists in a new method of determining the *area*, with superior accuracy, from the northings, southings, eastings, and westings, (without any regard to the plot or draught) by an easy computation.

As

* A gentleman well known for his ingenious publications on finance.

† The table is printed separate, that it may be purchased or not, as the surveyor sees convenient.

As the measuring a strait line with exactness is one of the greatest difficulties in surveying, I was much surprized to find many land-measurers using only a chain; a mode in which errors are multiplied without a possibility of their being discovered or corrected. I must not forget to mention here, that I have inserted in this part Mr. Break's method of surveying and planning by the plain table, the bearings being taken and protracted at the same instant in the field upon one sheet of paper; thus avoiding the trouble and inconvenience of shifting the paper: this is followed by a small sketch of maritime surveying, the use of the pantographer, the art of levelling, and a few astronomical problems, with the manner of using Hadley's quadrant and sextant; even here some suggestions will be found, that are new and useful.

I have now to name another gentleman, who has contributed to render this work more perfect than it would otherwise have been, and it is with pleasure I return my best thanks to *Mr. Landman*, (*Professor of Fortification and Artillery to the Royal Academy at Woolwich*,) for his communications, more particularly for the papers from which the course of practical geometry on the ground was extracted. If the professors of useful sciences would thus liberally co-operate for their advancement, the progress thereof would be rapid and extensive. This course will be found useful not only to the military officer, but would make a useful and entertaining part of every gentleman's education. I found it necessary to abridge the papers Mr. Landman lent me, and leave out the calculations, as the work had already swelled to a larger size than was originally intended, though printed on a page unusually full.

The work finishes with a small tract on perspective,

tive, and a description of two instruments, designed to promote and facilitate the practice of that useful art. It is hoped that the publication of these will prevent the public from being imposed upon by men, who, under the pretence of secrecy, enhance the value of their contrivances. I knew an instance where 40*l.* was paid for an instrument inferior to the most ordinary of the kind that are sold in the shops. Some pains have been taken, and no small expence incurred, to offer something to the public superior in construction, and easier to use, than any instrument of the kind that has been hitherto exhibited.

I have been anxious and solicitous not to neglect any thing that might be useful to the practitioner, or acceptable to the intelligent. In a work which embraces so many subjects, notwithstanding all the care that has been taken, many defects may still remain; I shall therefore be obliged to any one, who will favour me with such hints or observations as may tend towards it's improvement.

I have to apologize for some errors of the press; the most essential are, I hope, sufficiently obviated by the errata subjoined to this preface: they have arisen from the circumstances under which the work was composed, the languor of ill health, and the avocations of business.

A list of the authors I have seen is subjoined to this preface. I beg leave to return my thanks to the following gentlemen for their hints and valuable communications, the Rev. Mr. Hawkins, J. Priestley, Esq. Mr. Gale, Mr. Milne, Dr. Rotherham, Mr. Heywood, Mr. Landman, and Mr. Beck, a very ingenious artist.

The instrument represented in the frontispiece to these essays, was not finished in time to have it's description inserted in it's place; I am therefore
under

under the necessity of giving a short description thereof in this preface, referring for particulars to the essays.

Description of the theodolet figured in the frontispiece.

This instrument, like those described in pages 233 to 240, consists, 1. Of a telescope and it's level. 2. Of an horizontal limb and compass. 3. Of a vertical arch. 4. Of a telescope underneath the vertical arch. 5. Of the staff with it's parallel plates.

It is obvious, that when the line of sight or telescope is moved on one of the circles of the instrument, it's motion will be affected by every irregularity or deviation of that circle from a true plane; but on the contrary, if the motion be performed about axes well fitted, the extremity of the line of sight will in all situations remain directed to some point in the circle in which it may be placed. On this account the telescope in this construction has it's movements upon axes, instead of being governed by the plane of a circle. A further advantage is this, that the adjustments are made at the axes themselves, where they are neither so liable to be altered, nor so easily deranged, as when the adjustments are at the bottom of the supporters. The telescope moves sufficiently to reduce angles taken in an oblique plane to an horizontal one, without moving the semicircle of altitude, which is kept firmly in it's situation by a finger screw: as this instrument is used in the same manner as those that are described in pages 233 to 240, we shall have no occasion to notice any thing more than the mode of adjusting it, and the figure renders a minute enumeration of the parts unnecessary.

To adjust the horizontal circle, set the semicircle of altitude exactly to 0, then fix it firmly

there by the finger screw b, and turn the horizontal circle by the milled screw a, till the level is in a line with two of the staff screws; and by means of these screws, make the bubble in the level become stationary in the middle of the glass; turn the horizontal circle half round, and if the bubble remains in the middle, the level is well adjusted; if it does not, correct the position of the level till the bubble has moved half the distance it ought to come, to reach the middle, making it move the other half by the staff screws; return the horizontal circle to it's first position, and if the adjustments have been well made, the bubble will remain in the middle; if otherwise, the process of altering the level, and the staff screws with the reversing, must be repeated till it bears this proof of accuracy.

The telescope has a strong axis, whose circular ends turn in moveable sockets of hard metal, one of which is capable of a vertical, the other of an horizontal adjustment. To make the axis of the telescope parallel to the horizon, hook the arms of the hanging level upon the ends of the axis of the telescope, having previously placed the horizontal circle level by means of the capstan screw d beneath one end of the axis; cause the bubble of the hanging level to occupy the middle of it's glass; unhook the level, and hang it on the axis, but in a contrary position, so that it's right hand end may now be placed to the left; if the bubble still continues to occupy the middle of the glass, the axis of the level is truly parallel to the horizon; if not, alter the adjustment by the capstan screw, and of the axis by it's own screws, till the bubble stands in the middle in both positions.

The next adjustment is for the elevation level, or when the level is placed, as in the figure, on the

arm by the telescope, in order to bring the telescope level, for the purpose of taking angles of elevation and depression, or for levelling. The level having been adjusted by the preceding operation, you have only to fix the telescope, so that it cannot move, by passing the steel pin through the axis; if the bubble of the level is not in the middle, one half the error must be corrected by the screw of the bar on the outside of the telescope, and the other half by the horizontal screw that acts upon the piece through which the steel pin passes.

To make the vertical cross wire in the telescope at right angles to the axis, take out the small screws, e, f, g, h, and lift off the covers that confine the axis, direct the telescope to a distant well-defined object, and if the intersection of the wires coincides with the object in both positions, the line of sight is at right angles to the axis of motion; but if not, alter the position of the cross wires, by means of the two small screws near the eye end of the telescope, one of which must be unscrewed a little, and the other screwed up, till the desired proof of the accuracy of the position of the wires in the telescope be attained.

To make the line of collimation parallel to the horizon. The hanging level is to be first adjusted according to the preceding instructions, and then suspended to the bar on the outside of the telescope; turn the telescope on its axis till the bubble comes to the middle, and observe to what object the center of the cross wire is directed, invert the axis, bring the bubble again to the middle, and observe whether the telescope be directed to the same object; if not, alter it half way by turning the telescope about the axis, and then bring the bubble to the middle by the screw under the bar, and thus the rod, and consequently the level is placed parallel to the line of collimation. This operation should be repeated for greater certainty.

LIST

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Spiedeli.	Geometrical Extraction.	London, 1657
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E R R A T A.

- Page 54, line 18, *for they read those.*
 — 57, — 3, *for plate 6, read plate 4.*
 — 60, — 30, *for these read the.*
 — 62, — 26, *for G H read C H.*
 — 63, — 10, *for plate 4, read plate 5.*
 — 64, — 26, *for plate 4, read plate 5.*
 — 67, — 26, *for plate 4, read plate 5.*
 — 70, — 21, *for G D read c P.*
 — 71, — 3, *for a b read a V.*
 — 73, — 10, *for A D read A B.*
 — 74, and 75, *for plate 5, always read plate 6.*
 — 80, problem 49 and 50, *for plate 5, read plate 6.*
 — 88, last line, *for E C read A C.*
 — 89, line 5, *for A B read A D.*
 — Ibid. *for E D read C D.*
 — Ibid. line 7, *for C G read A E.*
 — 92, line 24, *for M N read M L.*
 — Ibid. line 30, *for C K read K I.*
 — 133, — 19, *for fig. 8, read fig. 9.*
 — 156, — 14, *for I C read A C.*
 — 159, — 3 from bottom, *for fig. 14, read fig. 13.*
 — 161, — 30, *for A a read c a.*
 — Ibid. line 31, *for A b read C b.*
 — 162, — 26, *for C a C b read C a, C b.*
 — 179, — 7, *for problem 1, read preceding problem.*

* Other errata not being so material, will be easily corrected by the reader.

ADVER.

(xii)

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GEOMETRICAL AND GRAPHICAL ESSAYS.

NECESSARY DEFINITIONS, *and* FIRST PRINCIPLES.

GEOMETRY is the science that treats of, and considers the properties of magnitude in general. In other words, extension and figure are the objects of geometry. It is a science in which human reason has the most ample field, and can go deeper, and with more certainty, than in any other. It is divided into two parts, theoretical and practical.

Theoretical geometry considers and treats of first principles abstractedly. Practical geometry applies these considerations to the purposes of life. Proclus asserts, that the end of geometry is to be referred to the energies of intellect, and that it is degraded when it is made subservient to the common purposes of a mere animal life. We, however, conceive him to be mistaken; for the true end of science is *use*, and the truths of geometry, sublime as they may appear, are only valuable as they conduct us to the practice. By practical

B geometry

geometry many operations are performed, of the utmost importance to society and the arts. "The effects thereof are extended through the principal operations of human skill: it conducts the soldier in the field, the seaman in the ocean: it gives strength to the fortrefs, and elegance to the palace."

It has been common to attribute the invention of geometry to the Egyptians, and to suppose that we were indebted to the inundations of the Nile for this sublime branch of human knowledge. But surely there is no occasion, in this part of literary history, to satisfy learned curiosity by tales and fables: for geometry is an art that must have grown with man; it is, in a great measure, natural to the human mind; we were born spectators of the universe, which is the kingdom of geometry, and are continually obliged to judge of heights, measure distances, and estimate the bulk of bodies.

A *point* is considered by the geometricians, as that which has no parts or magnitude.

A *line* is length without breadth.

A *strait line* is that which lies evenly between it's extreme points or ends.

A *superficies* is that which has only length and breadth.

A *plane angle* is an opening or corner made by two strait lines meeting one another.

When a strait line A B, *fig. 1, plate 4*, standing upon another C D, makes angles A B C, A B D, on each side equal to one another; each of these angles is called a *right angle*; and the line A B is said to be *perpendicular* to the line C D.

It is usual to express an angle by three letters, that placed at the angular point being always in the middle.

An.

An *obtuse angle* is that which is greater than a right angle.

An *acute angle* is that which is less than a right angle.

A line A B, *fig. 2, plate 4*, cutting another line C D in E, will make the opposite angles equal, namely, the angle A E C equal to B E D, and A E D equal to B E C.

A line A B, *fig. 3, plate 4*, standing any-how upon another C D, makes two angles C B A, A B D, which, taken together, are equal to two right angles.

A *plane triangle* is a figure bounded by three right lines.

An *equilateral triangle* is that which has three equal sides.

An *isosceles triangle* is that which has only two equal sides.

A *scalene triangle* is that which has all it's sides unequal.

A *right-angled triangle* is that which has one right angle.

In a right-angled triangle, the side opposite to the right angle is called the *hypotenuse*.

An *oblique-angled triangle* is that which has no right angle.

In the same triangle, opposite to the greater side, is the greater angle; and opposite to the greater angle, is the greater side.

If any side of a plane triangle be producèd, the outward angle will be equal to both the inward remote angles.

The three angles of any plane triangle taken together, are equal to two right angles.

Parallel lines are those which have no inclination towards each other, or which are every-where equidistant.

All plane figures, bounded by four right lines, are called *quadrangles*, or *quadrilaterals*.

A *square* is a quadrangle, whose sides are all equal, and it's angles all right angles.

A *rhombus* is a quadrangle, whose sides are all equal, but it's angles not right angles.

A *parallelogram* is a quadrangle, whose opposite sides are parallel.

A *rectangle* is a parallelogram, whose angles are all right angles.

A *rhomboid* is a parallelogram, whose angles are not right angles.

All other four-sided figures besides these, are called *trapeziums*.

A right line joining any two opposite angles of a four-sided figure, is called the *diagonal*.

All plane figures contained under more than four sides, are called *polygons*.

Polygons having five sides, are called *pentagons*; those having six sides, *hexagons*; with seven sides, *heptagons*, and so on.

A *regular polygon* is that whose angles and sides are all equal.

The *base* of any figure is that side on which it is supposed to stand, and the *altitude* is the perpendicular falling thereon from the opposite angle.

Parallelograms upon the same base, and between the same parallels, are equal.

Parallelograms having the same base, and equal altitudes, are equal.

Parallelograms having equal bases, and equal altitudes, are equal.

If a triangle and parallelogram have equal bases, and equal altitudes, the triangle is half the parallelogram.

A circle

A *circle* is a plane figure, bounded by a curve line called the *circumference*, every part whereof is equally distant from a point within the same figure, called the *center*.

Any part of the circumference of a circle, is called an *arch*.

Any right line drawn from the center to the circumference of a circle, is called a *radius*.

All the radii of the same circle are equal.

The circumference of every circle is supposed to be divided into 360 equal parts, called *degrees*; each degree into 60 equal parts, called *seconds*, &c.

A *quadrant* of a circle will therefore contain 90 degrees, being a fourth part of 360.

Equal angles at the centers of all circles, will intercept equal numbers of degrees, minutes, &c. in their circumferences.

The *measure* of every plane angle is an arch of a circle, whose center is the angular point, and is said to be of so many degrees, minutes, &c. as are contained in it's measuring arch.

All right angles, therefore, are of 90 degrees, or contain 90 degrees, because their measure is a quadrant.

The three angles of every plane triangle taken together, contain 180 degrees, being equal to two right angles.

In a right-angled plane triangle, the sum of it's two acute angles is 90 degrees.

The *complement* of an arch, or of an angle, is it's difference from a quadrant or a right angle.

The *supplement* of an arch, or of an angle, is it's difference from a semicircle, or two right angles.

The magnitudes of arches and angles are determined by certain strait lines, appertaining to a circle, called chords, sines, tangents, &c.

The *chord* of an arch is a strait line, joining it's extreme points.

A *diameter* is a chord passing through the center.

A *segment* is any part of a circle bounded by an arch and it's chord.

A *sector* is any part of a circle bounded by an arch, and two radii drawn to it's extremities,

The *sine* of an arch is a line drawn from either end of it, perpendicular to a diameter meeting the other end.

The *versed sine* of an arch is that part of the diameter intercepted between the sine and the end of the said arch.

The *tangent* of an arch is a line proceeding from either end, perpendicular to the radius joining it; it's length is limited by a line drawn from the center through the other end.

The *secant* of an arch is the line proceeding from the center, and limiting the tangent of the same arch.

The co-sine and co-tangent, &c. of any arch, is the sine and tangent, &c. of it's complement.

Thus in *fig. 4, plate 4*, FO is the chord of the arch FVO, and FR is the sine of the arches FV, FAD; RV, RD are the versed sines of the arches FV, FAD.

VT is the tangent of the arch FV, and it's supplement.

CT is the secant of the arch FV.

AI is the co-tangent, and CI the co-secant of the arch FV.

The chord of 60° , the sine of 90° , the versed sine of 90° , the tangent of 45 , and the secant of 0.0 , are all equal to the radius.

It is obvious, that in making use of these lines we must always use the same radius, otherwise there

there would be no settled proportions between them.

Whosoever considers the whole extent and depth of geometry, will find that the main design of all it's speculations is mensuration. To this the elements of Euclid are almost entirely devoted; and this has been the end of the most laboured geometrical disquisitions of either the ancients or moderns.

Now the whole mensuration of figures may be reduced to the measure of triangles, which are always the half of a rectangle of the same base and altitude, and consequently their area is obtained by taking the half of the product of the base multiplied by the altitude.

By dividing a polygon into triangles, and taking the value of these, that of the polygon is obtained; by considering the circle as a polygon, with an infinite number of sides, we obtain the measure thereof.

The theory of triangles is as it were the hinge upon which all geometrical knowledge turns.

All triangles are more or less similar, according as their angles are nearer to, or more remote from equality.

The similitude is perfect, when all the angles of the one are equal respectively to those of the other; the sides are then also proportional,

The angles and the sides determine both the relative and absolute size, not only of triangles, but of all things,

Strictly speaking, angles only determine the relative size; equiangular triangles may be of very unequal magnitudes, yet perfectly similar.

But when they are also equilateral, the one having it's sides equal to the homologous sides of

the other, they are not only similar and equiangular, but are equal in every respect.

The angles, therefore, determine the relative species of the triangle, the sides it's absolute size, and consequently that of every other figure, as all are resolvable into triangles.

Yet the essence of a triangle seems to consist much more in the angles than the sides, for the angles are the true, precise, and determined boundaries thereof; their equation is always fixed and limited to two right angles.

The sides have no fixed equation; but may be extended from the infinitely little, to the infinitely great, without the triangle changing it's nature and kind.

It is in the theory of isoperimetrical figures * that we feel how efficacious angles are, and how inefficacious lines, to determine not only the kind, but the size of the triangles, and all kinds of figures.

For the lines still subsisting the same, we see how a square decreases in proportion as it is changed into a more oblique rhomboid, and thus acquires more acute angles. The same observation holds good in all kinds of figures, whether plane or solid.

Of all isoperimetrical figures, the plane triangle and solid triangle, or pyramid, are the least capacious; and amongst these, those have the least capacity, whose angles are most acute.

But curved surfaces, and curved bodies, and among curves the circle and sphere, are those whose capacity are the largest, being formed, if we may so speak, of the most obtuse angles.

The

* Isoperimetrical figures are such as have equal circumferences.

The theory of geometry may therefore be reduced to the doctrine of angles, for it treats only of the boundaries of things, and by angles the ultimate bounds of all things are formed. It is the angles which gives them their figure.

Angles are measured by the circle; to these we may add parallels, which, according to the signification of the terms, are the source of all geometrical similitude and comparifon.

OF MATHEMATICAL DRAWING INSTRUMENTS.

The strictness of geometrical demonstration admits of no other instruments, than a rule and a pair of compaffes. But in proportion as the practice of geometry was extended to the different arts, either connected with, or dependent upon it, new instruments became neceffary, fome to answer peculiar purpofes, fome to facilitate operation, others to promote accuracy.

It is the bufinefs of this work to describe thefe instruments, and explain their various ufes. In performing this task, a difficulty arofe relative to the arrangement of the fubject, whether each instrument, with it's application, fhould be described feparately, or whether the description fhould be introduced under thofe problems, for whofe performance they were peculiarly defigned. After fome confideration, I determined to adopt neither rigidly, but to ufe either the one or the other, as they appeared to answer beft the purpofes of fci-
ence.

As almoft every artift, whofe operations are connected with mathematical defigning, furnifhes himfelf with a cafe of drawing instruments fuited to his peculiar purpofes, they are fitted up in various
modes,

modes, some containing more, others fewer instruments. The smallest collection put into a case, consists of a plane scale, a pair of compasses with a moveable leg, and two spare points, which may be applied occasionally to the compasses; one of these points is to hold ink, the other a porte crayon, for holding a piece of a black-lead pencil.

What is called a full pocket case, contains the following instruments.

A pair of large compasses with a moveable point, an ink point, a pencil point, and one for dotting; either of these points may be inserted in the compasses, instead of the moveable leg.

A pair of plain compasses, somewhat smaller than those with the moveable leg.

A drawing pen, with a protracting pin in the upper part.

A pair of bow compasses.

A sector.

A plain scale,

A protractor.

A parallel rule.

A pencil.

The plain scale, the protractor, and parallel rule, are sometimes so constructed as to form but one instrument; but it is a construction not to be recommended, as it injures the plain scale, and lessens the accuracy of the protractor. In a case with the best instruments, the protractor and plain scale are always combined. The instruments in most general use are those of six inches; instruments are seldom made longer, but often smaller. Those of six inches are however to be preferred in general before any other size; they will effect all that can be performed with the shorter ones, while

at the same time they are better adapted to large work.

Large collections are called, *magazine cases of instruments*; these generally contain

A pair of six inch compasses with a moveable leg, an ink point, a dotting point, the crayon point, so contrived as to hold a whole pencil, two additional pieces, to lengthen occasionally one leg of the compasses, and thereby enable them to measure greater extents, and describe circles of a larger radius.

A pair of hair compasses.

A pair of bow compasses.

A pair of triangular compasses.

A sector.

A parallel rule.

A protractor.

A pair of proportionable compasses, either with or without an adjusting screw.

A pair of wholes and halves.

Two drawing pens, and a pointrel.

A pair of small hair compasses, with a head similar to those of the bow compasses.

A knife, a file, key and screw driver for the compasses, in one piece.

A small set of Reeves's water colours.

To these some of the following instruments are often added.

A pair of beam compasses.

A pair of gunners callipers.

A pair of elliptical compasses.

A pair of spiral ditto.

A pair of perspective compasses.

A pair of compasses with a micrometer screw.

A rule for drawing lines, tending to a center at a great distance.

A pro-

A protractor and parallel rule, such as is represented at *fig. I K L, plate 2.*

One or more of the parallel rules represented, *plate 2.*

A pantographer.

A pair of sectoral compasses, forming at the same time a pair of beam and calliper compasses.

OF DRAWING COMPASSES:

Compasses are made either of silver or brass, but with steel points. The joints should always be framed of different substances; thus one side or part should be of silver or brass, and the other of steel. The difference in the texture and pores of the two metals, causes the parts to adhere less together, diminishes the wear, and promotes uniformity in their motion. The truth of the work is ascertained by the smoothness and equality of the motion at the joint, for all shake and irregularity is a certain sign of imperfection. The points should be of steel so tempered as neither to be easily bent or blunted, not too fine and tapering, and yet meeting closely when the compasses are shut.

As an instrument of art, compasses are so well known, that it would be superfluous to enumerate their various uses; suffice it then to say, that they are used to transfer small distances, measure given spaces, describe arches and circles.

If the arch or circle is to be described obscurely, the steel points are best adapted to the purpose; if it is to be in ink or black lead, either the drawing pen or crayon points are to be used.

To use a pair of compasses. Place the thumb and middle finger of the right hand in the opposite hollows of the shanks of the compasses, then press the compasses, and the legs will open a little way;
this

this being done, push the innermost leg with the third finger, elevating at the same time the furthestmost, with the nail of the middle finger, till the compasses are sufficiently opened to receive the middle and third finger; they may then be extended at pleasure, by pushing the furthestmost leg outwards with the middle, or pressing it inwards with the fore finger. In describing circles or arches, set one foot of the compasses on the center, and then roll the head of the compasses between the middle and fore finger, the other point pressing at the same time upon the paper. They should be held as upright as possible, and care should be taken not to press forcibly upon them, but rather to let them act by their own weight; the legs should never be so far extended, as to form an acute angle with the paper or plane on which they are used.

The ink and crayon points have a joint just under that part which fits into the compasses, by this they may be always so placed as to be set nearly perpendicular to the paper; the end of the shank of the best compasses is framed so as to form a strong spring to bind firmly the moveable points, and prevent them from shaking. This is found to be a more effectual method than that by a screw. *Fig. B, plate 1*, a pair of the best compasses with the plain point, *c* the ink, *d* the dotting, *e* the crayon point.

In small cases the crayon and ink points are joined by a middle piece, with a socket at each end to receive the points, which by this means only occupy one place in the case.

Two additional pieces, *fig. P, Q, plate 1*, are often applied to these compasses; these by lengthening the leg *b*, enable them to strike larger circles,

cles, or measure greater extents than they would otherwise perform, and that without the inconveniences attending longer compasses. When compasses are furnished with this additional piece, the moveable leg has a joint, as at b, that it may be placed perpendicular to the paper.

Of the hair compasses, fig. L, plate 1. They are so named on account of a contrivance in the shank, to set them with greater accuracy than can be effected by the motion of the joint alone. One of the steel points is fastened near the top of the compasses, and may be moved very gradually by turning the screw *n*, either backwards or forwards.

To use these compasses. 1. Place the leg to which the screw is annexed outermost; 2. set the fixed leg on that point from whence the extent is to be taken; 3. open the compasses as nearly as possible to the required distance, and then make the points accurately coincide therewith by turning the screw.

Of the bow compasses, fig. F, plate 1. These are a small pair, usually with a point for ink; they are used to describe small arches or circles, which they do much more conveniently than large compasses, not only on account of their size, but also from the shape of the head, which rolls with great ease between the fingers. It is, for this reason, customary to put into magazine cases of instruments, a small pair of hair compasses, fig. H, plate 1, with a head similar to the bows; these are principally used for repeating divisions of a small but equal extent, a practice that has acquired the name of *stepping*.

Of the drawing pen and protracting pin, fig. I, K, plate 1. The pen part of this instrument is used to draw strait lines; it consists of two blades with steel

steel points fixed to a handle, the blades are so bent that the ends of the steel points meet, and yet leave a sufficient cavity for the ink; the blades may be opened more or less by a screw, and being properly set, will draw a line of any assigned thickness. One of the blades is framed with a joint, that the points may be separated, and thus cleaned more conveniently; a small quantity only of ink should be put at one time into the drawing pen, and this should be placed in the cavity between the blades by a common pen, or the feeder; the drawing pen acts better, if the feeder or pen, by which the ink is inserted, be made to pass through the blades. To use the drawing pen, first feed it with ink, then regulate it to the thickness of the required line by the screw. In drawing lines, incline the pen a small degree, taking care, however, that the edges of both the blades touch the paper, keeping the pen close to the rule, and in the same direction during the whole operation; the blades should always be wiped very clean before the pen is put away.

These directions are equally applicable to the ink point of the compasses, only observing, that when an arch or circle is to be described of more than an inch radius, the point should be so bent, that the blades of the pen may be nearly perpendicular to the paper, and both of them touch it at the same time.

The protracting pen is only a short piece of steel wire with a very fine point, fixed at one end of the upper part of the handle of the drawing pen. It is used to mark the intersection of lines, or to set off divisions from the plotting scale and protractor.

The feeder, fig. O, plate 1, is a thin flat piece of metal;

metal ; it sometimes forms one end of a cap to fit on a pencil, or it is framed at the top of the tracing point, as in the figure. It serves to place the ink between the blades of the drawing pens, or to pass between them when the ink does not flow freely. *The tracing point, or pointrel*, is a pointed piece of steel fitted to a brass handle ; it is used to draw obscure lines, or to trace the out-lines of a drawing or print when an exact copy is required, an article that will be fully explained in the course of this work ; it forms the bottom part of the tracer O.

Of triangular compasses. A pair of these are represented at *fig. N, plate 1.* They consist of a pair of compasses, to whose head a joint and socket is fitted for the reception of a third leg, which may be moved in almost every direction.

These compasses, though exceedingly useful, are but little known ; they are very serviceable in copying all kinds of drawings, as from two fixed points they will always ascertain the exact position of a third point.

Fig. 12, plate 3, represents another kind, which has some advantages over the preceding. 1. That there are many situations so oblique, that the third point cannot be ascertained by the former, though it may by these. 2. It extends much further than the other in proportion to it's size. 3. The points are in all positions perpendicular to the paper.

Of wholes and halves, fig. R, plate 1. A name given to these compasses, because that when the longer legs are opened to any given line, the shorter ones will be opened to the half of that line.

Proportional compasses. These compasses are of two kinds, one plain, represented *fig. A, plate 1 ;* the other with an adjusting screw, of which there are

are two views, one edgeways, *fig. 8, plate 3*, the other in the front, *fig. 7, plate 3*: the principle on which they both act is exactly the same; those with an adjusting screw are more easily set to any given division or line, and are also more firmly fixed when adjusted.

There is a groove in each shank of these compasses, and the center is moveable, being constructed to slide with regularity in these grooves, and when properly placed, is fixed by a nut and screw; on one side of these grooves are placed two scales, one for lines, the other for circles. By the scale of lines, a right line may be divided into any number of equal parts expressed on the scale. By the scale for circles a regular polygon may be inscribed in a circle, provided the sides do not exceed the numbers on the scale.

To divide a given line into a proposed number (11) of equal parts. 1. Shut the compasses. 2. Unscrew the milled nut, and move the slider until the line across it coincides with the 11th division on the scale. 3. Tighten the screw, that the slider may be immoveable. 4. Open the compasses, so that the longer points may take in exactly the given line, and the shorter will give you $\frac{1}{11}$ th of that line.

To inscribe in a circle a regular polygon of 12 sides. 1. Shut the compasses. 2. Unscrew the milled nut, and set the division on the slider to coincide with the 12th division on the scale of circles. 3. Fasten the milled nut. 4. Open the compasses, so that the longer legs may take the radius, and the distance between the shorter legs will be the side of the required polygon.

To use the proportional compasses with an adjusting screw. The application being exactly the same

same as the simple one, we have nothing more to describe than the use and advantage of the adjusting screw. 1. Shut the legs close, slacken the screws of the nuts *g* and *f*; move the nuts and slider *k* to the division wanted, as near as can be readily done by the hand, and screw fast the nut *f*: then by turning the adjuster *b*, the mark on the slider *k* may be brought exactly to the division: screw fast the nut *g*. 2. Open the compasses; gently lift the end *e*, of the screw of the nut *f*, out of the hole in the bottom of the nut *g*; move the beam round it's pillar *a*, and slip the point *e*, into the hole in the pin *n*, which is fixed to the under leg; slacken the screw of the nut *f*; take the given line between the longer points of the compasses, and screw fast the nut *f*: then may the shorter points of the compasses be used without any danger of the legs changing their position; this being one of the inconveniencies that attended the proportional compasses before this ingenious contrivance.

Fig. 10, plate 3, represents a pair of *beam compasses*; they are used for taking off and transferring divisions from a diagonal or nonius scale, describing large arches, and bisecting lines or arches. It is the instrument upon which Mr. Bird principally depended in dividing those instruments, whose accuracy has so much contributed to the progress of astronomy. These compasses consist of a long beam, made of brass or wood, furnished with two brass boxes, the one fixed at the end, the other sliding along the beam, to any part of which it may be firmly fixed by the screw *p*. An adjusting screw and micrometer are adapted to the box *A* at the end of the beam; by these the point connected therewith may be moved with extreme regularity

regularity and exactness, even less than the thousandth part of an inch.

Fig. 13, plate 3, is a small pair of beam compasses, with a micrometer and adjusting screw, for accurately ascertaining and laying down small distances.

Fig. 11, plate 3, represents a scale of equal parts, constructed by Mr. Sisson; that figured here contains two scales, one of three chains, the other of four chains in an inch, being those most frequently used; each of these is divided into 10 links, which are again subdivided by a nonius into single links; the index carries the protracting pin for setting off the lengths of the several station lines on the plan. By means of an instrument of this kind, the length of a station line may be laid down on paper with as much exactness, as they can be measured on land.

OF PARALLEL RULES.

Parallel lines occur so continually in every species of mathematical drawing, that it is no wonder so many instruments have been contrived to delineate them, with more expedition than could be effected by the general geometrical methods: of the various contrivances for this purpose, the following are those most approved.

1. *The common parallel rule, fig. A, plate 2.* This consists of two strait rules which are connected together, and always maintained in a parallel position by the two equal and parallel bars, which move very freely on the center or rivets by which they are fastened to the strait rules.

2. *The double parallel rule, fig. B, plate 2.* This instrument is constructed exactly upon the same principles

principles as the foregoing, but with this advantage, that in using of it the moveable rule may always be so placed, that it's ends may be exactly over or even with the ends of the fixed rule, whereas in the former kind they are always shifting away from the ends of the fixed rule.

This instrument consists of two equal flat rules, and a middle piece; they are connected together by four brass bars, the ends of the two bars are rivetted on the middle line of one of the strait rules, the ends of the other two bars are rivetted on the middle line of the other strait rule; the other ends of the brass bars are taken two and two, and rivetted on the middle piece, as is evident from the figure; it would be needless to observe, that the brass bars move freely on their rivets, as so many centers.

3. *Of the improved double parallel rule, fig. C, plate 2.* The motions of this rule are more regular than those of the preceding one; it's construction is evident from the figure; it was contrived by the ingenious Mr. Haywood, a man well known to most mechanics of any name or abilities.

4. *The cross barred parallel rule, fig. D, plate 2.* In this two strait rules are joined by two bars which cross each other, and turn on their intersection as on a center; one end of each bar moves on a center, the other slides in a groove, as the rules recede from each other.

As the four parallel rules above described, are all used in the same way, one problem will serve for them all; ex. gr. a right line being given to draw a line parallel thereto by either of the foregoing instruments:

Set the edge of the uppermost rule to the given line; press the edge of the lower rule tight to the
paper

paper with one hand, and with the other move the upper rule, till it's edge coincides with the given point; and a line drawn along the edge through the point, is the line required.

5. *Of the rolling parallel rule.* This instrument was contrived by Mr. Eckhardt, and the simplicity of the construction does credit to the inventor; it must however be owned, that it requires some practice and attention to use it with success.

Fig. E, plate 2, represents this rule; it is a rectangular parallelogram of black ebony, with slips of ivory laid on the edges of the rule, and divided into inches and tenths. The rule is supported by two small wheels, which are connected together by a long axis, the wheels being exactly of the same size, and their rolling surfaces being parallel to the axis; when they are rolled backwards or forwards, the axis and rule will move in a direction parallel to themselves. The wheels are somewhat indented, to prevent their sliding on the paper; small ivory cylinders are sometimes affixed to the rollers, as in this figure; they are called rolling scales. The circumference of these is so adjusted, that they indicate with exactness, the parts of an inch moved through by the rule.

In rolling these rules, one hand only must be used, and the fingers should be placed nearly in the middle of the rule, that one end may not have a tendency to move faster than the other. The wheels only should touch the paper when the rule is moving, and the surface of the paper should be smooth and flat.

In using the rule with the rolling scales, to draw a line parallel to a given line at any *determined* distance, adjust the edge of the rule to the given line, and pressing the edge down, raise the wheels

a little from the paper, and you may turn the cylinders round, to bring the first division to the index; then if you move the rule towards you, look at the ivory cylinder on the left hand, and the numbers will shew in tenths of an inch, how much the rule moves. If you move the rule from you, then it will be shewn by the numbers on the right hand cylinder.

To raise a perpendicular from a given point on a given line. Adjust the edge of the rule to the line, placing any one of the divisions on the edge of the rule to the given point; then roll the rule to any distance, and make a dot or point on the paper at the same division on the edge of the rule; through this point draw the perpendicular.

To let fall a perpendicular from any given point to a given line. Adjust the rule to the given line, and roll it to the given point; then observing what division or point on the edge of the rule the given point comes to, roll the rule back again to the given line, and the division or point on the edge of the rule will shew the point on the given line, to which the perpendicular is to be drawn.

By this method of drawing perpendiculars, squares and parallelograms may be easily drawn of any dimensions.

To divide any given line into any number of equal parts. Draw a right line from either of the extreme points of the given line, making any angle with it. By means of the rolling scales, divide that line into as many inches, or parts of an inch, as will equal the number of parts into which the given line is to be divided. Join the last point of division, and the extreme point of the given line: to that line draw parallel lines through the other points

points of division, and they will divide the given line into equal parts.

6. *Of the square parallel rule.* The evident advantages of the T square and drawing board, over every other kind of parallel rule, gave rise to a variety of contrivances to be used, when a drawing board was not at hand, or could not, on account of the size of the paper, be conveniently used; among these are, 1. The square parallel rule. 2. The parallel rule and protractor, both of which I contrived some years since, as substitutes to the T square. The square parallel rule, besides its use as a parallel rule, is peculiarly applicable to the mode of plotting recommended by *Mr. Gale*. Its use as a rule for drawing parallel lines, at given distances from each other, for raising perpendiculars, forming squares, rectangles, &c. are evident from a view of the figure alone; so that what has been already said of other rules, will be sufficient to explain how this may be used. It is also evident, that it will plot with as much accuracy as the beam, *fig. 11, plate 3.*

Fig. FGH, plate 2, represents this instrument; the two ends FG are lower than the rest of the rule, that weights may be laid on them to steady the rule, when both hands are wanted. The two rules are fastened together by the brass ends, the frame is made to slide regularly between the two rules, carrying at the same time, the rule H in a position at right angles to the edges of the rule EG. There are slits to the frame a, b, with marks to coincide with the respective scales on the rules while the frame is moving up or down: c is a point for pricking off with certainty divisions from the scales. The limb or rule H is made to take off, that other rules with different scales of equal parts may be applied;

when taken off, this instrument has this further advantage, that if the distances to which the parallels are to be drawn, exceed the limits of the rules, the square part, when taken off, may be used with any strait rule, by applying the perpendicular part against it.

7. *Fig. I K L, plate 2, represents the protracting parallel rule*; the uses of this in drawing parallel lines in different directions, are so evident from an inspection of the figure, as to render a particular description unnecessary. It answers all the purposes of the T square and bevil, and is peculiarly useful to surveyors for plotting and protracting, which will be seen when we come to treat of those branches

MNO is a parallel rule upon the same principle as the former: it was contrived by Mr. Haywood, to whom I can pay no greater compliment than by saying he is the Hook of this age; equal to the performance of every mechanical engine, within the reach of human abilities. Each of the rules MN turns upon a center; it's use as a parallel rule is evident from the figure, but it would require more pages than can be spared to describe all the uses of which it is capable; it forms the best kind of calipers, or guage; serves for laying down divisions and angles with peculiar accuracy; answers as a square or bevil; indeed scarce any artist can use it without reaping considerable advantage from it, and finding uses peculiar to his own line of business.

8. *Of the German parallel rule, fig. 1, plate 3.* This was probably one of the first instruments invented to facilitate the drawing of parallel lines. It has, however, only been introduced within these few years among the English artists; and as this introduction

introduction probably came from some German work, it has thence acquired it's name. It consists of a square and a strait rule, the edge of the square is moved in use by one hand against the rule, which is kept steady by the other, the edge having been previously set to the given line; it's use and construction is obvious from *fig. 1, plate 3*. Simple as it is in it's principle, it has undergone some variations, two of which I shall mention, the one by Mr. Jackson, of Tottenham; the other by Mr. Marquois, of London.

Mr. Jackson's, fig. 6, plate 3, consists of two equal triangular pieces of brass ivory, or wood, ABC, DEF, right angled at A and E; the edges AB and AC are divided into any convenient number of equal parts, the divisions in each equal; BC and DE are divided into the same number of equal parts as AC, one side of DEF may be divided as a protractor.

To draw a line GH, fig. V. plate 2, parallel to a given line, through a point P, or at a given distance.

Place the edge DE upon the given line IK and let the instrument form a rectangle, then slide the upper piece till it comes to the given point or distance, keeping the other steady with the left hand, and draw the line GH. By moving the piece equal distances by the scale on BC, any number of equidistant parallels may be obtained.

If the distance, *fig. X, plate 2*, between the given and required lines be considerable, place AB upon IK, and EF against AC, then slide the pieces alternately till DE comes to the required point; in this manner it is easy also to construct any square or rectangle, &c.

From any given point or angle P, *fig. S, plate 2*, to let fall or raise a perpendicular on a given line.

Place

Place either of the edges AC, upon GP, and slide AB upon it, till it comes to the point P, and draw PH.

To divide a line into any proposed number of equal parts, *fig. T, plate 2.* Find the proposed number in the scale BC, and let it terminate at G, then place the rules in a rectangular form, and move the whole about the point G, till the side DE touches H; now move D one, two, or three divisions, according to the number and size of the required divisions, down BC, and make a dot at I, where DE cuts the line for the first part; then move one or more divisions as before, make a second dot, and thus proceed till the whole be completed.

Of Marquois's parallel scales. These consist of a right angled triangle, whose hypotenuse or longest side is three times the length of the shortest, and two rectangular scales. It is from this relative length of the hypotenuse that these scales derive their peculiar advantages, and it is this alone that renders them different from the common German parallel rule: for this we are much indebted to Mr. Marquois.

What has been already said of the German rule, applying equally to those of Mr. Marquois's, I shall proceed to explain their chief and peculiar excellence. On each edge of the rectangular rule are placed two scales, one close to the edge, the other within this. The outer scale, Mr. Marquois terms the artificial scale, the inner one the natural scale: the divisions on the outer are always three times longer than those on the inner scales, as to derive any advantage from this invention, they must always bear the same proportion to each other, that the shortest side of the right angled triangle does to the longest. The

triangle has a line near the middle of it, to serve as an index or pointer; when in use, this point should be placed so as to coincide with the O division of the scales; the numbers on the scales are reckoned both ways from this division, consequently, by confining the rule and sliding the triangle either way, parallel lines may be drawn on either side of a given line, at any distance pointed out by the index on the triangle. The advantages of this contrivance are, 1. That the sight is greatly assisted as the divisions on the outer scale are so much larger than those of the inner ones, and yet answer the same purpose, for the edge of the right angled triangle only moves through one third of the space passed over by the index. 2. That it promotes accuracy, for all error in the setting of the index or triangle is diminished one third.

Mr. Marquois recommends the young student to procure two rules of about two feet long, having one of the edges divided into inches and tenths, and several triangles with their hypotenuse in different proportions to their respective perpendiculars. Thus, if you would make it answer for a scale of twenty to an inch, the hypotenuse must be twice the length of the perpendicular; if a scale of 30 be required three times, of 40 four times, of 50 five times, and so on. Thus also for intermediate proportions, if a scale of 25 is wanted, the hypotenuse must be in the proportion of 25 to 2; if 35, of 7 to 42, &c. Or a triangle may be formed in which the hypotenuse may be so set as to bear any required proportion with the perpendicular.

Of the protractor. This is an instrument used to protract or lay down an angle containing any number of degrees, or to find how many degrees are contained

contained in any given angle. There are two kinds put into cases of mathematical drawing instruments, one in the form of a semicircle, the other in the form of a parallelogram. The circle is undoubtedly the only natural measure of angles; when a strait line is therefore used, the divisions thereon are derived from a circle or it's properties, and the strait line is made use of for some relative convenience: it is thus the parallelogram is often used as a protractor instead of the semicircle, because it is in some cases more convenient, and that other scales, &c. may be placed upon it.

The semicircular protractor, fig. 2, plate 3, is divided into 180 equal parts or degrees, which are numbered at every tenth degree each way, for the conveniency of reckoning either from the right towards the left, or from the left towards the right, or the more easily to lay down an angle from either end of the line, beginning at each end with 10, 20, &c. and proceeding to 180 degrees. The edge is the diameter of the semicircle, and the mark in the middle points out the center. Fig. 3, plate 3, is a protractor in the form of a parallelogram: the divisions are here as in the semicircular one, numbered both ways; the blank side represents the diameter of the circle. The side of the protractor to be applied to the paper is made flat, and that whereon the degrees are marked is chamfered or sloped away to the edge, that an angle may be more easily measured, and the divisions set off with greater exactness.

Protractors of horn, are from their transparency very convenient in measuring angles and raising perpendiculars. When they are out of use, they should be kept in a book to prevent their warping.

Upon some protractors the numbers denoting the angle for regular polygons are laid down, to avoid

avoid the trouble of a reference to a table, or the operation of dividing; thus, the number 5 for a pentagon is set against 72° ; the number 6 for a hexagon against 60; the number 7 for a heptagon against $51\frac{1}{2}^{\circ}$, and so on.

Protractors for the purposes of surveying will be described in their proper place.

Application of the protractor to use. 1. A number of degrees being given, to protract or lay down an angle, whose measure shall be equal thereto.

Thus, to lay down an angle of 60 degrees from the point A of the line AB, *fig. 14, plate 3*, apply the diameter of the protractor to the line AB, so that the center thereof may coincide exactly with the point A; apply the diameter of the protractor to the line AB then with a protracting pin make a fine dot at C against 60 upon the limb of the protractor; now remove the protractor, and draw a line from A through the point C, and the angle CAB contains the given number of degrees.

2. To find the number of degrees contained in a given angle BAC.

Place the center of the protractor upon the angular point A, and the fiducial edge or diameter exactly upon the line AB; then the degree upon the limb that is cut by the line CA, will be the measure of the given angle, which in the present instance is found to be 60 degrees.

3. From a given point A, in the line AB, to erect a perpendicular to that line.

Apply the protractor to the line AB, so that the center may coincide with the point A, and the division marked 90 may be cut by the line AB, then a line DA drawn against the diameter of the protractor will be perpendicular to AB.

Further uses of this instrument will occur in most parts of this work, particularly it's use in constructing

structing of polygons, which will be found under their respective heads. Indeed the general use being explained, the particular application must be left to the practitioner, or this work would be unnecessarily swelled by a tedious detail and continual repetitions.

OF THE PLANE SCALE.

The divisions laid down on the plane scale are of two kinds, the one having more immediate relation to the circle and it's properties, the other being merely concerned with dividing of strait lines.

It has been already observed, that though arches of a circle are the most natural measures of an angle, yet in many cases right lines were substituted, as being more convenient; for the comparison of one right line with another, is more natural and easy, than the comparison of a right line with a curve; hence it is usual to measure the quantities of angles not by the arch itself, which is described on the angular point, but by certain lines described about that arch. See definitions.

The lines laid down on the plane scales for the measuring of angles, or the protracting scales, are, 1. A line of *chords* marked CHO. 2. A line of *sines* marked SIN. of *tangents* marked TAN. of *semitangents* marked ST. and of *secants* marked SEC. this last is often upon the same line as the *sines*, because it's gradations do not begin till the *sines* end.

There are two other scales, namely, the *rhumbs*, marked RU. and *long*, marked LON. Scales of latitude and hours were formerly put upon the plane scale; but as dialling is now but very little studied, they are generally left out.

The divisions used for measuring strait lines are called *scales of equal parts*, and are of various lengths
for

for the convenience of delineating any figure of a large or smaller size, according to the fancy or purposes of the draughts-man. They are indeed nothing more than a measure in miniature for laying down upon paper, &c. any known measure, as chains, yards, feet, &c. each part on the scale answering to one foot, one yard, &c. and the plan will be larger or smaller as the scale contains a smaller or a greater number of parts in an inch. Hence a variety of scales is useful to lay down lines of any required length, and of a convenient proportion with respect to the size of the drawing. If none of the scales happen to suit the purpose, recourse should be had to the *line of lines* on the sector; for by the different openings of that instrument, a line of any length may be divided into as many equal parts as any person chuses.

Scales of equal parts are divided into two kinds, the one simple, the other diagonally divided.

Six of the simply divided scales are generally placed one above another upon the same rule; they are divided into as many equal parts as the length of the rule will admit of; the numbers which are placed on the right hand shew how many parts in an inch each scale is divided into. The upper scale is sometimes shortened for the sake of introducing another line of chords.

The first of the larger or primary divisions on every scale is subdivided into 10 equal parts, which small parts are those which give a name to the scale; thus it is called a scale of 20, when 20 of these divisions are equal to one inch. If, therefore, these lesser divisions be taken as units, and each represent one league, one mile, one chain, or one yard, &c. then will the larger divisions be so many tens;

tens; but if the subdivisions are supposed to be tens, the larger divisions will be hundreds.

To illustrate this, suppose it were required to set off from either of the scales of equal parts, $\frac{1}{10}$, 36, or 360 parts either miles or leagues. Set one foot of your compasses on 3 among the larger or primary divisions, and open the other point till it falls on the 6th subdivision, reckoning backwards or towards the left hand. Then will this extent represent $\frac{1}{10}$, 36, or 360 miles or leagues, &c. and bear the same proportion in the plan as the line measured does to the thing represented.

To adapt these scales to feet and inches, the first primary division is often duodecimally divided by an upper line; therefore to lay down any number of feet and inches, as for instance, 8 feet 8 inches, extend the compasses from 8 of the larger to eight of the upper small ones, and that distance laid down on the plan will represent 8 feet 8 inches.

Of the scale of equal parts diagonally divided. The use of this scale is the same as those already described. But by it a plane may be more accurately divided than by the former; for any one of the larger divisions may by this be subdivided into 100 equal parts, and therefore if the scale contains 10 of the larger divisions, any number under 1000 may be laid down with accuracy.

The diagonal scale is seldom placed on the same side of the rule with the other plotting scales. The first division of the diagonal scale, if it be a foot long, is generally an inch divided into 100 equal parts, and at the opposite end there is usually half an inch divided into 100 equal parts. If the scale be six inches long, one end has commonly half an inch, the other a quarter of an inch subdivided into 100 equal parts.

The

The nature of this scale will be better understood by considering its construction. For this purpose.

First. Draw eleven parallel lines at equal distances, divide the upper of these lines into such a number of equal parts, as the scale to be expressed is intended to contain ; from each of these divisions draw perpendicular lines through the eleven parallels.

Second. Subdivide the first of these divisions into ten equal parts, both in the upper and lower lines.

Thirdly. Subdivide again each of these subdivisions, by drawing diagonal lines from the 10th below to the 9th above, from the 8th below to the 7th above, and so on, till from the 1st below to the 0 above: by these lines each of the small divisions is divided into ten parts, and consequently the whole first space into 100 equal parts; for as each of the subdivisions is one tenth part of the whole first space or division, so each parallel above it is one tenth of such subdivision, and consequently one hundredth part of the whole first space; and if there be ten of the larger divisions, one thousandth part of the whole space.

If therefore the larger divisions be accounted as units, the first subdivisions will be tenth parts of an unit, and the second, marked by the diagonal upon the parallels, hundredth parts of the unit. But if we suppose the larger divisions to be tens, the first subdivisions will be units, and the second tenths. If the larger are hundreds, then will the first be tens, and the second units.

The numbers, therefore, 576. 57.6. 5.76. are all

all expreffible by the fame extent of the compaffes : thus fetting one foot in the number 5 of the larger divifions, extend the other along the 6th parallel, to the feventh diagonal. For if the 5 larger divifions be taken for 5 hundred, 7 of the firft fubdivifions will be 70, which upon the fixth parallel, taking in fix of the fecond fubdivifions for units, makes the whole number 576. Or, if the five larger divifions be taken for 5 tens or 50, feven of the firft fubdivifions will be 7 units, and the 6 fecond fubdivifions upon the 6th parallel, will be fix tenths of an unit. Laftly, if the five larger divifions be only efteemed as five units, then will the feven firft fubdivifions be 7 tenths, and the fix fecond fubdivifions be the fix hundredths parts of an unit.

Of the ufe of the fcales of equal parts. Though what I have already faid on this head may be deemed fufficient, I fhall not fcruple to introduce a few more examples, in order to render the young practitioner more perfect in the management of an instrument, that will be continually occurring to him in practical geometry. He will have already obferved, that by fcales of equal parts lines may be laid down, or geometrical figures conftituted, whofe right-lined fides fhall be in the fame proportion as any given numbers.

Example 1. To take off the number 4.79 from a diagonal fcale. Set one foot of the compaffes on the point where the fourth vertical line cuts the feventh horizontal line, and extend the other foot to the point where the ninth diagonal cuts the feventh horizontal line.

Example 2. To take off the number 76.4. Obferve the points where the fixth horizontal cuts

cuts the seventh vertical, and fourth diagonal line, the extent between these points will represent the number 764.

In the first example each primary division is taken for 1, in the second it is taken for 10.

Example 3. To lay down a line of 7.85 chains by the diagonal scale. Set one point of your compasses where the eighth parallel, counting upwards, cuts the seventh vertical line; and extend the other point to the intersection of the same eighth parallel with the fifth diagonal. Set off the extent 7.85 thus found on the line.

Example 4. To measure by the diagonal scale a line that is already laid down. Take the extent of the line in your compasses, place one foot on the first vertical line that will bring the other foot among the diagonals; move both feet upwards till one of them fall into the point, where the diagonal from the nearest tenth cuts the same parallel as is cut by the other on the vertical line. Then one foot shews the chain, and the other the hundredth parts or odd links. Thus, if one foot is on the 8th diagonal, of the 4th parallel, while the other is on the same parallel, but coincides with the 12th vertical, we have 12 chains 48 links, or 12.48 chains.

Example 5. Three adjacent parts of any right-lined triangle being given, to form the plan thereof. Thus suppose the base of a triangle 40 chains, the angle A B C equal 36 deg. and angle B A C equal 41 deg.

Draw the line A B, *fig. 15, plate 3*, and from any of the scales of equal parts take off 40, and set it on the same line from A to B for the base of the triangle; at the points A, B, make the angle A B C equal to 36 degrees, and B C A to 41, and the

D 2

triangle

triangle will be formed ; then take in your compasses the length of the side A C, and apply it to the same scale, and you will find it's length to be 24 chains ; do the same by the side B C, and you will find it measure 27 chains, and the protractor will shew that the angle A C B contains 103 degrees.

Example 6. Given the base A B, *fig. 16, plate 3,* of a triangle 327 yards, the angle C A B 44,30, and the side A C 208 yards, to constitute the said triangle, and find the length of the other sides.

Draw the line A B at pleasure, then take 327 parts from the scale, and lay it from A to B ; set the center of the protractor to the point A, lay off $44\frac{1}{2}$ degrees, and by that mark draw A C ; then take with the compasses from the same scale 208, and lay it from A to C, and join C B, and the parts of the triangle in the plan will bear the same proportion to each other as the real parts in the field do.

OF THE REMAINING LINES ON THE PLAIN SCALE.

OF THE PROTRACTING SCALES.

I. *Of the line of chords.* This line is used to set off an angle from a given point in any right line, or to measure the quantity of an angle already laid down.

Thus to draw a line A C, *fig. 14, plate 3,* that shall make with the line A B an angle, containing a given number of degrees, suppose 40 degrees.

Open your compasses to the extent of 60 degrees upon the line of chords, (which is always equal to the radius of the circle of projection,) and

and setting one foot in the angular point, with that extent describe an arch; then taking the extent of 40 degrees from the said chord line, set it off from the given line on the arch described; a right line drawn from the given point, through the point marked upon the arch, will form the required angle.

The degrees contained in an angle already laid down, are found nearly in the same manner; for instance, to measure the angle C A B. From the center describe an arch with the chord of 60 degrees, and the distance C B, measured on the chords, will give the number of degrees contained in the angle.

If the number of degrees are more than 90 degrees, they must be measured upon the chords at twice: thus, if 120 degrees were to be practised, 60 may be taken from the chords, and those degrees be laid off twice upon the arch. Degrees taken from the chords are always to be counted from the beginning of the scale.

Of the rhumb line. This is, in fact, a line of chords constructed to a quadrant divided into eight parts or points of the compass, in order to facilitate the work of the navigator in laying down a ship's course.

Thus, supposing the ship's course to be NNE, and it be required to lay down that angle. Draw the line A B, *fig. 14, plate 3*, to represent the meridian, and about the point A sweep an arch with the chord of 60 degrees; then take the extent to the third rhumb, from the line of rhumbs, and set it off on the arch from B to e, and draw the line A e, and the angle B A e will represent the ship's course. The third rhumb was taken, because NNE is the third point from the north.

Of the line of longitudes. The line of longitudes is a line divided into sixty unequal parts, and so applied to the line of chords, as to shew by inspection, the number of equatorial miles contained in a degree on any parallel of latitude. The graduated line of chords is necessary, in order to shew the latitudes; the line of longitude shews the quantity of a degree on each parallel in 60th parts of an equatorial degree, that is, miles. The use of this line will be evident from the following example. A ship in latitude 44. 12. N. sails E. 79 miles, required her difference of longitude: opposite to $44\frac{1}{2}$, equal to the latitude on the line of chords, stands 43 on the line of longitude, which is therefore the number of miles in a degree of longitude in that latitude. Whence $43 : 60 :: 79 : 110$ miles the difference of longitude.

The *lines of tangents, semitangents, and secants* serve to find the centers and poles of projected circles in the stereographical projection of the sphere. The *line of sines* is principally used for the orthographic projection of the sphere; but as the application of these lines is the same as that of similar lines on the sector, we shall refer the reader to the explanation of those lines in our description of that instrument.

The lines of latitudes and hours are used conjointly, and serve very readily to mark the hour lines in the construction of dials; they are now but seldom used, and therefore very often not put on the scales, &c.

NAMES OF THE INSTRUMENTS REPRESENTED IN PLATES 1, 2, 3.

Plate 1, fig. A, is a pair of proportionable compasses without an adjusting screw.

B, a pair of the best drawing compasses; *b*, the plain point with a joint; *c*, the ink point; *d*, the dotting point; *e*, the pencil or crayon point; P Q, additional pieces fitting into the place of the moveable point *b*, and to which the other parts are fitted.

F, a pair of bow compasses for ink; G, a ditto for a pencil; H, a pair of ditto with a plain point for stepping minute divisions; *b*, a screw to one of the legs thereof, which acts like the spring leg of the hair compasses.

L, the hair compasses; *n*, the screw that acts upon the spring leg.

I K, the drawing pen; I, the upper part; *k*, the protracting pin thereof; K, the lower or pen part.

N, a pair of triangular compasses.

O, the feeder and tracing point.

R, a pair of wholes and halves.

S, a small protracting pin.

T, a knife, screw driver, and key in one piece.

Plate 2, *fig. A*, the common parallel rule.

B, the double barred ditto.

C, the improved double barred parallel rule.

D, the cross barred parallel rule. Of these rules, that figured at *c* is the most perfect.

E, Eckhardts, or the rolling parallel rule.

F G H, the rectangular parallel rule.

I K L, the protracting parallel rule.

M N O, Haywood's parallel rule.

Plate 3, *fig. 1*, the German parallel rule.

Fig. 2, a semicircular protractor; *fig. 3*, a rectangular ditto.

Fig. 4 and 5, the two faces of a sector.

Fig. 6, Mr. Jackson's parallel rule.

Fig. 7 and 8, two views of a pair of proportionable compasses with an adjusting screw.

Fig. 9, a pair of sectoral compasses. In this instrument are combined the sector, beam elliptical, and calliper compasses; *fig. 9, a*, the square for ellipses; *b c*, the points to work therein; *d e* the calliper points.

Fig. 10, a pair of beam compasses.

Fig. 11, Sisson's protracting scale.

Fig. 12, improved triangular compasses.

Fig. 13, a pair of small compasses with a beam and micrometer.

OF THE SECTOR.

Amidst the variety of mathematical instruments that have been contrived to facilitate the art of drawing, there is none so extensive in it's use, or of such general application as the *sector*. It is an universal scale, uniting as it were angles and parallel lines, the rule and the compass, which are the only means that geometry makes use of for measuring, whether in speculation or practice. The real inventor of this valuable instrument is unknown; yet of so much merit has the invention appeared, that it was claimed by Galileo, and disputed by nations.

This instrument derives it's name from the tenth *definition* of the third book of Euclid, where he defines the sector of a circle. It is formed of two equal rules, (*fig. 4 and 5, plate 3,*) *AB, DB*, called *legs*; these legs are moveable about the center *C* of a joint *d e f*, and will, consequently, by their different opening, represent every possible variety of plane angles. The distance of the extremities of these rules are the subtenses or chords, or the arches they describe.

Sectors

Sectors are made of different sizes, but their length is usually denominated from the length of the legs when the sector is shut. Thus a sector of six inches, when the legs are close together, forms a rule of 12 inches when opened; and a foot sector is two feet long when opened to it's greatest extent. In describing the lines usually placed on this instrument, I refer to those usually laid down on the best 6 inch brass sectors. But as the principles are the same in all, and the differences little more than in the number of subdivisions, it is to be presumed that no difficulty will occur in the application of what is here said to sectors of a larger radius.

Of this instrument, Dr. Priestley thus speaks in his *Treatise on Perspective*. "Besides the small sector in the common pocket cases of instruments, I would advise a person who proposes to learn to draw, to get another of one foot radius. Two sectors are in many cases exceedingly useful, if not absolutely necessary; and I would not advise a person to be sparing of expence in procuring a very good instrument, the uses of which are so various and important."

The scales, or lines graduated upon the faces of the instrument, and which are to be used as *sectoral lines*, proceed from the center; and are, 1. Two scales of equal parts, one on each leg, marked LIN. or L. each of these scales, from the great extensiveness of it's use, is called the *line of lines*. 2. Two lines of *chords*, marked CHO. or C. 3. Two lines of *secants*, marked SEC. or S. A line of *polygons*, marked POL. Upon the other face, the *sectoral lines* are, 1. Two lines of *fines*, marked *sin.* or S. 2. Two lines of *tangents*, marked *tan.* 3. Between the line of tangents and fines, there is another

another line of tangents to a lesser radius, to supply the defect of the former, and extending from 45 to 75°.

Each pair of these lines (except the line of polygons) is so adjusted as to make equal angles at the center, and consequently at whatever distance the sector be opened, the angles will be always respectively equal. That is, the distance between 10 and 10 on the line of lines, will be equal to 60 and 60 on the line of chords, 90 and 90 on the line of sines, and 45 and 45 on the line of tangents.

Besides the sectoral scales, there are others on each face, placed parallel to the outward edges, and used as those of the common plane scale.

1. There are on the face, *fig. 5*, a line of inches,
2. A line of latitudes. 3. A line of hours. 4.
A line of inclination of meridians. 5. A line of
chords. On the face, *fig. 4*, three logarithmic
scales, namely, 1 of numbers, 1 of sines, and 1
of tangents; these are used when the sector is fully
opened, the legs forming one line; their use will
be explained when we treat of trigonometry.

To read and estimate the divisions on the sectoral lines. The value of the divisions on most of the lines, are determined by the figures adjacent to them; these proceed by tens, which constitute the divisions of the first order, and are numbered accordingly; but the value of the divisions on the line of lines, that are distinguished by figures, is entirely arbitrary, and may represent any value that is given to them; hence the figures 1, 2, 3, 4, &c. may denote either 10, 20, 30, 40, or 100, 200, 300, 400, and so on.

The line of lines is divided into ten equal parts, numbered 1, 2, 3, to 10; these may be called divisions

visions

visions of the first order; each of these are again subdivided into 10 other equal parts, which may be called divisions of the second order; each of these is divided into two equal parts, forming divisions of the third order.

The divisions, on all the scales, are contained between four parallel lines; those of the first order extend to the most distant; those of the third, to the least; those of the second, to the intermediate parallel.

When the whole line of lines represents 100, the divisions of the first order, or those to which the figures are annexed, represent tens; those of the second order, units; those of the third order, the halves of these units. If the whole line represent ten, then the divisions of the first order are units; those of the second, tenths; the thirds, twentieths.

In the line of tangents, the divisions to which the numbers are affixed, are the degrees expressed by those numbers. Every fifth degree is denoted by a line somewhat longer than the rest; between every number and each fifth degree, there are four divisions, longer than the intermediate adjacent ones, these are whole degrees; the shorter ones, or those of the third order, are 30 minutes.

From the center, to 60 degrees, *the line of sines* is divided like the line of tangents, from 60 to 70; it is divided only to every degree, from 70 to 80, to every two degrees, from 80 to 90; the division must be estimated by the eye.

The divisions on *the line of chords* are to be estimated in the same manner as the tangents.

The lesser line of tangents is graduated every two degrees, from 45 to 50; but from 50 to 60, to every degree; from 60 to the end, to half degrees.

The

The *line of secants* from 0 to 10, are to be estimated by the eye, from 20 to 50; it is divided to every two degrees; from 50 to 60, to every degree; from 60 to the end, to every half degree.

OF THE GENERAL LAW, OR FOUNDATION, OF
SECTORAL LINES.

Let CA, CB, *fig. 17, plate 3*, represent a pair of sectoral lines, (ex. gr. those of the line of lines,) forming the angle ACB; divide each of these lines into four equal parts; in the points H, D, F, I, F, G; draw the lines HI, DE, FG, AB. Then because CA, CB, are equal, their sections are also equal, the triangles are equiangular, having a common angle at C, and equal angles at the base; and therefore, the sides about the equal angles will be proportional; for as CH to CA, so is HI to AB, and therefore, CA to CH, so is AB to HI, and consequently, CH to HI, so is CA to AB; and thence if CH be one fourth of CA, HI will be one fourth of AB, and so of all other sections.

Hence, as in all operations on the sectoral lines, there are two triangles, both isosceles and equi-angled; isosceles, because the pairs of sectoral lines are equal by construction; equi-angled, because of the common angle at the center; the sides encompassing the equal angles, are therefore proportional.

Hence also, if the lines CA, CB, represent the line of chords, sines, or tangents, that is, if CA, AB be the radius, and the line CF the chord, sine, or tangent, of any proposed number of degrees, then the line FG will be the chord, sine, or tangent, of the same number of degrees, to the radius AB.

OF THE GENERAL MODE OF USING SECTORAL LINES.

It is necessary to explain, in this place, a few terms, either used by other writers in their description of the sector, or such as we may occasionally use ourselves.

The solution of questions on the sector is said to be *simple*, when the work is begun and ended on the same line; *compound*, when the operation begins on one line, and is finished on the other.

The operation varies also by the manner in which the compasses are applied to the sector. If a measure be taken on any of the sectoral lines, beginning at the center, it is called *a lateral distance*. But if the measure be taken from any point in one line, to it's corresponding point on the line of the same denomination on the other leg, it is called *a transverse or parallel distance*.

The divisions of each sectoral line are bounded by three parallel lines; the innermost of these is that on which the points of the compasses are to be placed, because this alone is the line which goes to the center, and is alone therefore the sectoral line.

We shall now proceed to give a few general instances of the manner of operating with the sector, and then proceed to practical geometry, exemplifying it's use further in the progress of the work, as occasion offers.

Multiplication by the line of lines. Make the lateral distance of one of the factors the parallel distance of 10; then the parallel distance of the other factor is the product.

Example. Multiply 5 by 6, extend the compasses from the center of the sector to 5 on the primary divisions, and open the sector till this distance

distance become the parallel distance from 10 to 10 on the same divisions; then the parallel distance from 6 to 6, extended from the center of the sector, shall reach to 3, which is now to be reckoned 30. At the same opening of the sector, the parallel distance of 7 shall reach from the center to 35, that of 8 shall reach from the center to 40, &c.

Division by the line of lines. Make the lateral distance of the dividend the parallel distance of the divisor, the parallel distance of 10 is the quotient. Thus, to divide 30 by 5, make the lateral distance of 30, viz. 3 on the primary divisions, the parallel distance of 5 of the same divisions; then the parallel distance of 10, extended from the center, shall reach to 6.

Proportion by the line of lines. Make the lateral distance of the second term the parallel distance of the first term, the parallel distance of the third term is the fourth proportional. *Example.* To find a fourth proportional to 8, 4, and 6, take the lateral distance of 4, and make it the parallel distance of 8; then the parallel distance of 6, extended from the center, shall reach to the fourth proportional 3.

In the same manner, a third proportional is found to three numbers. Thus, to find a third proportional to 8 and 4, the sector remaining as in the former example, the parallel distance of 4, extended from the center, shall reach to the third proportional 2. In all these cases, if the number to be made a parallel distance be too great for the sector, some aliquot part of it is to be taken, and the answer is to be multiplied by the number by which the first number was divided. Thus, if it were required to find a fourth proportional to 4, 8, and

8, and 6; because the lateral distance of the second term 8 cannot be made the parallel distance of the first term 4, take the lateral distance of 4, viz: the half of 8, and make it the parallel distance of the first term 4; then the parallel distance of the third term 6 shall reach from the center to 6, viz: the half of 12. Any other aliquot part of a number may be used in the same way. In like manner, if the number proposed be too small to be made the parallel distance, it may be multiplied by some number, and the answer is to be divided by the same number.

To protract angles by the line of chords. Case 1. When the given degrees are under 60. 1. With any radius AB, fig. 14, plate 3, on A as a center, describe the arch BG. 2. Make the same radius a transverse distance between 60 and 60 on the line of chords. 3. Take out the transverse distance of the given degrees, and lay this on the arch from B towards G, which will mark out the angular distance required.

Case 2. When the given degrees are more than 60. 1. Open the sector, and describe the arch as before. 2. Take $\frac{1}{2}$ or $\frac{2}{3}$ of the given degrees, and take the transverse distance of this $\frac{1}{2}$ or $\frac{2}{3}$, and lay it off from B towards G, twice if the degrees were halved, three times if the third was used as a transverse distance.

Case 3. When the required angle is less than 6 degrees, suppose 3. 1. Open the sector to the given radius, and describe the arch as before. 2. Set off the radius from B to C. 3. Set off the chord of 57 degrees backward from G to f, which will give the arc fb of 3 degrees.

SOME USES OF THE SECTORAL SCALES OF SINES,
TANGENTS, AND SECANTS.*

Given the radius of a circle, (suppose equal to 2 inches,) required the sine, and tangent of $28^{\circ} 30'$ to that radius.

Solution. Open the sector so that the transverse distance of 90 and 90 on the sines, or of 45 and 45 on the tangents, may be equal to the given radius, viz. two inches; then will the transverse distance of $28^{\circ} 30'$, taken from the sines, be the length of that sine to the given radius; or if taken from the tangents, will be the length of that tangent to the given radius.

But if the secant of $28^{\circ} 30'$ was required?

Make the given radius two inches, a transverse distance to 0 and 0, at the beginning of the line of secants; and then take the transverse distance of the degrees wanted, viz. $28^{\circ} 30'$.

A tangent greater than 45 degrees (suppose 60 degrees) is found thus.

Make the given radius, suppose 2 inches, a transverse distance to 45 and 45 at the beginning of the scale of upper tangents; and then the required degrees $60^{\circ} 00'$ may be taken from this scale.

The scales of upper tangents and secants do not run quite to 76 degrees; and as the tangent and secant may be sometimes wanted to a greater number of degrees than can be introduced on the sector, they may be readily found by the help of the annexed table of the natural tangents and secants of the degrees above 75; the radius of the circle being unity.

Degrees

* Robertson on Mathematical Instruments,

Degrees.	Nat. Tangent.	Nat. Secant.
76	4,011	4,133
77	4,331	4,445
78	4,701	4,810
79	5,144	5,241
80	5,671	5,759
81	6,314	6,392
82	7,115	7,185
83	8,144	8,205
84	9,514	9,567
85	11,430	11,474
86	14,301	14,335
87	19,081	19,107
88	28,636	28,654
89	57,290	57,300

Measure the radius of the circle used, upon any scale of equal parts. Multiply the tabular number by the parts in the radius, and the product will give the length of the tangent or secant sought, to be taken from the same scale of equal parts.

EXAMPLE. *Required the length of the tangent and secant of 80 degrees to a circle, whose radius, measured on a scale of 25 parts to an inch, is $47\frac{1}{2}$ of those parts?*

	tangent.	secant.
Against 80 degrees stands	5,671	5,759
The radius is	47,5	47,5
	28355	28795
	39697	40313
	22684	23036
	269,3725	273,5525

So the length of the tangent on the twenty-fifth scale will be $269\frac{1}{5}$ nearly. And that of the secant about $273\frac{1}{5}$.

E

Or

Or thus. The tangent of any number of degrees may be taken from the sector at once; if the radius of the circle can be made a transverse distance to the complement of those degrees on the lower tangent.

EXAMPLE. *To find the tangent of 78 degrees to a radius of 2 inches.*

Make two inches a transverse distance to 12 degrees on the lower tangents; then the transverse distance of 45 degrees will be the tangent of 78 degrees.

In like manner the secant of any number of degrees may be taken from the fines, if the radius of the circle can be made a transverse distance to the co-fine of those degrees. Thus making two inches a transverse distance to the fine of twelve degrees; then the transverse distance of 90 and 90 will be the secant of 78 degrees.

From hence it will be easy to find the degrees answering to a given line, expressing the length of a tangent or secant, which is too long to be measured on those scales, when the sector is set to the given radius.

Thus, for a tangent, make the given line a transverse distance to 45 and 45 on the lower tangents; then take the given radius and apply it to the lower tangents; and the degrees where it becomes a transverse distance is the co-tangent of the degrees answering to the given line.

And for a secant, make the given line a transverse distance to 90 and 90 on the fines. Then the degrees answering to the given radius, applied as a transverse distance on the fines, will be the co-fine of the degrees answering to the given secant line.

Given

Given the length of the sine, tangent, or secant, of any degrees; to find the length of the radius to that sine, tangent, or secant.

Make the given length a transverse distance to it's given degrees on it's respective scale: Then,

In the sines. The transverse distance of 90 and 90 will be the radius sought.

In the lower tangents. The transverse distance of 45 and 45, near the end of the sector, will be the radius sought.

In the upper tangents. The transverse distance of 45 and 45, taken towards the center of the sector on the line of upper tangents, will be the center sought.

In the secant. The transverse distance of 0 and 0, or the beginning of the secants, near the center of the sector, will be the radius sought.

Given the radius and any line representing a sine, tangent, or secant; to find the degrees corresponding to that line.

SOLUTION. Set the sector to the given radius, according as a sine, or tangent, or secant is concerned.

Take the given line between the compasses; apply the two feet transversely to the scale concerned, and slide the feet along till they both rest on like divisions on both legs; then will those divisions shew the degrees and parts corresponding to the given line.

To find the length of a versed sine to a given number of degrees, and a given radius.

Make the transverse distance of 90 and 90 on the sines, equal to the given radius.

Take the transverse distance of the sine complement of the given degrees.

If the given degrees are less than 90, the difference between the fine complement and the radius, gives the versed fine.

If the given degrees are more than 90, the sum of the fine complement and the radius, gives the versed fine.

To open the legs of the sector, so that the corresponding double scales of lines, chords, sines, tangents, may make each a right angle.

On the lines, make the lateral distance 10, a distance between 8 on one leg, and six on the other leg.

On the sines, make the lateral distance 90, a transverse distance from 45 to 45; or from 40 to 50; or from 30 to 60; or from the sine of any degrees, to their complement.

Or on the sines, make the lateral distance of 45 a transverse distance between 30 and 30.

SELECT GEOMETRICAL PROBLEMS.

Science may suppose, and the mind conceive things as possible, and easy to be effected, in which art and practice often find insuperable difficulties. "Pure science has to do only with ideas; but in the application of it's laws to the use of life, we are constrained to submit to the imperfections of matter, and the influence of accident." Thus practical geometry shews how to perform, what theory supposes; in the theory, however, it is sufficient to have only a right conception of the objects on which the demonstrations are founded; drawing or delineations being of no further use, than to assist the imagination, and lessen the exertions of the mind. But in practical geometry, we not only consider the things as possible to be effected,

fected, but are to teach the ways, the instruments, and the operations by which they may be actually performed. It is not sufficient here to shew, that a right line may be drawn between two points, or a circle described about a center, and then demonstrate their properties; but we must actually delineate them, and exhibit the figures to the senses: and it will be found, that the drawing of a strait line, which occurs in all geometrical operations, and which in theory is conceived as easy to be effected, is in practice attended with considerable difficulties.

To draw a strait line between two points upon a plane, we lay a rule so that the strait edge thereof may just pass by the two points; then moving a fine pointed needle or drawing pen along this edge, we draw a line from one point to the other, which for common purposes is sufficiently exact; but where great accuracy is required, it will be found extremely difficult to lay the rule equally, with respect to both the points, so as not to be nearer to one point than the other. It is difficult also so to carry the needle or pen, that it shall neither incline more to one side than the other of the rule; and thirdly, it is very difficult to find a rule that shall be perfectly strait. If the two points be very far distant, it is almost impossible to draw the line with accuracy and exactness; a circular line may be described more easily, and more exactly, than a strait or any other line, though even then many difficulties occur, when the circle is required to be of a large radius.

It is from a thorough consideration of these difficulties, that geometricians will not allow those lines to be geometrical, which in their description require the sliding of a point along the edge of a

rule, as in the ellipse, and several other curve lines, whose properties have been as fully investigated, and as clearly demonstrated, as those of the circle.

From hence also we may deduce some of those maxims, which have been introduced into practice by Bird and Smeaton, which will be seen in their proper place. And let no one consider these reflections as the effect of too scrupulous exactness, or as an unnecessary aim at precision; for as the foundation of all our knowledge in geography, navigation, and astronomy, is built on observation, and all observations are made with instruments, it follows, that the truth of the observations, and the accuracy of the deductions therefrom, will principally depend on the exactness with which the instruments are made and divided; and that these sciences will advance in proportion as they are less difficult in their use, and more perfect in the performance of their respective operations.

There is scarce any thing which proves more clearly the distinction between mind and body, and the superiority of the one over the other, than a reflection on the rigid exactness of speculative geometry, and the inaccuracy of practice, that is not directed by theory on one hand, and its approximation to perfection on the other, when guided by a just theory.

In theory, most figures may be measured to an almost infinite exactness, yet nothing can be more inaccurate and gross, than the ordinary methods of mensuration; but an intelligent practice finds a medium, and corrects the imperfection of our mechanical organs, by the resources of the mind. If we were more perfect, there would be less room
for

for the exertions of our mind, and our knowledge would be less.

If it had been easy to measure all things with exactness, we should have been ignorant of many curious properties in numbers, and been deprived of the advantages we derive from logarithms, sines, tangents, &c. If practice were perfect, it is doubtful whether we should have ever been in possession of theory.

We sometimes consider with a kind of envy, the mechanical perfection and exactness, that is to be found in the works of some animals; but this perfection, which does honour to the Creator, does little to them; they are so perfect, only because they are beasts.

The imperfection of our organs is abundantly made up by the perfection of the mind, of which we are ourselves to be the artificers.

If any wish to see the difficulties of rendering practice as perfect as theory, and the wonderful resources of the mind, in order to attain this degree of perfection, let him consider the operations of *General Roy*, at Hounslow-heath; operations that cannot be too much considered, nor too much praised by every practitioner in the art of geometry.

PROBLEM I. *To erect a perpendicular at or near the end of a given right line, CD, fig. 5, plate 4.*

Method I. On *c*, with the radius *CD*, describe a faint arc *ef* on *D*; with the same radius, cross *ef* at *G*, on *G* as a center; with the same radius, describe the arc *DF*; set off the extent *DG* twice, that is, from *D* to *E*, and from *E* to *F*. Join the points *D* and *F* by a right line, and it will be the perpendicular required.

Method 2, fig. 5. On any point G, with the radius DG, describe an arch CFD; then a rule laid on C and G, will cut this arc in F, a line joining the points F and D will be the required perpendicular.

Method 3. 1. From the point c, *fig. 6, plate 4,* with any radius describe the arc *rn*, cutting the line AC in r. 2. From the point r, with the same radius, cross the arc in n, and from the point n, cross it in m. 3. From the points n and m, with the same or any other radius, describe two arcs, cutting each other in S. 4. Through the points s and C, draw the line SC, and it will be the perpendicular required.

Method 4. By the line of lines on the sector, fig. 7, plate 4. 1. Take the extent of the given line AC. 2. Open the sector, till this extent is a transverse distance between 8 and 8. 3. Take out the transverse distance between 6 and 6, and from A with that extent, sweep a faint arc at B. 4. Take out the distance between 10 and 10, and with it from c, cross the former arc at B. 5. A line drawn through A and B, will be the perpendicular required; the numbers 6, 8, 10, are used as multiples of 3, 4, 5.

By this method, a perpendicular may be easily and accurately erected on the ground.

Method 5. Let AC, *fig. 7, plate 4,* be the given line, and A the given point. 1. At any point D, with the radius DA, describe the arch EAB. 2. With a rule on E and D, cross this arch at B. 3. Through A and B draw a right line, and it will be the required perpendicular.

PROBLEM 2. To raise a perpendicular from the middle, or any other point G, of a given line AB.

1. On

1. On G, *fig. 8, plate 4*, with any convenient distance within the limits of the line, mark, or set off the points n and m. 2. From n and m, with any radius greater than GA, describe two arcs intersecting at c. 3. Join CG by a line, and it will be the perpendicular required.

PROBLEM 3. *From a given point C, fig. 8, plate 4, out of a given line AB, to let fall a perpendicular.*

When the point is nearly opposite to the middle of the line, this problem is the converse of the preceding one. Therefore, 1. From c, with any radius, describe the arcs n, m. 2. From n m, with the same, or any other radius, describe two arcs intersecting each other at S. 3. Through the points CS, draw the line CS, which will be the required line.

When the point is nearly opposite to the end of the line, it is the converse of method 5, problem 1, *fig. 7, plate 4*.

1. Draw a faint line through B, and any convenient point E, of the line AC. 2. Bisect this line at D. 3. From D with the radius DE describe an arc cutting AC at A. 4. Through A and B draw the line AB, and it will be the perpendicular required.

Another method. 1. From A, *fig. 9, plate 4*, or any other point in AB, with the radius AC, describe the arc CD. 2. From any other point n, with the radius nC, describe another arc cutting the former in D. 3. Join the point CD by a line CGD, and CG will be the perpendicular required.

PROBLEM 4. *Through a given point C, to draw a line parallel to a given strait line AB, fig. 10, plate 6.*

1. On any point D, (within the given line, or without

without it, and at a convenient distance from C,) describe an arc passing through C, and cutting the given line in A. 2. With the same radius describe another arc cutting AB at B. 3. Make BE equal to AC. 4. Draw a line CE through the point C and E, and it will be the required parallel.

This problem answers whether the required line is to be near to, or far from the given line; or whether the point D be situated on AB, or any where between it and the required line.

PROBLEM 5. *At the given point D, to make an angle equal to a given angle ABC, fig. 12, plate 4.*

1. From B, with any radius, describe the arc nm, cutting the legs Bn Bm, in the points n and m. 2. Draw the line Dr, and from the point D, with the same radius as before, describe the arc rs. 3. Take the distance mn, and apply it to the arc rs, from r to s. 4. Through the points D and s, draw the line Ds, and the angle rDs will be equal to the angle mBn, or ABC, as required.

PROBLEM 6. *To extend with accuracy a short strait line to any assignable length; or, through two given points at a small distance from each other, to draw a strait line.*

It frequently happens that a line as short as that between A and B, fig. 11, plate 4, is required to be extended to a considerable length, which is scarce attainable by the help of a rule alone; but may be performed by means of this problem, without error. Let the given line be AB, or the two points A and B; then from A as a center, describe an arch CBD; and from the point B, lay off BC equal to BD; and from C and D as centers, with any radius, describe two arcs intersecting at E, and from F two more intersecting the arch FEG at F and G, and

and from F and G, &c. then a strait line drawn through E will pass through A, and in the same manner the line may be extended to any assignable length.

OF THE DIVISION OF STRAIT LINES.

PROBLEM 7. *To bisect or divide a given strait line AB into two equal parts, fig. 13, plate 4.*

1. On A and B as centers, with any radius greater than half AB, describe arcs intersecting each other at C and D. 2. Draw the line CD, and the point F, where it cuts AB, will be the middle of the line.

If the line to be bisected be near the extreme edge of any plane, describe two pair of arcs above the given line, as at C and E; then a line C produced, will bisect AB in E.

By the line of lines on the sector. 1. Take AB in the compasses. 2. Open the sector till this extent is a transverse distance between 10 and 10. 3. The extent from 5 to 5 on the same line, set off from A or B, gives the half required: by this means a given line may be readily divided into 2, 4, 8, 16, 32, 64, 128, &c. equal parts.

PROBLEM 8. *To divide a given strait line AB into any number of equal parts, for instance 5.*

Method 1, fig. 14, plate 4. Through A, one extremity of the line AB, draw AC, making any angle therewith. 2. Set off on this line from A to H as many equal parts of any length as AB is to be divided into. 3. Join HB. 4. Parallel to HB draw lines through the points D, E, F, G, and these will divide the line AB into the parts required.

Second method, fig. 15, plate 4. 1. Through B draw LD, forming any angle with AB. 2. Take any point

point D either above or below A B, and through D draw D K parallel to A B. 3. On D set off five equal parts D F, F G, G H, H I, I K. 4. Through A and K draw A K, cutting B D in L. 5. Lines drawn through L, and the points F, G, H, I, K, will divide the line A B into the required number of parts.

Third method, fig. 17, plate 4. 1. From the ends of the line A B, draw two lines A C, B D, parallel to each other. 2. In each of these lines, beginning at A and B, set off as many equal parts less one, as A B is to be divided into, in the present instance four equal parts, A I, I K, K L, L M, on A C; and four, B E, E F, F G, G H, on B D. 3. Draw lines from M to E, from L to F, K to G, I to H, and A B will be divided into five equal parts.

Fourth method, fig. 16, plate 4. 1. Draw any two lines C D, E F parallel to each other. 2. Set off on each of these lines, beginning at C and E, any number of equal parts. 3. Join each point in C E with it's opposite point in D F. 4. Take the extent of the given line in your compasses. 5. Set one foot of the compasses opened to this extent in D, and move the other about till it crosses N G in I. 6. Join D I, which being equal to A B, transfer the divisions of D I to A B, and it will be divided as required. H M is a line of a different length to be divided in the same number of parts.

These foregoing methods are introduced, on account, not only of their own peculiar advantages, but because they also are the foundation of several mechanical methods of division.

PROBLEM 9. *To cut off from a given line A B any odd part, as $\frac{1}{3}$ d, $\frac{1}{4}$ th, $\frac{1}{5}$ th, $\frac{1}{7}$ th, &c. of that line, fig. 18, plate 4.*

1. Draw

1. Draw through either end A, a line AC, forming any angle with AB. 2. Make AC equal to AB. 3. Through C and B draw the line CD. 4. Make BD equal to CB. 5. Bisect AC in a. 6. A rule on a and D will cut off aB equal $\frac{1}{3}$ d of AB.

If it be required to divide AB into five equal parts, 1. Add unity to the given number, and halve it, $5+1=6$, $\frac{6}{2}=3$. 2. Divide AC into three parts; or, as AB is equal to AC, set off Ab equal Aa. 3. A rule on D, and b will cut off bB $\frac{1}{3}$ th part of AB. 4. Divide Ab into four equal parts by two bisections, and AB will be divided into five equal parts.

To divide AB into seven equal parts, $7+1=8$, $\frac{8}{2}=4$. 1. Now divide AC into four parts, or bisect aC in c, and cC will be the $\frac{1}{4}$ th of AC. 2. A rule on c and D cuts off cB $\frac{1}{4}$ th of AB. 3. Bisect Ac, and the extent cB will divide each half into 3 equal parts, and consequently the whole line into 7 equal parts.

To divide AB into 9 equal parts, $9+1=10$, $\frac{10}{2}=5$. Here, 1. Make Ad equal to Ab, and dC will be $\frac{1}{5}$ of AC. 2. A rule on D and d cuts off dB $\frac{1}{5}$ of AC. 3. Bisect Ad. 4. Halve each of these bisections, and Ad is divided into 4 equal parts. 5. The extent dB will bisect each of these, and thus divide AB into nine equal parts.

If any odd number can be subdivided, as 9 by 3; then first divide the given line into 3 parts, and take the third as a new line, and find the third thereof as before, which gives the ninth part required.

Method 2. Let DB, fig. 19, plate 4, be the given line. 1. Make two equilateral triangles ADB, CDB, one on each side of the line DB. 2. Bi-
sect

sect AB in G . 3. Draw CG , which will cut off HB equal $\frac{1}{3}$ d of DB . 4. Draw DF , and make GF equal to DG . 5. Draw HF which cuts off Bh equal $\frac{1}{4}$ of AB , or DB . 6. Ch cuts DB in i one fifth part. 7. Fi cuts AB in k equal $\frac{1}{6}$ th of DB . 8. Ck cuts DB in l equal $\frac{1}{7}$ th of DB . 9. Fl cuts AB in m equal $\frac{1}{8}$ th of DB . 10. Cm cuts DB at n equal one ninth part thereof.

Method 3. Let AB , *fig. 12, plate 5*, be the given line to be divided into it's aliquot parts $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$. 1. On AB erect the square $ABCD$. 2. Draw the two diagonals AC , DB , which will cross each other at E . 3. Through E draw FEG parallel to AD , cutting AB in G . 4. Join DG , and the line will cut the diagonal AC at H . 6. Through H draw IHK parallel to AD . 5. Draw DH crossing AC in L . 7. And through L draw MLN parallel to AD , and so proceed as far as necessary. AG is $\frac{1}{2}$, AK $\frac{1}{3}$, AN $\frac{1}{4}$ of AB .*

Method 4. Let AB , *fig. 13, plate 5*, be the given line to be subdivided. 1. Through A and B draw CD , FE parallel to each other. 2. Make CA , AD , FB , FE equal to each other. 3. Draw CE , which shall divide AB into two equal parts at G . 4. Draw AE , DB , intersecting each other at H . 5. Draw GH intersecting AB at I , making AI $\frac{1}{3}$ d of AB . 6. Draw DF cutting AE in K . 7. Join CK , which will cut AB in L , making AL equal $\frac{1}{4}$ of AB . 8. Then draw Dg , cutting AE in M , and proceed as before.

COROLLARY. Hence a given line may be accurately divided into any prime number whatsoever, by first cutting off the odd part, then dividing the remainder by continual bisections.

PROBLEM

PROBLEM 10. *An easy, simple, and very useful method of laying down a scale for dividing lines into any number of equal parts, or for reducing plans to any size less than the original.*

If the scale is for dividing lines into two equal parts, constitute a triangle, so that the hypothenuse may be twice the length of the perpendicular. Let it be three times for dividing them into 3 equal parts, four for four parts, and so on: *fig. 11, plate 4*, represents a set of triangles so constituted.

To find the third of the line by this scale. 1. Take any line in your compasses, and set off this extent from A towards $\frac{1}{3}$, on the line marked one third; then close the compasses so as to strike an arch that shall touch the base AC, and this distance will be the $\frac{1}{3}$ of the given line. Similar to this is what is termed the angle of reduction, or proportion, described by some foreign writers, and which we shall introduce in it's proper place.

PROBLEM 11. *To divide by the sector a given strait line into any number of equal parts.*

Case 1. When the given line is to be divided into a number of equal parts that are multiples of 2.

In this case the operations are best performed by continual bisection; let it be required to divide AB, *fig. 6, plate 5*, into 16 equal parts. 1. Make AB a transverse distance between 10 and 10 on the line of lines. 2. Take out from thence the distance between 5 and 5, and set it from A or B to 8, and AB will be divided into two equal parts. 3. Make A 8 a transverse distance between 10 and 10, and 4 the transverse distance between 5 and 5, will bisect 8 A, and 8 B at 4 and 12; and thus AB is divided into four equal parts in the points 4, 8, and 12. 4. The extent A 4, put between 10 and 10,

10, and then the distance between 5 and 5 applied from A to 2, from 4 to 6, from 8 to 10, and from 12 to 14, will bisect each of those parts, and divide the whole line into 8 equal parts. 5. To bisect each of these, we might take the extent of A 2, and place it between 10 and 10 as before; but as the spaces are too small for that purpose, take three of them in the compasses, and open the sector at 10 and 10, so as to accord with this measure. 6. Take out the transverse measure between 5 and 5, and one foot of the compasses in A will give the point 3, in 4 will fall on 7 and 1, on 8 will give 5 and 11, on 12 gives 9 and 15, and on B will give 13. Thus we have, in a correct and easy manner, divided AB into 16 equal parts by a continual bisection.

If it were required to bisect each of the foregoing divisions, it would be best to open the sector at 10 and 10, with the extent of five of the divisions already obtained; then take out the transverse distance between 5 and 5, and set it off from the other divisions, and they will thereby be bisected, and the line divided into 32 equal parts.

Case 2. When the given number is not a multiple of 2.

Let the given line be AB, *fig. 7, plate 4*, to be divided into 14 equal parts, a number which is not a multiple of 2.

1. Take the extent AB, and open the sector to it on the terms 10 and 10, and the transverse distance of 5 and 5, set from A or B to 7, will divide AB into two equal parts, each of which are to be subdivided into 7, which may be done by dividing A 7 into 6 and 1, or 4 and 3, which last is preferable to the first, as by it, the operation may be finished with only two bisections.

2. There

2. Therefore open the sector in the terms 7 and 7, with the extent A 7; then take out the transverse distance between 4 and 4, this laid off from A, gives the point 4, from 7 gives 3 and 11, and from B gives 10.

3. Make A 4 a transverse distance at 10 and 10, then the transverse distance between 5 and 5 bisects A b, and 10 B in 2 and 12, and gives the point 6 and 8; then one foot in 3, gives 1 and 5, and from 11, 13, and 9: lastly, from 4 it gives 6, and from 10, 8; and thus the line A B is divided into 14 equal parts.

PROBLEM 12. *To make a scale of equal parts containing any given number in an inch.*

Example. To construct a scale of feet and inches in such a manner, that 25 of the smallest parts shall be equal to one inch, and 12 of them represent one foot.

By the line of lines on the sector. 1. Multiply the given numbers by four, the products will be 100, and 48. 2. Take one inch between your compasses, and make it a transverse distance between 100 and 100, 2. the distance between 48 and 48 will be twelve of these 25 parts in an inch; this extent set off from A to 1, fig. 3, plate 5, from 1 to 2, &c. to 12 at B divides A B into a scale of 12 feet. 3. Set off one of these parts from A to a, to be subdivided into 12 parts to represent inches. 4. To this end divide this into three parts; thus take the extent A 2 of two of these parts, and make it a transverse distance between 9 and 9. 5. Set the distance between 6 and 6 from b to e, the same extent from 1 gives g, and from e gives n, thus dividing Aa into 3 equal parts in the points n and g. 6. By two bisections each of these may be

F
subdi-

subdivided into four equal parts, and thus the whole space into 12 equal parts.

When a small number of divisions are required, as 1, 2, or 3, instead of taking the transverse distance near the center of the sector, the division will be more accurately performed by using the following method.

Thus, if 3 parts are required from A, of which the whole line contains 90, make AD, *fig. 4, plate 5*, a transverse distance between 90 and 90; then take the distance between 87 and 87, which set off from D to E backwards, and AE will contain the 3 desired parts.

Example 2. Supposing a scale 6 inches to contain 140 poles, to open the sector so that it may answer for such a scale; divide 140 by 2, which gives 70, the half of 6 equal to 3; because 140 was too large to be set off on the line of lines. Make three inches a transverse distance between 70 and 70, and your sector becomes the required scale.

Example 3. To make a scale of 7 inches that shall contain 180 fathom; $\frac{180}{2} = 90 = 9 \cdot 2 = 3 \frac{1}{2}$, therefore make $3 \frac{1}{2}$ a transverse between 9 and 9, and you have the required scale.

OF PROPORTIONAL LINES.

PROBLEM 13. To cut a given line AD, *fig. 14, plate 5*, into two unequal parts that shall have any given proportion, ex. gr. of C to D.

1. Draw AG forming any angle with AD.
2. From A on AG, set off AC equal to C, and CE equal to D.
3. Draw ED, and parallel to it CB, which will cut AD at B in the required proportion.

To

To divide by the sector, the line AB, *fig. 1, plate 5*, in the proportion of 3 to 2. Now as 3 and 2 would fall near the center, multiply them by 2, thereby forming 6 and 4, which use instead of 3 and 2. As the parts are to be as 6 to 4, the whole line will be 10; therefore make AB a transverse distance between 10 and 10, and then the transverse distance between 6 and 6, set off from B to e, is $\frac{3}{5}$ ths of AB; or the distance between 4 and 4, will give Ae $\frac{2}{5}$ ths of AB; therefore AB is cut in the proportion of 3 to 2.

Example 2. To cut AB, *fig. 2, plate 5*, in the proportion of 4 to 5; here we may use the numbers themselves; therefore with AB, open the sector at 9 and 9, the sum of the two numbers; then the distance between 5 and 5 set off from B to c, or between 4 and 4, set off A to c, and it divides AB in the required proportion.

Note. If the numbers be too small, use their equimultiples; if too large, subdivide them.

Corollary. From this problem we obtain another mode of dividing a strait line into any number of equal parts.

PROBLEM 14. To estimate the proportion between two or more given lines, as AB, CD, EF, *fig. 9, plate 4*.

Make AB a transverse distance between 10 and 10, then take the extents severally of CD and EF, and carry them along the line of lines, till both points rest exactly upon the same number; in the first it will be found to be 85, in the second 87. Therefore AB is to CD as 85 to 100, to EF as 67 to 100, and of CD to EF as 67 to 85.

PROBLEM 15. To find a third proportional to two given right lines A and B, *fig. 15, plate 5*.

F 2

1. From

1. From the point D draw two right lines DE, DF, making any angle whatever. 2. In these lines take DG equal to the first term A, and DC DH, each equal to the second term B. 3. Join GH, and draw CF parallel thereto; then DF will be the third proportional required, that is, DG (A) to DC, (B,) so is DH (B) to DF.

By the sector. 1. Make AB, fig. 5, plate 5, a transverse distance between 100 and 100. 2. Find the transverse distance of EF, which suppose 50. 3. Make EF a transverse distance between 100 and 100. 4. Take the extent between 50 and 50, and it will be the third proportional CD required.

PROBLEM 16. *To find a fourth proportional to three given right lines A, B, C, fig. 16, plate 5.*

1. From the point a draw two right lines making any angle whatever. 2. In these lines make ab equal to the first term A, ac equal to the second B, and ad equal to the third C. 3. Join bc, and draw dc parallel thereto, and ae will be the fourth proportional required; that is, ab (A) is to ac (B), so is ad (C) to ae.

By the sector. 1. Make the line A a transverse distance between 100 and 100. 2. Find the transverse measure of B, which is 60. 3. Make c the third line a transverse measure between 100 and 100. 4. The measure between 60 and 60, will be the fourth proportional.

PROBLEM 17. *To find a mean proportional between two given strait lines A and B, fig. 17, plate 5.*

1. Draw any right line, in which take CE equal to A, and EA equal to B. 2. Bisect ac in B, and with BA, or BC as a radius, describe the semicircle ADC. 3. From the point E, draw EG perpendicular to AC, and it will be the mean proportional required.

By the sector. Join the lines together, and get the sum of them, 130; then find the half of this sum 65, and half the difference 25. Open the line of lines, so that they may be at right angles to each other; then take with the compasses the lateral distance 65; and apply one foot to the half difference 25, and the other foot will reach to 60, the mean proportional required; for 40 to 60, so is 60 to 90.

PROBLEM 18. *To cut a given line AB into extreme and mean proportion, fig. 18, plate 5.*

1. Extend AB to C. 2. At A erect a perpendicular AD, and make it equal to AB. 3. Set the half of AD or AB from A to F. 4. With the radius FD describe the arc DG, and AB will be divided into extreme and mean proportion. AG is the greater segment.

By the sector. Make AB a transverse distance between 60 and 60 of the line of chords. 2. Take out the transverse distance between the chord of 36, which set from A to G, gives the greatest segment.

Or, make AB, a transverse distance between 54 and 54 of the line of lines, then is the distance between 30 and 30 the greater segment, and 18 and 18 the lesser segment.

PROBLEM 19. *To divide a given strait line in the same proportion as another given strait line is divided.*

Let AB, or CD, fig. 10, plate 5, be two given strait lines, the first divided into 100, the second into 60 equal parts; it is required to divide EF into 100, and GH into 60 equal parts.

Make EF a transverse distance in the terms 100 and 100, then the transverse measure between 90 and 90 set from E to 90, and from F to 10; the measure between 80 and 80 set from E to 80, and from F to 20, and the measure between 70 and 70

set from E gives 70, from F 30. The distance between 60 and 60, gives us 60 and 40; and lastly, the transverse measure between 50 and 50 bisects the given line in the point 50, and we shall, by five transverse extents, have divided the line EF into 10 equal parts, each of which are to be subdivided into 10 smaller divisions by problems 11 and 12.

To divide GH into 60 parts, as we have supposed CD to be divided, make GH a transverse distance in the terms of 60, then work as before.

PROBLEM 20. To find the angular point of two given lines AB, cP, fig. 22, plate 5, which incline to each other without producing either of them.

Through A draw at pleasure AN, yet so as not to cut cP too obliquely.

1. Draw the parallel lines $A\nu$, $G\epsilon$, $K\delta$. 2. Take any number of times the extents, AN, GO, KP, and set them on their respective lines, as from N to γ , and ν , from o to β and ϵ , from P to α and δ , and a line through δ , ϵ , ν , and another through α , β , γ , will tend to the same point as the lines AB, GD.

Method 2. 1. Through AB and CD, fig. 21, plate 5, draw any two parallel lines, as GH and FE. 2. Set off the extent BD twice, from B to G, and D to H; and the extent AC twice, from A to F, and from C to E. 3. A line passing through F and G will intersect another line passing through E and H in I, the angular point required.

The extent FA, GB, may be multiplied as divided, so as to suit peculiar circumstances.

Corollary. Hence, if any two lines be given that tend to the same angular point, a third, or more lines may be drawn that shall tend to the same point, and yet pass through a given point.

Solution by the sector. Case 1. When the proposed point e is between the two given lines VL and ab, fig. 23, plate 5:

Through

Through e draw a line av , cutting ab at a , and VL at V , then from any other point b , in ab , the further the better, draw bx parallel to ab , and cutting VL in x , make av a transverse distance between 100 and 100, or the line of lines; take the extent ev , and find it's transverse measure, which suppose to be 60; now make xb a transverse distance between 100 and 100, and take out the transverse distance of the terms 60, which set off from x to f , then a line drawn through the points e and f , shall tend to the same inaccessible point q with the given lines ab , VL .

Case 2. When the proposed point e is without the given lines ab and VL .

Through e draw any line ev , cutting ab in a , and VL in v ; and from any other point b in ab draw xF parallel to ev , and cutting VL in x , make ve a transverse distance in the terms of 100; find the transverse measure of av , which suppose 72; make xb a transverse measure of 72, and take out the distance between the terms of 100, which set off from x to f , and a line ef drawn through e and f , will tend to the same point with the line ab .

If the given or required lines fall so near each other that neither of them can be measured on the terms of 100, then use any other number, as 80, 70, 60, &c. as a transverse measure, and work with that as you did with 100.

This problem is of considerable use in many geometrical operations, but particularly so in perspective; for we may consider VL as a vanishing line, and the other two lines as images on the picture; hence having any image given on the picture that tends to an inaccessible vanishing point, as many more images of lines tending to the same point as may be required are readily drawn. This problem

is more fully illustrated, and all the various cases investigated, in another part of this work.

PROBLEM 21. *Upon a given right line AB, fig. 19, plate 5, to make an equilateral triangle.*

1. From A and B, with a radius equal to AB, describe arcs cutting in C. 2. Draw AC and BC, and the figures ACB is the triangle required.

An isosceles triangle may be formed in the same manner.

PROBLEM 22. *To make a triangle, whose three sides shall be respectively equal to three given lines, A, B, C, fig. 20, plate 5, provided any two of them be greater than the third.*

1. Draw a line BC, equal to the line B. 2. On B, with a radius equal to C, describe an arc at A. 3. On C, with a radius equal to A, describe another arc, cutting the former at A. 4. Draw the lines AC and AB, and the figure ABC, will be the triangle required.

PROBLEM 23. *Upon a given line AB, fig. 1, plate 6, to describe a square.*

1. From the point B, draw BD perpendicular, and equal to AB. 2. On A and D, with the radius AB, describe arcs cutting in C. 3. Draw AC and CD, and the figure ABCD, is the required square.

PROBLEM 24. *To describe a rectangle or parallelogram, whose length and breadth shall be equal to two given lines A and B, fig. 2, plate 6.*

1. Draw CD equal to A, and make DE perpendicular thereto, and equal to B. 2. On the points E and C, with the radii A and B, describe arcs cutting in F. 3. Join CF, and EF; then CDEF will be the rectangle required.

PROBLEM 25. *Upon a given line AB, to construct a rhombus, fig. 3, plate 6.*

1. On

1. On B, with the radius AB, describe an arc at D. 2. On A, with the same radius, describe an arch at C. 3. On C, but still with the same radius, make the intersection D. 4. Draw the lines AD, DC, CB, and you have the required figure.

Having two given lines AB, AD, and a given angle, to construct a rhomboides.

Make the angle ACD equal to the given angle, and set off CD equal to AD, and AC equal to AD; then from A, with the distance AB, describe an arc at B; intersect this arc with the extent AD, set off from D; join AB, AD, and the figure is completed.

PROBLEM 26. *Having the diagonal AD, and four sides AB, BD, CD, AC, to construct a trapezium, fig. 5, plate 6.*

Draw an occult line AD, and make it equal to the given diagonal. Take AB in the compasses, and from A strike an arch at B; intersect this arch from D with the extent DB, and draw AB, DB; now with the other two lines AC, CD, and from A and D, make an intersection at C; join DC, AC, and the figure is completed.

PROBLEM 27. *Having the four sides, and one angle, to construct a trapezium, fig. 5, plate 6.*

Make the line AB equal to it's given side, and at A make the angle CAB equal to the given angle, and AC equal to the given side AC; then with the extent BD from B, describe an arch at D, intersect this from c, with the extent CD; join the several lines, and the figure is obtained.

PROBLEM 28. *To find the center of a circle, fig. 6, plate 5.*

1. Draw

1. Draw any chord AB, and bisect it with the chord CD. 2. Bisect CD by the chord EF, and their intersection o, will be the center of the circle.

PROBLEM 29. *To describe the circumference of a circle through any three given points ABC, fig. 7, plate 5, provided they are not in a strait line.*

1. From the middle point B, draw the chords BA, and BC. 2. Bisect these chords with the perpendicular lines mO, nO. 3. From the point of intersection O, and radius OA, OB, or OC, you may describe the required circle ABC. By this problem, a portion of the circumference of a circle may be finished, by assuming three points.

PROBLEM 30. *To draw a tangent to a given circle, that shall pass through a given point A, fig. 8 and 9, plate 5.*

Case 1. When the point A is in the circumference of the circle, fig. 8, plate 5.

1. From the center O, draw the radius OA, 2. Through the point A, draw CD perpendicular to OA, and it will be the required tangent.

Case 2. When the point A is without the circle, fig. 9, plate 5.

1. From the center O, draw OA, and bisect it in n. 2. From the point n, with the radius nA, or nO, describe the semicircle ADO, cutting the given circle in D. 3. Through the points A and D, draw AB, the tangent required.

PROBLEM 31. *To cut off from a circle, fig. 10, plate 5, a segment containing any proposed angle, ex. gr. 120° .*

Let F be the point from whence it is required to draw a chord which shall contain an angle of 120° .

1. Through F, draw FR a tangent to the circle, 2. From F, draw FA, making an angle of 60 degrees,

degrees, with the tangent FR , and FCA is the segment required.

PROBLEM 32. *On a given line AB , to describe the segment of a circle capable of containing a given angle, fig. 11, plate 6.*

Draw AC , and BC , making the angles ABC , ACB , each equal to the given angle. Draw AD perpendicular to AC , and BD perpendicular to BC ; with the center D , and radius DA , or DB , describe the segment AEB , and any angle made in this segment, will be equal to the given angle. A more easy solution of this problem will be given, when we come to apply it to practice.

PROBLEM 33. *To describe an arch of a circle that shall contain any number of degrees without compasses, or without finding the center of the circle, fig. 12, plate 5.*

Geometrically by finding points through which the arch is to pass, let AB be the given chord.

1. Draw AF , making any angle with BA . 2. At any point F , in AF , make the angle EFG , equal to the given angle. 3. Through B , draw BE parallel to FG , and the intersection gives the point E , in the same manner as many points D , C , &c. may be found, as will be necessary to complete the arch.

This subject will be found fully investigated hereafter.

PROBLEM 34. *To inscribe a circle in a given triangle, ABC , fig. 13, plate 6.*

1. Bisect the angles A and B with the lines AO and BO . 2. From the point of intersection O , let fall the perpendicular ON , and it will be the radius of the required circle.

PROBLEM 35. *To inscribe a pentagon, a hexagon, or a decagon, in a given circle, fig. 15, plate 6.*

1. Draw the diameters AB and CE at right angles to each other. 2. Bisect DB at G. 3. On G, with the radius GC, describe the arc CF. 4. Join C and F, and the line CF will be one side of the required pentagon.

The two sides DC, FD of the triangle FDC, enable us to inscribe a hexagon or decagon in the same circle; for DC is the side of the hexagon, DF that of a decagon.

PROBLEM 36. *To inscribe a square or an octagon in a given circle, fig. 16, plate 6.*

1. Draw the diameters AC, BD, at right angles to each other. 2. Draw the lines AD, BA, BC, CD, and you obtain the required square.

FOR THE OCTAGON.

Bisect the arc AB of the square in the point F, and the line AE being carried 8 times round, will form the octagon required.

PROBLEM 37. *In a given circle to inscribe an equilateral triangle, an hexagon, or a dodecagon, fig. 17, plate 6.*

FOR THE EQUILATERAL TRIANGLE.

1. From any point A as a center, with a distance equal to the radius AO, describe the arc FOB. 2. Draw the line BF, and make BD equal to BF. 3. Join DF, and BDF will be the equilateral required.

FOR THE HEXAGON.

Carry the radius AO six times round the circumference, and you obtain a hexagon.

FOR

FOR THE DODECAGON.

Bisect the arc AB of the hexagon in the point n, and the line An, being carried twelve times round the circumference, will form the required dodecagon.

PROBLEM 38. *Another method to inscribe a dodecagon in a circle, or to divide the circumference of a given circle, into 12 equal parts, each of 30 degrees, fig. 18, plate 6.*

1. Draw the two diameters I, 7, 4, 10, perpendicular to each other. 2. With the radius of the circle, and on the four extremities I, 4, 7, 10, as centers, describe arcs through the center of the circle; these arcs will cut the circumference in the points required, dividing it into 12 equal parts, at the points marked with the numbers.

PROBLEM 39. *To find the angles at the center, and the sides of a regular polygon.*

Divide 360 by the number of sides in the proposed polygon, thus $\frac{360}{5}$ gives 72, for the angle at the center of a pentagon. To find the angle formed by the sides, subtract the angle at the center from 180, and the remainder is the angle required; thus 72° from 180° , gives 108° for the angle of a pentagon.

A TABLE, shewing the angles at the centers, and circumferences of regular polygons, from three to twelve sides inclusive.

Names.	Sides.	Angles at Center	Angles at Cir.
Trigon	3	$120^\circ 00'$	$60^\circ 00'$
Square	4	$90^\circ 00'$	$90^\circ 00'$
Pentagon	5	$72^\circ 00'$	$108^\circ 00'$
Hexagon	6	$60^\circ 00'$	$120^\circ 00'$
Heptagon	7	$51^\circ 25\frac{1}{7}'$	$128^\circ 34\frac{2}{7}'$
Octagon	8	$45^\circ 00'$	$135^\circ 00'$
Nonagon	9	$40^\circ 00'$	$140^\circ 00'$
Decagon	10	$36^\circ 00'$	$144^\circ 00'$
Endecagon	11	$32^\circ 43\frac{7}{11}'$	$147^\circ 16\frac{4}{11}'$
Dodecagon	12	$30^\circ 00'$	$150^\circ 00'$

This

This table is constructed, by dividing 360, the degrees in a circumference, by the number of sides in each polygon, and the quotients are the angles at the centers; the angle at the center subtracted from 180 degrees, leaves the angle at the circumference.

PROBLEM 40. *In a given circle to inscribe any regular polygon, fig. 14, plate 6.*

1. At the center c , make an angle equal to the center of the polygon, as contained in the preceding table, and join the angular points $A B$. 2. The distance $A B$ will be one side of the polygon, which being carried round the circumference, the proper number of times will complete the figure.

Another method, which approximates very nearly the truth, fig. 19, plate 6.

1. Divide the diameter $A B$ into as many equal parts as the figure has sides. 2. From the center O raise the perpendicular $O m$. 3. Make $m n$ equal three fourths of $O m$. 4. From n draw $n C$, through the second division of the diameter. 5. Join the points $A C$, and the line $A C$ will be the side of the required polygon, in this instance a pentagon.

PROBLEM 41. *About any given triangle $A B C$, to circumscribe a circle, fig. 21, plate 6.*

1. Bisect any two sides $A B$, $B C$, by the perpendiculars $m o$, $n o$. 2. From the point of intersection o , with the distance $O A$ or $O B$, describe the required circle.

PROBLEM 42. *About a given circle to circumscribe a pentagon, fig. 20, plate 6.*

1. Inscribe a pentagon within the circle. 2. Through the middle of each side draw the lines $O A$, $O B$, $O C$, $O D$, and $O E$. 3. Through the point n draw the tangent $A B$, meeting $O A$ and $O B$

OB in A and B. 4. Through the points A and m, draw the line AmC, meeting OC in c. 5. In like manner draw the lines CD, DE, and EB, and ABCDE will be the pentagon required.

In the same manner you may about a given circle circumscribe any polygon.

PROBLEM 43. *About a given square to circumscribe a circle, fig. 22, plate 6.*

1. Draw the two diagonals AD, BC, intersecting each other at o. 2. From O, with the distance OA or OB, describe the circle ABCD, which will circumscribe the square.

PROBLEM 44. *On a given line AB, to make a regular hexagon, fig. 24, plate 6.*

1. On AB make the equilateral triangle AOB. 2. From the point O, with the distance OA or OB, describe the circle ABCDEF. 3. Carry AB six times round the circumference, and it will form the required hexagon.

PROBLEM 45. *On a given line AB, to form a regular polygon of any proposed number of sides, fig. 14, plate 6.*

1. Make the angles CAB, CBA, each equal to half the angles at the circumference; see the preceding table. 2. From the point of intersection c, with the distance CA, describe a circle. 3. Apply the chord AB to the circumference, the proposed number of times, and it will form the required polygon.

PROBLEM 46. *On a given line AB, fig. 25, plate 6, to form a regular octagon.*

1. On the extremities of the given line AB, erect the indefinite perpendiculars AF and BE. 2. Produce AB both ways to s and w, and bisect the angle nAs and oBw by the lines AH, BC. 3. Make AH and BC each equal to AB, and draw

draw the line HC. 4. Make ov equal to on , and through v draw GD parallel to HC . 5. Draw HG and CD parallel to AF and BE , and make cE equal to cD . 6. Through E draw EF , parallel to AB , and join the points GF and DE , and $ABCDEFGH$ will be the octagon required.

PROBLEM 47. *On a given right line AB , fig. 26, plate 6, to describe a regular pentagon.*

1. Make Bm perpendicular, and equal to AB . 2. Bisect AB in n . 3. On n , with distance nm , cross AB , produced in O . 4. On A and B , with radius AO , describe arcs intersecting at D . 5. On D , with radius AB , describe the arc EC , and on A and B , with the same extent, intersect this arc at E and C . 6. Join AE , ED , DC , CB , and you complete the figure.

PROBLEM 48. *Upon a given right line AB , fig. 27, plate 6, to describe a triangle, similar to the triangle CDE .*

1. At the end A , of the given line AB , make an angle FAB , equal to the angle ECD . 2. At B make the angle ABF , equal to the angle CDE . 3. Draw the two sides AF , BF , and ABF will be the required triangle.

PROBLEM 49. *To describe a polygon, similar to a given polygon $ABCDEF$, one of it's sides ab being given, fig. 28, plate 5.*

1. Draw AC , AD , AF . 2. Set off ab on AB , from a to $r3$; draw rg parallel to BE , meeting AF in g . 4. Through the point g draw gh , parallel to FC , meeting AC in h . 5. Thro' the point h draw the parallel hi . 6. Through i draw ik , parallel to ED , and the figure $Arghik$ will be similar to the figure $ABCDEF$.

PROBLEM 50. *To reduce a figure by a scale, fig. 28 and 29, plate 5.*

1. Mea-

1. Measure each side of the figure $A B C D E$ with the scale $G H$. 2. Make $a b$ as many parts of a smaller scale $K L$, as $A B$ was of the larger. 3. $b c$ as many of $K L$, as $B C$ of $G H$, and $a c$ of $K L$, $A C$, &c. by which means the figure will be reduced to a smaller one.

OF THE TRANSFORMATION AND REDUCTION OF FIGURES.

PROBLEM 51. *To change a triangle into another of equal extent, but different height.*

Let $A B C$, *fig. 1, 2, 3, 4, plate 7*, be the given triangle, D a point at the given height.

Case 1. Where the point D , *fig. 1 and 2, plate 7*, is either in one of the sides, or in the prolongation of a side. 1. Draw a line from D to the opposite angle C . 2. Draw a line $A E$, parallel thereto from A , the summit of the given triangle. 3. Join $D E$, and $B D E$ is the required triangle.

Case 2. When the point D , *fig. 3 and 4, plate 7*, is neither in one of the sides, nor in the prolongation thereof. 1. Draw an indefinite line $B D a$, from B through the point D . 2. Draw from A , the summit of the given triangle, a line $A a$, parallel to the base $B C$, and cutting the line $B D$ in a . 3. Join $a c$, and the triangle $B a c$ is equal to the triangle $B A C$; and the point D being in the same line with $B a$. 4. By the preceding case, find a triangle from D , equal to $B a C$; i. e. join $D C$, draw $a E$ parallel thereto, then join $D E$, and $B D E$ is the required triangle.

Corollary. If it be required to change the triangle $B A C$ into an equal triangle, of which the height and angle $D B E$ are given: 1. Draw the indefinite line $B D A$, making the required angle with G BC.

B C. 2. Take on B D a point D at the given height; and, 3. Construct the triangle by the foregoing rules.

PROBLEM 52. *To make an isosceles triangle AEB, fig. 5, plate 7, equal to the scalene triangle ACB.*

1. Bisect the base in D. 2. Erect the perpendicular DE. 3. Draw CE parallel to AB. 4. Draw AE, EB, and AEB is the required triangle.

PROBLEM 53. *To make an equilateral triangle equal to a given scalene triangle ABC, fig. 7, plate 7.*

1. On the base AB make an equilateral triangle ABD. 2. Prolong BD towards E. 3. Draw CE parallel to AB. 4. Bisect DE at I, on DI describe the semicircle DFE. 5. Draw BF, the mean proportional between BE, BD. 6. With BF, from B, describe the arch FGH; with the same radius from G, intersect this arch at H, draw BH, GH, and BGH is the triangle required.

Corollary. If you want an equilateral triangle equal to a rectangle, or to an isosceles triangle, find a scalene triangle respectively equal to each, and then work by the foregoing problem.

PROBLEM 54. *To reduce a rectilinear figure ABCDE, fig. 8 and 9, plate 7, to another equal to it, but with one side less.*

1. Join the extremities E, C, of two sides, DE, DC, of the same angle D. 2. From D, draw a line DF, parallel to EC. 3. Draw EF, and you obtain a new polygon ABFE, equal to ABCDE, but with one side less.

Corollary. Hence every rectilinear figure may be reduced to a triangle, by reducing it successively to a figure with one side less, until it is brought to one with only three sides.

For

For example; let it be required to reduce the polygon $ABCDEF$, *fig. 10* and *11*, *plate 7*, into a triangle IAH , with it's summit at A , in the circumference of the polygon, and it's base on the base thereof prolonged.

1. Draw the diagonal DF . 2. Draw EG , parallel to DF . 3. Draw FG , which gives us a new polygon, $ABCGF$, with one side less. 4. To reduce $ABCGF$, draw AG , and parallel thereto FH ; then join AH , and you obtain a polygon $ABCH$, equal to the preceding one $ABCGF$. 5. The polygon $ABCH$ having a side AH , which may serve for a side of the triangle, you have only to reduce the part ABC , by drawing AC , and parallel thereto BI ; join AI , and you obtain the required triangle IAH .

N. B. In figure 10 the point A is taken at one of the angular points of the given polygon; in figure 11 it is in one of the sides, in which case there is one reduction more to be made, than when it is at the angular point.

Corollary. As a triangle may be changed into another of any given height, and with the angle at the base equal to a given angle; if it be required to reduce a polygon to a triangle of a given height, and the angle at the base also given, you must first reduce it into a triangle by this problem, and then change that triangle into one, with the required data, by the *problem 51*.

Corollary. If the given figure is a parallelogram, *fig. 12*, *plate 7*, draw the diagonal EC , and DF parallel thereto; join EF , and the triangle EBF is equal to the parallelogram $EBCD$.

OF THE ADDITION OF FIGURES.

1. If the figures to be added are triangles of the same height as AMB , BNC , COD , DPE , *fig. 14, plate 7*, make a line AE equal to the sum of their bases, and constitute a triangle AME thereon, whose height is equal to the given height, and AME will be equal to the given triangles.

2. If the given figures are triangles of different heights, or different polygons, they must first be reduced to triangles of the same height, and then these may be added together.

3. If the triangle into which they are to be summed up, is to be of a given height, and with a given angle at the base, they must first be reduced into one triangle, and then that changed into another, by the preceding rules.

4. The triangle obtained may be changed into a parallelogram by the last corollary.

MULTIPLICATION OF FIGURES.

1. To multiply AMB , *fig. 13, plate 7*, by a given number, for example by 4; or more accurately, to find a triangle that shall be quadruple the triangle AMB . Lengthen the base AE , so that it may be four times AB ; join ME , and the triangle AME will be quadruple the triangle AMB . 2. By reducing any figure to a triangle, we may obtain a triangle which may be multiplied in the same manner.

SUBTRACTION OF FIGURES.

1. If the two triangles BAC , dac , *fig. 15, plate 7*, are of the same height; take from the base BC of the first a part DC , equal to the base dc

dc of the other, and join AB : then will the triangle ABD be the difference between the two triangles.

If the two triangles be not of the same height, they must be reduced to it by the preceding rules, and then the difference may be found as above; or if a polygon is to be taken from another, and a triangle found equal to the remainder, it may be easily effected, by reducing them to triangles of the same height.

2. A triangle may be taken from a polygon, by drawing a line within the polygon, from a given point F on one of it's sides. To effect this, let us suppose the triangle to be taken from the polygon $ABCDE$, *fig. 16, plate 7*, has been changed into a triangle MOP , *fig. 26*, whose height above it's base MP , is equal to that of the given point F , above AB ; this done, on AB , (prolonged if necessary) lay off AG equal to OP , join FG , and the triangle AFG is equal to the triangle MOP . There are, however, three cases in the solution of this problem, which we shall therefore notice by themselves.

If the base MP does not exceed AB , *fig. 16, plate 7*, the point G will fall thereon, and the problem will be solved.

But if the base MP exceeds the base AB , G will be found upon AB prolonged, *fig. 17 and 18, plate 7*; join FB , and draw GH parallel thereto; from the situation of this point arise the other two cases.

Case 1. When the point H , *fig. 17, plate 7*, is found on the side BC , contiguous to the side AB , join FH , and the quadrilateral figure $FABH$ is equal to the triangle MOP .

Case 2. When H , *fig. 18, plate 7*, meets BC , prolonged; from F , draw FC and HI parallel thereto;

thereto ; then join FI , and the pentagon $FABCI$ is equal to the triangle MOP .

DIVISION OF RECTILINEAR FIGURES.

1. To divide the triangle AME , *fig. 13, plate 7*, into four equal parts ; divide the base into four equal parts by the points B, C, D ; draw MD, MC, MB , and the triangle will be divided into four equal parts.

2. If the triangle AME , *fig. 19, plate 7*, is to be divided into four equal parts from a point m , in one of it's sides, change it into another Ame , with it's summit at m , and then divide it into four equal parts as before ; if the lines of division are contained in the triangle AME , the problem is solved ; but if some of the lines, as mD , terminate without the triangle, join mE , and draw dD parallel thereto ; join md , and the quadrilateral $mCED$ is equal CmD one fourth of AME , and the triangle is divided into four equal parts.

3. To divide the polygon $ABCDEF$, *fig. 20, plate 6*, into a given number of equal parts, ex. gr. 4, from a point G , situated in the side AF ; 1. Change the polygon into a triangle AGM , whose summit is at G . 2. Divide this triangle into as many equal triangles AGH, HGI, IGK, KGM , as the polygon is required to be divided into. 3. Subtract from the polygon a part equal to the triangle AGH ; then a part equal to the triangle AGI , and afterwards a part equal to the triangle AGK , and the lines GH, GR, GO , drawn from the point G , to make these subtractions, will divide the polygon into four equal parts, all which will be sufficiently evident from the figure.

PROBLEM

PROBLEM 55. *Three points, fig. 21 to 25, plate 7, N, O, A, being given, arranged in any manner on a strait line; to find two other points, B, b, in the same line so situated, that as*

$$NO : AB :: AB : NB$$

$$NO : Ab :: Ab : N$$

Make $AP = \frac{NO}{4}$, and $PL = \frac{NO}{4}$, placing

them one after the other, so that $AL = \frac{NO}{2}$.

Observing, 1. That in the two figures 21 and 22, A is placed between N and O; and that AP is taken on AO, prolonged if necessary.

2. In fig. 23, where the point O is situated between N and A, AP should be put on the side opposite to AO.

3. In fig. 24, where the point N is situated between A and O, if AP be smaller than AN, it must be taken on the side opposite to AN.

4. In fig. 25, where the point N is also placed between A and O, if AP be greater than AN, it must be placed on AN prolonged.

Now by problem 17 make MN in all the five figures a mean proportional between NO, NP, and carry this line from L to B, and from L to b. and NO will be : AB :: AB : NB.

$$NO : Ab :: Ab : NB.$$

PROBLEM 56. *Two lines EF, GH, fig. 27, 28, 29, plate 7, intersecting each other at A, being given, to draw from C a third line BD, which shall form with the other two a triangle DAB equal to a given triangle X.*

1. From C draw CN parallel to EF. 2. Change the triangle X into another CNO, whose summit is at the point C. 3. Find on GH a point B, so that $NO : AB :: AB : NB$, and from this

G 4

point

point B, draw the line CB, and DAB shall be the required triangle.

Scholium. This problem may be used to retrench one rectilinear figure from another, by drawing a line from a given point.

Thus, if from a point C, without or within the triangle EAH, *fig. 30*, a right line is required to be drawn, that shall cut off a part DAB, equal to the triangle X, *fig. 30*; it is evident this may be effected by the preceding problem.

If it be required to draw a line BD from a point c, which shall cut off from the quadrilateral figure EFGH, a portion DFG B, *fig. 31*, equal to the triangle X; if you are sure, that the right line BD will cut the two opposite sides EF, GH, prolong EF, GH, till they meet; then form a triangle Z, equal to the two triangles Z and FAG; and then take Z from AEH by a line BD from the point C, which is effected by the preceding problem.

If it be required to take from a polygon Y, a part DFH B equal to a triangle X; and that the line BD is to cut the two sides EF, GH; prolong these sides till they meet in A; then make a triangle Z, equal to the triangle X, and the figure AFH; and then retrench from the triangle EAG the triangle DAB equal to Z, by a line BD, from a given point C.

As all rectilinear figures may be reduced to triangles, we may, by this problem, take one rectilinear figure from another by a strait line drawn from a given point.

PROBLEM 57. To make a triangle equal to any given quadrilateral figure ABCD, *fig. 33, plate 7.*

1. Draw the diagonal BD. 2. Draw CE parallel thereto, intersecting AD produced in E. 3. Join EC, and ACE is the required triangle.

PROBLEM

PROBLEM 58. *To make a rectangle, or parallelogram, equal to a given triangle A C E, fig. 33, plate 7.*

1. Bisect the base A E in D. 2. Through C draw C B parallel to A B. 3. Draw E D, B A, parallel to each other, and either perpendicular to C G, or making any angle with it. And the rectangle or parallelogram A B C D will be equal to the given triangle.

PROBLEM 59. *To make a triangle equal to a given circle, fig. 34, plate 7.*

Draw the radius O B, and tangent A B perpendicular thereto; make A B equal to three times the diameter of the circle, and $\frac{1}{7}$ more; join A O, and the triangle A O B will be nearly equal the given circle.

PROBLEM 60. *To make a square equal to a given rectangle, A B C D, fig. 35, plate 7.*

Produce one side A B, till B L be equal to the other B C. 2. Bisect A L in O. 3. With the distance A O, describe the semicircle L F A. 4. Produce B C to F. 5. On B F make the square B F G H, which is equal to the rectangle A B C D.

ADDITION AND SUBTRACTION OF SIMILAR FIGURES.

PROBLEM 61. *To make a square equal to the sum of any number of squares taken together, e. gr. equal to three given squares, whose sides are equal to the lines A B C, fig. 36, plate 7.*

1. Draw the indefinite lines E D, D F, at right angles to each other. 2. Make D G equal to A, and D H equal to B. 3. Join G and H, and G H will be the side of a square, equal the two squares whose sides are A and B. 4. Make D L equal G H, D K equal C, and join K L; then will K L be the side of a square equal the three given squares. Or,
after

after the same manner may a square be constructed equal any number of given squares.

PROBLEM 62. *To describe a figure equal to the sum of any given number of similar figures, fig. 36, plate 7.*

This problem is similar to the foregoing: 1. Form a right angle. 2. Set off thereon two homologous sides of the given figures, as from D to G, and from D to H. 3. Draw G H, and thereon describe a figure similar to one of the given ones, and it will be equal to their sum. In the same manner you may go on, adding a greater number of similar figures together.

If the similar figures be circles, take the radii or diameters for the homologous lines.

PROBLEM 63. *To make a square equal to the difference of two given squares, whose sides are A B, C D, fig. 37, plate 7.*

1. On one end B of the shortest line, raise a perpendicular B F. 2. With the extent C D from A, cut B F in F, and B F will be the side of the required square.

In the same manner the difference between any two similar figures may be found.

PROBLEM 64. *To make a figure which shall be similar to, and contain a given figure, a certain number of times. Let M N be an homologous side of the given figure, fig. 38, plate 7.*

1. Draw the indefinite line B Z. 2. At any point D, raise D A perpendicular to B Z. 3. Make B D equal to M N, and B C as many times a multiple of B D, as the required figure is to be of the given one. 4. Bisect B C, and describe the semicircle B A C. 5. Draw A C, B C. 6. Make A E equal M N. 7. Draw E F parallel to B C, and E F will be the homologous side of the required figure.

PROBLEM

PROBLEM 65. *To reduce a complex figure from one scale to another, mechanically, by means of squares.*

1. Divide the given figure by cross lines into as many squares as may be thought necessary. 2. Divide another paper into the same number of squares, either greater, equal, or less, as required. 3. Draw in every square what is contained in the correspondent square of the given figure, and you will obtain a copy tolerably exact; see fig. 39, plate 7.

PROBLEM 66. *To enlarge a map or plan, and make it twice, or 3, 4, 5, &c. times larger than the original, fig. 12, plate 8.*

1. Draw the indefinite line *ab*. 2. Raise a perpendicular at *a*. 3. Divide the original plan into squares by the preceding problem. 4. Take the side of one of the squares, which set off from *a* to *d*, and on the perpendicular from *a* to *e*, finish the square *aefd*, which is equal to one of the squares of the proposed plan. 5. Take the diagonal *de*, set it off from *a* to *g*, and from *a* to *l*; complete the square *alng*, and it will be double the square *aefd*. To find one three times greater, take *dg*, and with that extent form the square *amoh*, which will be the square required. With *dh* you may form a square that will contain the given one *aefd* four times. The line *dl* gives a square five times larger than the original square.

PROBLEM 67. *To reduce a map $\frac{1}{2}$, $\frac{1}{3}$ d, $\frac{1}{4}$ th, $\frac{1}{5}$ th, &c. of the original, fig. 5, plate 8.*

1. Divide the given plan into squares by problem 65. 2. Draw a line, on which set off from *A* to *B* one side of one of these squares. 3. Divide this line into two equal parts at *F*, and on *F* as a center with *FA* or *FB*, describe the semicircle *AHB*.

4. To obtain $\frac{1}{2}$ the given square, at *F* erect the perpendicular *FH*, and draw the right line *AH*, which

which will be the side of the required square. 5. For $\frac{1}{3}$ d divide AB into three parts; take one of these parts, set it off from A to C , at C raise the perpendicular CI , through I draw AI , and it will be the side of the required square. 6. For $\frac{1}{4}$ divide AB into four equal parts, set off one of these from B to E , at E make EG perpendicular to AB , join BG , and it will be the side of a square $\frac{1}{4}$ th of the given one.

PROBLEM 68. *To make a map or plan in proportion to a given one, e. gr. as 3 to 5, fig. 6, plate 8.*

The original plan being divided into squares, 1. Draw AM equal to the side of one of these squares. 2. Divide AM into five equal parts, 3. At the third division raise the perpendicular CD , and draw AD , which will be the side of the required square,

PROBLEM 69. *To reduce figures by the angle of reduction.* Let ab be the given side on which it is required to construct a figure similar to $ACDE$, fig. 1, 2, 3, plate 8.

1. Form an angle LMN at pleasure, and set off the side AB from M to I . 2. From I , with a , cut MN in K . 3. Draw the line IK , and several lines parallel to, and on both sides of it. The angle LMN is called the angle of proportion or reduction. 4. Draw the diagonal lines AC , AD , AE , BD . 5. Take the distance BC , and set it off from M towards L on ML . 6. Measure it's corresponding line CK . 7. From b describe the arc no . 8. Now take AC , set it off on ML , and find it's correspondent line fg . 9. From a , with the radius fg , cut the former arc no in c , and thus proceed till you have completed the figure.

PROBLEM 70. *To enlarge a figure by the angle of reduction.* Let $abcde$ be the given figure, and AB the given side, fig. 3, 1, and 4, plate 8.

6

1. Form,

1. Form, as in the preceding problem, the angle LMN, by setting off ab from M to H, and from H with AB, cutting MN in I. 2. Draw HI, and parallels to, and on both sides of it. 3. Take the diagonal bc, set it off from M towards L, and take off it's corresponding line qr. 4. With qr as a radius, on B describe the arc mn. 5. Take the same correspondent line to ac, and on A cut mn in c, and so on for the other sides.

CURIOUS PROBLEMS ON THE DIVISION OF LINES AND CIRCLES.

PROBLEM 71. *To cut off from any given arch of a circle a third, a fifth, a seventh, &c. odd parts, and thence to divide that arch into any number of equal parts, fig. 7, plate 8.*

Example 1. To divide the arch AKB into 3 equal parts, CA being the radius, and C the center of the arch.

Bisect AB in K, draw the two radii CK, CB, and the chord AB; produce AB at pleasure, and make BL equal AB; bisect AC at G; then a rule on G and L will cut CB in E, and BE will be $\frac{1}{3}d$, and CE $\frac{2}{3}ds$ of the radius CB; on CB with CE describe the arc Eed: lastly, set off the extent Ee or De from B to a, and from a to b, and the arc AKB will be divided into three equal parts.

Corollary. Hence having a sextant, quadrant, &c. accurately divided, $\frac{1}{2}$ the chord of any arc set off upon any other arc of $\frac{1}{2}$, that radius will cut off an arc similar to the first, and containing the same number of degrees.

Also $\frac{1}{3}d$, $\frac{1}{4}th$, $\frac{1}{5}th$, &c. of a larger chord, will constantly cut similar arcs on a circle whose radius is $\frac{1}{3}th$, $\frac{1}{4}th$, $\frac{1}{5}th$, &c. of the radius of the first arch.

2. Let it be required to divide the arc AKB into 5 equal parts, or to find the $\frac{1}{5}th$ part of the arc AB.

Having

Having bisected the given arc AB in K , and drawn the three radii CA , CK , CB , and having found the 5th part of IB , of the radius CB , with radius CI describe the arc InM , which will be bisected in n , by the line CK ; then take the extent In , or it's equal Mn , and set it off twice from A to B , that is, first, from A to d , and from d to o , and oB will be one fifth of the arc AB . Again, set off the same extent from B to m , and from m to c , and the arch AB will be accurately divided into five equal parts.

3. To divide the given arc AB into seven equal parts. AB being bisected as before, and the radii CA , CK , CB drawn, find by problem 9, the seventh part of PB , of the radius CB , and with the radius CP describe the arc PrN ; then set off the extent Pr twice from A to 3, and from 3 to 6, and $6B$ will be the 7th part of the given arc AB ; the compasses being kept to the same opening Pr , set it from B to 4, from 4 to 1; then the extent $A1$ will bisect 1, 3 into 2, and 4, 6 into 5, and thus divide the given arch into 7 equal parts.

PROBLEM 72. *To inscribe a regular heptagon in a circle, fig. 8, plate 8.*

In fig. 8, let the arch BD be one eighth part of the given circle, and AB the radius of the circle. Divide AB into 8 equal parts, then on center A , with radius AC , describe the arc CE , bisect the arc BD in a , and set off this arc Ba from C to b , and from b to c ; then through A and c , draw Ace cutting BD in e , and Be will be $\frac{1}{7}$ th part of the given circle.

Corollary 1. Hence we have a method of finding the 7th part of any given angle; for, if from the extremities of the given arch, radii be drawn to the center, and one of these be divided into 8 equal

equal parts, and seven of these parts be taken, and another arch described therewith, the greater arch will be to the lesser as 8 to 7, and so of any other proportion.

Corollary 2. If an arch be described with the radius, it will be equal to the $\frac{1}{5}$ part of a circle whose radius is AB , and to the seventh part of a circle whose radius is AC , and to the sixth part of a circle whose radius is AL , &c.

Corollary 3. Hence also a pentagon may be derived from a hexagon. Let the given circle be, *fig. 9, plate 8*, $ABCDEF$, in which a pentagon is to be inscribed; with the radius AC set off AF equal $\frac{1}{5}$ th of the circle, divide AC into 6 equal parts; then CG will be 5 of these parts; with radius CG describe the arc GH , bisect AF in q , and make GP and PH each equal to Aq ; then through C and H draw the semi-diameter Cw , cutting the given circle in w , join Aw , and it will be one side of the required pentagon.

Corollary 4. Hence as radius divides a circle into 6 equal parts, each equal 60 degrees, twice radius gives 120 degrees, or the third part of the circumference.

Once radius gives 60 degrees, and that arc bisected gives 30 degrees, which added to 60, divides the circumference into 4 equal parts; whence we divide it into 2, 3, 4, 5, 6 equal parts; the preceding corollary divides into 5 equal parts, the arch of a quadrant bisected, divides it into 8 equal parts. By problem 71, we obtain the 7th part of a circle, and by this method divide it into any number of equal parts, even a prime number; for the odd unit may be cut off by the preceding problem, and the remaining part be subdivided by continual bisection

bisection till another prime number arises to be cut off in the same manner.

PROBLEM 73. *To divide a given right line, or an arch of a circle into any number of equal parts, by the help of a pair of beam or other compasses, the distance of whose points shall not be nearer to each other than the given line.*

From this problem, published by Clavius the Jesuit, in 1611, in a treatise on the construction of a dialling instrument, it is clear, that he was the original inventor of that species of division called nonius, and which many moderns have, with an affectation of learning, called vernier.

Let AB , *fig. 11, plate 8*, be the given line or circular arch, to be divided into a number of equal parts. Produce them at pleasure. Then take the extent AB , and set it off on the prolonged line, as many times as the given line is to be divided into smaller parts, BC , CD , EF , FG . Then divide the whole line AG into as many equal parts as are required in AB , as GH , HI , IK , KL , LA , each of which contains the given line, and one of those parts into which the given line is to be divided. For AG is to AL , as AF to AB ; in other words, AL is contained five times in AG , as AB in AF ; therefore, since AG contains AF , and $\frac{1}{5}$ of AF , AL will contain AB , and $\frac{1}{5}$ of AB ; therefore, BL is the $\frac{1}{5}$ of AB . Then as GH contains AB plus, FH which is $\frac{1}{5}$ of AB , EL will be $\frac{2}{5}$ of AB , DK $\frac{3}{5}$, CL $\frac{4}{5}$. Therefore, if we set off the interval GH from F and H , we obtain two parts at L and I , set off from the points near E , gives three parts between DK , from the four points at D and K , gives 4 parts at CL , and the next setting off one more of these parts; so that lastly, the extent

extent GH set off from the points between C and L , divides the given line as required.

To divide a given line AC , or arch of any circle, into any number of equal parts, suppose 30, *fig. 11, plate 8.*

1. Divide it into any number of equal parts less than 30, yet so that they may be aliquot parts of 30; as for example, AG is divided into 6 equal parts, AB, BC, CD, DE, EF, FG , each of which are to be subdivided into 5 equal parts. 2. Divide the first part AB into 5 parts, by means of the interval AL or GH , as taught in this problem. If now one foot of the compasses be put into the point A , (the extent AL remaining between them unaltered,) and then into the point next to A , and so on to the next succeeding point, the whole line AG will be divided by the other foot of the compasses into thirty equal parts.

Or if the right line or arch be first divided into five equal parts, each of these must be subdivided into 6 parts, which may be effected by bisecting each part, and then dividing the halves into 3 parts.

Or it may be still better to bisect 3 of the first five parts, and then to divide 4 of these into 3, which being set off from every point, will complete the division required.

COROLLARY. It frequently happens that so many small divisions are required, that notwithstanding their limited number, they can be hardly taken between the points of the compasses without error. In this case, use the following method.

If the whole number of smaller parts can be subdivided, take so many of the small parts in the given line, as each is to be subdivided into, yet so that they may together make up the whole

H

of

of the given line. For if the first of these parts be cut into as many smaller parts as the proposed number requires every one of them to contain, and the same is also done in the remaining parts, we shall obtain the given number of smaller parts.

If 84 parts are to be taken in the proposed line, first bisect it, and each half will contain 42; bisect these again, and you have 4 parts, each of which are to contain 21; and these divided into 3, give 12 parts, each of which are to contain 7 parts; subdivide these into 7 each.

But if the proposed number of small parts cannot be thus subdivided, it will be necessary to take a number a little less or greater, that will be capable of subdivision; for if the superfluous parts are rejected, or those wanting, added, we shall obtain the proposed number of parts. Thus if 74 parts are to be cut from any given line of 80 parts. 1. Bisect the given line, and each $\frac{1}{2}$ will contain 40. 2. Bisect these again, and you have 4 parts to contain 20 each. 3. Each of these bisected, you have 8 parts to contain 10 each. 4. Bisect these, and you obtain 16 parts, each to be divided into 5 parts. 6. Reject 6 of the parts, and the remainder is the 74 parts proposed.

Or if 72 parts be proposed, divide the line into 24 equal parts, and each of these into 3 parts, and you obtain 72; to which adding two, you will obtain the number of 74.

PROBLEM 74. *To cut off from the circumference of any given arch of a circle, any number of degrees and minutes.*

1. Let it be proposed to cut off from any arc 57 degrees; with the radius of the given arc or circle describe a separate arch as A B, *fig. 3, plate 9*, and having set off the radius from A to C, bisect

A C

A C in E, then A E and E C will be each of them an arc of 30 degrees. Make C B equal to A E, and A B will be a quadrant or 90 degrees, and will also be divided into 3 equal parts; next divide each of these into 3 by the preceding rules, and the quadrant will be divided into 9 equal parts, each containing 10 degrees. Lastly, divide the first of these into 10 degrees, then set one foot of the compasses into the 7th single degree, and extend the other to the 50th, and the distance between the points of the compasses will contain 57 degrees, which transfer to the given arc. Or at two operations, first take 50 degrees, and then from the first 10 take 7, and you have 57 degrees.

2. Let it be required to cut off from any given arc of a circle, 45 degrees, 53 minutes.

Divide the arc of 53 degrees of the quadrant A B, *fig. 3, plate 9*, whose radius is A C (or rather it's equal arc F G) into 60 equal parts, first into 5, and then one of these into 3; or first into 3, and one of these into 5. Again, one of these bisected, and this bisection again bisected, gives the 60th part of an arc of 53 degrees.

For the 5th part of the arc F G is F H, containing 12 parts; it's third part is F I, containing 4 parts; the $\frac{1}{2}$ of F I is N, which contains 2 parts; and F N again bisected in K, leaves F the 60th part of the arc F G; consequently F K comprehends 53 minutes; therefore, add the arc F K to 45 degrees, and the arc A F will contain $45^{\circ} 53'$.

Corollary. If we describe a separate arch L M with the radius A C, and set off thereon the extent L M of 61 degrees of the given arch A B, and divide L M into 60 equal parts; thus, first into 2, then both of these into 3, and then the first of these 3 into 10 by the former rules, which gives the 60th

part of the arc LM. And as one division of the arc LM contains by construction one degree of the quadrant AB, and one sixtieth part of a degree more, that is, one minute; therefore, two divisions of LM contain two degrees, and two minutes over; 3 divisions exceed three degrees by three minutes, and so of all the rest.

Whence if one division of LM be set off from any degree on AB, it will add one minute to that degree, 2 adds two minutes, 3 three minutes, and so on.

When the division is so small that the compasses will hardly take it in without error, take two, three, four, or more of the parts on L, set them off from as many degrees back from the degree intended, and you will obtain the degree and minute required.

PROBLEM 75. *To divide a circle into any uneven number of equal parts.*

Example 1. Let it be required to divide a circle into 346 $\frac{2}{3}$ equal parts.

Reduce the whole into 3d parts, which gives us 1040; find the greatest multiple of 3 less than 1040, which may be bisected; this number will be found in a double geometrical progression, whose first term is 3, as in the margin; 768 the ninth number, is the number sought, as in the margin. Subtract 768 from 1040, the remainder is 272, then find how many degrees and minutes this remainder contains by the rule of three. As 1040 is to 360 degrees, so is 272 to 94° 9' 23". Now set off 94° 9' 23" upon the circle to be divided, and divide the remaining part of that circle by continual bisections, till you come to the number 3, which will be one of the required divisions

3	2
6	2
12	2
24	2
48	2
96	2
192	2
384	2
768	2

of

of the 346 equal parts, by which dividing the arc of $94^{\circ} 9' 23''$ you will have the whole circle divided into $346\frac{2}{3}$ equal parts; for there will be 256 divisions in the greatest arc, and $9\frac{1}{3}$ in the other.

Example 2. Let it be required to divide a circle into 179 equal parts. Find the greatest number not exceeding 179, which may be continually bisected to unity, which you will find to be 128. Subtract 128 from 179, the remainder is 51; then find what part of the circle this remainder will occupy by the following proportion, as 179 is to 360, so is 51 to $102^{\circ} 34' 11''$; set off from the circle an arc of $102^{\circ} 34' 11''$, and divide the remaining part of the circle by continual bisections, seven of which will be unity in this example; by which means this part of the circle will be divided into 128 equal parts, and the remaining 51 may be obtained, by using as many of the former bisections as the space will contain, so that the whole circumference will be divided into 179 equal parts.

Example 3. Let it be required to divide a circle into $29\frac{1}{2}$ equal parts, to represent the days of the moon's age.

Reduce the given number into halves, which gives 59 parts; seek the greatest number, not exceeding 59, which may be continually bisected to unity, which you will find to be 32. Subtract this from 59, the remainder is 27; and find, as before, the angle equal to the remainder by this proportion, as 59 is to 360, so is 27 to $164^{\circ} 44' 44''$; set off $164^{\circ} 44' 44''$, divide the remaining part of the circle by continual bisections, which will divide this portion into 32 parts, and from that, the rest into $27\frac{1}{2}$ equal parts.

Example 4. Let it be required to divide a circle

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into

into $365^{\circ} 5' 49''$ equal parts, the length of a tropical year.

Reduce the whole into minutes, which will be 525949; then seek the greatest multiple of 1440, the minutes in a solar day, that may be halved, and is at the same time less than 525949; this you will find to be 368640, which subtracted from 525949, leaves 157309. To find the number of degrees that is to contain this number, use the following proportion; as 525949 is to 157309, multiplied by 360, so is 157309 to $107^{\circ} 40' 27'' 49'''$.

Now set off an angle of $107^{\circ} 40' 27'' 49'''$ upon the circle to be divided, and divide the remaining part of that circle by continual bisections, till you come to the number 1440, which in this case is unity, or one natural solar day; by which, dividing the arc of $107^{\circ} 40' 27'' 49'''$, the whole circle will be divided into $365^{\circ} 5' 49''$; for there will be 256 divisions or days in the greater arc, and $109^{\circ} 5' 49''$ in the lesser arc.

Example 5. Let it be required to divide a circle into $365\frac{1}{4}$ equal parts, which is the quantity of a Julian year.

Reduce the whole into four parts, which gives us 1461; 1024 is the greatest multiple of 2, less than 1461; when subtracted from 1461, we have for a remainder 437. Then by the following proportion, as 1461 is to 437×360 , so is 407 to $107^{\circ} 40' 46'' 49'''$, the degrees to be occupied by this remainder.

Set off an angle of $107^{\circ} 40' 47''$ upon the circle to be divided, and divide the remainder by continual bisections, until you arrive at unity, by which dividing the arc, you will have the whole circle divided in $365\frac{1}{4}$ parts.

PROBLEM

PROBLEM 76. *To divide a quadrant or circle into degrees.*

Let A B, fig. 10, plate 8, be the quadrant, C the center thereof. With the radius A C describe the two arcs A D, B E, and the quadrant will be divided into three equal parts, each equal to 30 degrees; then divide each of these into 5 parts, by the preceding rules, and the quadrant is divided into 15 equal parts; bisect these parts, and then subdivide as already directed, and the quadrant will be divided into 90 degrees. Other methods will be soon explained more at large.

PROBLEM 77. *To find what part any smaller line or arch is of a greater, as for example, any angle is of a semicircle.*

Take the smaller with a pair of compasses, and with this opening step the greater. With the remainder or surplus, step one of the former steps; with the remainder of this, step one of the last steps, setting down the number of steps each time. About five times will measure angles to 5 seconds.

Then to find the fraction, expressing what part of the whole the smaller part is,

Suppose the number of steps each time to be e. d. c. b. a.

9. 7. 8. 2. 5. Then $5 \times 2 + 1 = 11$, and $11 \times 8 + 5 = 93$, and $93 \times 7 + 11 = 662$, and $662 \times 9 + 93 = 6051$; so that $\frac{6051}{662}$ is the fraction required.

If we call the number of steps a b c d e beginning at the last, the rule may run thus: multiply a by b, and add 1; multiply that sum by c, and add a; multiply this sum by d, and add the preceding sum; multiply this sum by e, and add the preceding sum; then the two last sums are the terms of the fraction.

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Example

d. c. b. a.

Example 1. Suppose the steps are 3. 5. 1. 9. then $9 \times 1 + 1 = 10$, $10 \times 5 + 9 = 59$, and $59 \times 3 + 10 = 187$; hence the terms are $\frac{187}{187}$. Now $180^\circ \times 59 = 10620$, this divided by 187 gives 56° , with a remainder of 148, $148 \times 60 = 8880$, $\frac{8880}{187}$, gives 47, &c. so that the measure required is $56^\circ 47' 29''$.

Example 3. Take a semicircle 3 inches radius, and let the angle be 2 in 1. Then the steps will be 4. 2. 1. 2. 3. 2. the answer $41^\circ 10' 50'' 9'''$.

The whole circle continued and stepped with the same opening, gave 8. 1. 3. 2. 3. for the same, yet the answers agreed to the tenth of a second.

By the same method any given line may be measured, and the proportion it bears to any other strait line found. Or it will give the exact value of any strait line, ex. gr. the opening of a pair of compasses by stepping any known given line with it, and this much nearer than the eye can discern, by comparing it with any other line, as a foot, a yard, &c.

This method will be found more accurate than by scales, or even tables of sines, tangents, &c. because the measure of a chord cannot be so nicely determined by the eye, with extreme exactness.

There may be some apparent difficulty attending the rule when put in practice, it being impossible to assign any example, which another person can repeat with perfect accuracy, on account of the inequality in the scales, by which the same steps or line will be measured by different persons. There will, therefore, be always some small variation in the answer; it is, however, demonstrably true, that the answer given by the problem is most accurately the measure of the given angle, although you can never delineate another angle or line exactly equal

to the given one, first measured by way of example, and this arising from the inequality of our various scales, our inattention in measuring, and the imperfection of our eyes. Hence, though to all appearance two angles may appear perfectly equal to each other, this method will give the true measure of each, and assign the minutest difference between them.

Figure 1, plate 9, will illustrate clearly this method; thus to measure the angle A C B, take A B between your compasses, and step B a, a b, b c, there will be c D over.

Take c D, and with it step A e, e f, and you will have f b over; with this opening step A g, g h, and you will have h e over, and so on.

PROBLEM 78. *To divide a large quadrant or circle.*

We shall here give the principal methods used by instrument-makers, before the publication of Mr. Bird's method by the Board of Longitude, leaving it to artists to judge of their respective merits, and to use them separately, or combine them together, as occasion may require; avoiding a minute detail of particulars, as that will be found when we come to describe Mr. Bird's method. It will be necessary, however, previously to mention a few circumstances, which, though in common use, had not been described until Mr. Bird's and Mr. Ludlam's comment thereon were published.

"In all mathematical-instruments divided by hand, and not by an engine or pattern, the circles or lines which bound the divisions, are not those which are actually divided by the compasses."

"A faint circle is drawn very near the bounding circle;

circle; it is this that is originally divided. It has been termed the *primitive circle*."

"The divisions made upon this circle are faint arches, struck with the beam compasses; fine points or conical holes are made by the prick punch or pointing tool, at the points where these arches cross the primitive circle; these are called *original points*."

"The *visible divisions* are transferred from the original points to the space between the bounding circles, and are cut by the beam compasses; they are therefore always arches of a circle, though so short, as not to be distinguished from strait lines."

Method 1. The faint or primitive arch is first struck; the exact measure of the radius thereof is then obtained upon a standard scale with a nonius division of 1000 parts of an inch, which if the radius exceeds 10 inches, may be obtained to five places of figures. This measure is the chord of 60. The other chords necessary to be laid off are computed by the subjoined proportion,* and then taken off from the standard scale to be laid down on the quadrant.

Set off the chord of 60° , then add to it the chord of 30, and you obtain the 90th degree.

Mr. Bird, to obtain 90° , bisects the chord of 60° , and then sets off the same chord from 30 to 90° , and not of 30 from 60° to 90° . Some of the advantages that arise from this method are these; for whether the chord of 30 be taken accurately or not from the scale of equal parts, yet the arch of 60 will be truly bisected, (see remarks on bisection hereafter); and if the radius unaltered be set off from the point of bisection, it will give 90 true; but if the chord 30,

as

* As radius to the given angle, so is the measure of radius to half the required chord.

as taken from the scale, be laid off from 60 to 90, then an error in that chord will make an equal error in the place of 90°.

60° is divided into 3 parts, by setting off the computed chord of 20 degrees, and the whole quadrant is divided to every 10 degrees, by setting off the same extent from the other points.

30° bisected by the computed chord of 15, gives 15°, which stepped from the points already found, divides the quadrant to every fifth degree.

The computed chord of 6° being laid off, divides 30 degrees into five parts; and set off from the other divisions, subdivides the quadrant into single degrees.

Thus with five extents of the beam compasses, and none of them less than 6 degrees, the quadrant is divided into 90 degrees.

15° bisected, gives 7° 30', which set off from the other divisions, divides the quadrant into half degrees.

The chord of 6° 40' divides 20° into 3 parts, and set off from the rest of the divisions, divides the whole instrument to every ten minutes.

The chord of 10° 5' divides the degrees into 12 parts, each equal to 5 minutes of a degree.

Method 2. The chords are here supposed to be computed as before, and taken off from the nonius scale.

1. Radius bisected, divides the quadrant into 3 parts, each equal to 30 degrees.

2. The chord of 10, gives 9 parts, each equal to 10 degrees.

3. 30° bisected and set off, gives 18 parts, each equal to 5 degrees.

4. 30° into 5, by the chord of 6°; then set off as before, gives 90 parts, each equal to 1 degree.

5. The

5. The chord of $6^{\circ} 40'$ gives 270 parts, each equal to 20 minutes.

6. The chord of $7^{\circ} 30'$ gives 540 parts, each equal to 10 minutes.

7. The chord of $7^{\circ} 45'$ gives 1080 parts, each equal to 5 minutes.

Or, 3d, the computed chords supposed,

1.	60	into $\frac{1}{2}$	gives	3 parts,	equal 30°
2.	30	$\frac{1}{2}$	equal 15	6	15 $^{\circ}$
3.	15	3	equal 5	18	5 $^{\circ}$
4.	20	5	equal 6	90	1 $^{\circ}$
5.	20	3	equal $6^{\circ} 40'$	270	20'
6.	15	$\frac{1}{2}$	equal $7^{\circ} 30'$	540	10'
7.	15.30	$\frac{1}{2}$	equal $7^{\circ} 45'$	1080	5'

Thus may the practitioner vary his numbers for any division whatsoever, and yet preserve a sufficient extent between the points of his compasses.

If the quadrant be divided as above to every 15 degrees, and then the computed arch of 16 degrees set off, this arch may be divided by continual bisection into single degrees. If from the arch of 45° , $2^{\circ} 20'$ be taken, or $1^{\circ} 10'$ from $22^{\circ} 30'$, we may obtain every 5th minute by continual bisection. If to the arch of $7^{\circ} 10'$ be added the arch of 62 minutes, the arch of every single minute may be had by bisection.

OF MR. BIRD'S METHOD OF DIVIDING.

Fig. 2, plate 9.

In 1767, the Commissioners of Longitude proposed an handsome reward to Mr. Bird, on condition, among other things, that he should publish an account of his method of dividing astronomical instruments; which was accordingly done; and a tract describing his method of dividing, was writ-

ten by him, and published by order of the Commissioners of Longitude in the same year; some defects in this publication were supplied by the Rev. Mr. Ludlam, one of the gentlemen who attended Mr. Bird, to be instructed by him in his method of dividing, in consequence of the Board's agreement with him. Mr. Ludlam's tract was published in 1787.

I shall use my endeavours to render this method still clearer to the practitioner, by combining and arranging the subject of both tracts.

MR. BIRD'S METHOD.

1. One of the first requisites is a scale of inches, each inch being subdivided into 10 equal parts.
2. Contiguous to this line of inches, there must be a nonius, in which 10.1 inches is divided into 100 equal parts, thus shewing the thousandth part of an inch. By the assistance of a magnifying glass of one inch focus, the 30000th part may be estimated.
3. Six beam compasses are necessary, furnished with magnifying glasses of not more than one inch focus. The longest beam is to measure the radius or chord of 60; the second for the chord of 42.40; the third for the chord of 30; the 4th for 10.20; the 5th for 4.40; the 6th for the chord of 15 degrees.
4. Compute the chords by the rules given, and take their computed length from the scale in the different beam compasses.
5. Let these operations be performed in the evening, and let the scale and the different beam compasses be laid upon the instrument to be divided, and remain there till the next morning.
6. The

6. The next morning, before sun-rise, examine the compasses by the scale, and rectify them, if they are either lengthened or shortened by any change in the temperature of the air.

7. The quadrant and scale being of the same temperature, describe the faint arc *bd*, or primitive circle; then with the compasses that are set to the radius, and with a fine prick punch, make a point at *a*, which is to be the 0 point of the quadrant; see *fig. 2, plate 9*.

8. With the same beam compass unaltered, lay off from *a* to *e* the chord of 60° , making a fine point at *e*.

9. Bisect the arc *ae* with the chord of 30° .

10. Then from the point *c*, with the beam compasses containing 60, mark the point *r*, which is that of 90° degrees.

11. Next, with the beam compasses containing 15° , bisect the arc *er* in *n*, which gives 75° .

12. Lay off from *n* towards *r*, the chord of $10^\circ 20'$, and from *r* towards *n* the chord of $4^\circ 40'$; these two ought to meet exactly at the point *g* of $85^\circ 20'$.

13. Now as in large instruments each degree is generally subdivided into 12 equal parts of 5 minutes each, we shall find that $85^\circ 20'$ contains 1024 such parts, because $20'$ equal 4 of these parts, and 85×12 makes 1020; now 1024 is a number divisible by continual bisection.

The last computed chord was $42^\circ 40'$, with which *ag* was bisected in *o*, and *ao*, *og*, were bisected by trials. Though Mr. Bird seems to have used this method himself, still he thinks it more advisable to take the computed chord of $21^\circ 20'$, and by it find the point *g*; then proceed by continual bisections till you have 1024 parts. Thus the

are $85^{\circ} 20'$, by ten bisections, will give us the arcs $42^{\circ} 40'$, $21^{\circ} 20'$, $10^{\circ} 40'$, $5^{\circ} 20'$, $2^{\circ} 40'$, $1^{\circ} 20'$, $40'$, $20'$, $10'$, $5'$.

14. To fill up the space between g and r , $85^{\circ} 20'$, and 90° , which is $4^{\circ} 40'$, or $4 \times 12 + 8$ equal to 56 divisions; the chord of 64 divisions was laid off from g towards d , and divided like the rest by continual bisections, as was also from a towards b . If the work is well performed, you will again find the points 30, 45, 60, 75, and 90, without any sensible difference. It is evident that these arcs, as well as those of 15° , are multiples of the arc of $5'$; for one degree contains 12 arcs of $5'$ each, of which 15° contains 180; the arc of 30° contains 360; the arc of 60° , 720; that of 75° , 900, and therefore 90° contains 1080.

Mr. Graham, in 1725, applied to the quadrant divided into 90° , or rather into 1080 parts of 5 minutes each, another quadrant, which he divided into 96 equal parts, subdividing each of these into 16 equal parts, forming in all 1536. This arc is a severe check upon the divisions of the other; but Bird says, that if his instructions be strictly followed, the coincidence between them will be surprising, and their difference from the truth exceeding small.

The arch of 96 is to be divided first into 3 equal parts, in the same manner as the arc of 90° ; each third contains 512 divisions, which number is divisible continually by 2, and gives 16 in each 96th part of the whole.

The next step is to cut the linear divisions from the points obtained by the foregoing rules. For this purpose a pair of beam compasses is to be used, both of whose points are conical and very sharp. Draw a tangent to the arc bd , suppose at e , it will intersect the arc xy in q , this will be the distance between the points of the beam compass to cut the

the divisions nearly at right angles to the arc. The point of the beam compasses next the right hand is to be placed in the point *r*, the other point to fall freely into the arc *xy*; then pressing gently upon the screw head which fastens the socket, cut the divisions with the point toward the right hand, proceeding thus till you have finished all the divisions of the limb.

FOR THE NONIUS.

1. Chuse any part of the arch where there is a coincidence of the 90 and 96 arches; for example, at *e*, the point of 60° . Draw the faint arc *st*, and *ik*, which may be continued to any length towards *A*; upon these the nonius divisions are to be divided in points. The original points for the nonius of the 90th arch, are to be made upon the arc *st*; the original points for the nonius of the 96th arch, are to be made upon the arc *ik*.

Because 90 is to 96, as 15 to 16, there will be a coincidence at 15° and 16pts, 30° and 32pts, 45° and 48pts, 60° and 64pts, 90° and 96pts.

2. Draw a tangent line to the primitive circle as before, intersecting the arc *d*, which gives the distance of the points of the beam compass, with which the nonius of the 90th arc must be cut.

3. Let us suppose then that the nonius is to subdivide the divisions of the limb to half a minute, which is effected by making 10 divisions of the nonius equal to 11 divisions of the limb; measure the radius of the arc, and compute the chord of 16, or rather 32 of the nonius division, which may easily be obtained by the following proportion: if 10 divisions of the nonius plate make 55 minutes of a degree, what will 32 of those divisions make? the answer is $2^\circ 56'$, the chord of which must

must be computed, and taken from the scale of equal parts; but as different subdivisions by the nonius may be required, let n be the number of nonius divisions, m the number of minutes taken in by the nonius b , 16, 32 or 64, and X the arc sought; then as $n:m :: b:x$.

4. Lay off with the beam compass, having the length of the tangent 90 between the points, the point q from e , q being a point in the arc st , and e an original point in the primitive circle, and the chord of 32 from q towards the left hand, (the chord of 32 being the chord which subtends 32 divisions on the nonius plate, or the chord of $2^\circ 56'$; this chord to be computed from the radius with which the faint arc st was struck, and taken off the scale of equal parts,) and divide by continual bisections; ten of those divisions, counting from q to the left, will be the required points.

The nonius belonging to the 96 arch is subject to no difficulty, as the number should always be 16, 32, &c. that the extremes may be laid off from the divisions of the limb without computation. To be more particular, the length of the tangent line or radius with which the divisions of the nonius of the 96 arch are to be cut, must be found in the way before directed for the nonius of the 90 arch; the arch ik standing instead of the arch st . Having the tangential distance between the points of the compass from one of the original points in the primitive 96, lay off a point on the faint arc ik towards the left hand; count from that point on the primitive 96 circle 17 points to the left hand, and lay off from thence another point on the faint arch ik ; the distance between those two points in the faint arch ik is to be subdivided

I by

by bisections into 16 parts, and those parts pointed; from these points the visible divisions of the nonius are to be cut.

Mr. Ludlam thinks, that instead of laying the extent of the nonius single, it would be better to lay it off double or quadruple; thus instead of 17 points to the left hand, count 34 or 68, and that both ways to the right, and to the left, and lay off a point from each extreme on the faint arch ik; subdivide the whole between these extreme points by continual bisections, till you get 16 points together on the left hand side the middle, answering to the extent of the nonius. No more of the subdivisions are to be completed than are necessary to obtain the middle portion of 16 points as before.

The nonius points obtained, *the next process* is to transfer them on the nonius plate, which plate is chamfered on both edges; on the inner edge is the nonius for the 90 arch, on the outer edge is the nonius for the 96 arch, in the middle between the two chamfers is a flat part parallel to the under surface of the nonius plate; upon the flat part the faint line now going to be mentioned, is to be drawn. *To find the place where the nonius is to begin upon the chamfered edge of the nonius plate*, measure the distance of the center of the quadrant from the axis of the telescope; this distance from the axis of the telescope at the eye end, will be the place for the first division of the nonius; then draw a faint line from the center on the flat part of the nonius plate.

Fasten the nonius plate to the arch with two pair of hand vices; then with one point of a beam compass, in the center of the quadrant, and the other at the middle of the nonius plate, draw a faint arch from end to end; where this arch cuts the faint

faint line before-mentioned, make a fine point; from this point lay off on each side another point, which may be at any distance in the arch, only care must be taken that they be equally distant from the middle point; from the two last make a faint intersection as near as possible to either of the chamfered edges of the nonius plate; through this intersection the first division of the nonius must be cut.

MR. BIRD'S METHOD OF DIVIDING HIS SCALE OF
EQUAL PARTS.

Let us suppose that we have 90 inches to divide into 900 equal parts, take the third of this number, or 300; now the first power of 2 above this is 512, therefore take $\frac{1}{512}$ th of an inch in one pair of beam compasses, $\frac{2}{512}$ in another, $\frac{4}{512}$ in a third, and $\frac{8}{512}$ in a fourth; then lay the scale from which these measures were taken, the scale to be divided, and the beam compasses near together, in a room facing the north; let them lie there the whole night; the next morning, correct your compasses, and lay off $\frac{1}{512}$ three times; then with the compasses $\frac{2}{512}$, $\frac{4}{512}$, $\frac{8}{512}$, bisect these three spaces as expeditiously as possible; the space 64 is so small that there is no danger from any partial or unequal expansion, therefore the remainder may be finished by continual bisections. The linear divisions are to be cut from the points with a beam compass as before described.

The nonius of this scale is $\frac{1}{100}$ th of an inch long, which is to be divided into 100 equal parts, as 100 is to 101, so is 256 : 258,56 tenths of an inch, the integer being $\frac{1}{10}$. Suppose the scale to be numbered at every inch from left to right; then make a fine point exactly against $\frac{1}{10}$, to the left of

1 2

0, from

o, from this lay off 258.56 to the right hand, which divide after the common method.

If you are not furnished with a scale long enough to lay off 258.56, then set off $\frac{240}{16}$, and add 8.56, from a diagonal scale.

OF MR. BIRD'S POINTING TOOL, AND METHOD OF POINTING.

The pointing tool consisted of a steel wire $\frac{1}{8}$ inch diameter, inserted into a brass wire $\frac{1}{4}$ inch diameter, the brass part $2\frac{1}{2}$ long, the steel part stood out $\frac{1}{4}$, whole length $3\frac{1}{4}$ inches. The angle of the conical point about 20 or 25 degrees, somewhat above a steel temper; the top of the brass part was rounded off, to receive the pressure of the finger; the steel point should be first turned, hardened, and tempered, and then whetted on the oil-stone, by turning the pointrel round, and at the same time drawing it along the oil-stone, not against, but from the point; this will make a sharp point, and also a kind of very fine teeth along the slant side of the cone, and give it the nature of a very fine countersink.

In striking the primitive circle by the beam compass, the cutting point raises up the metal a little on each side the arch; the metal so thrown up, forms what is called the bur. When an arch is struck across the primitive circle, this bur will be in some measure thrown down; but if that circle be struck again ever so lightly, the bur will be raised up again; the arches struck across the primitive circle have also their bur. Two such rasures or trenches across each other, will of course have four salient or prominent angles within, and as the sides of the trenches slope, so do also the lines which terminate the four solid angles. You may therefore

therefore *feel* what you cannot *see*; when the conical points bears against all four solid angles, they will guide, and keep the point of the tool in the center of decussation, while keeping the tool upright, pressing it gently with one hand, and turning it round with the other, you make a conical hole, into which you can at any time put the point of the beam compass, and feel, as well as see, when it is lodged there.

RULES OR MAXIMS LAID DOWN BY MR. BIRD.

1. The points of the beam compasses should never be brought nearer together than 2 or 3 inches, except near the end of the line or arch to be divided; and then spring dividers with round moveable points may be used.

2. The prick punch used to mark the points, should be very sharp and round, the conical point being formed to a very acute angle; the point to be made by it ought not to exceed the one thousandth of an inch. When lines or divisions are to be traced from these points, a magnifying glass of $\frac{1}{2}$ an inch focus must be used, which will render the impression or scratch made by the beam compasses sufficiently visible; and if the impression be not too faint, feeling will contribute as well as seeing towards making the points properly.

3. The method of finding the principal points by computing the chords, is preferable to other methods; as by taking up much less time, there is much less risk of any error from the expansion of the instrument or beam compasses.

4. To avoid all possible error from expansion, Mr. Bird never admitted more than one person, and him only as an assistant; nor suffered any fire

in the room, till the principal points were laid down.

Mr. Bird guards, by this method, against any inequality that might possibly happen among the original points, by first setting out a few capital points, distributed equally through the arch, leaving the intervals to be filled up afterwards; he could by this method check the distant divisions with respect to each other, and shorten the time of the most essential operations.

5. Great care is to be observed in pointing intersections, which is more difficult, than in pointing from a single line made by one point of the compass. For in bisections, the place to be pointed is laid off from the right to the left, and from the left to the right. If any error arises from an alteration of the beam compass, it will be shewn double; even if the chord be taken a little too long, or too short, it will not occasion any inequality, provided the point be made in the middle, between the two short lines traced by the compass.

Now as Mr. Ludlam observes, if the bisecting chord be taken exactly, the two fore-mentioned faint arches will intersect each other in the primitive circle, otherwise the intersection will fall above or below it. In either case the eye, assisted by a magnifier, can accurately distinguish on the primitive circle, the middle between these two arches, and a point may be made by the pointing tool.

In small portions of the primitive circles the two faint arches will intersect in so acute an angle, that they will run into one another, and form as it were a single line; yet even here, though the bisecting chord be not exact, if the intersection be
pointed

pointed as before, the point will fall in the middle of the portion to be bisected.

If in the course of bisecting you meet with a hole already made with the pointrel, the point of the compass should fall exactly into that hole, both from the right hand and the left, and you may readily feel what you cannot see, whether it fit or no; if it fits, the points of the compass will have a solid bearing against the bottom of the conical hole, and strike a solid blow against it; if it does not exactly coincide with the center of the hole, the slant part of the point will slide down the slant side of the hole, drawing or pushing the other point of the compass from it's place.

Mr. Bird's method of transferring the divisions by the beam compass from the original points, is founded on this maxim, *that a right line cannot be cut upon brass, so as accurately to pass through two given points*; but that a circle may be described from any center to pass with accuracy through a given point. It is exceeding difficult in the first place to fix the rule accurately, and keep it firmly to the two points; and secondly, supposing it could be held properly, yet as the very point of the knife which enters the metal and ploughs it out, cannot bear against the rule, but some other part above that point will bear against it; it follows, that if the knife be held in a different situation to or from the rule, it will throw the cutting point out or in; besides this, any hardness or inequality of the metal will turn the knife out of it's course, for the rule does not oppose the knife in departing from it, and the force of the hand cannot hold it to it. For these reasons it is almost impossible to draw a knife a second time against the rule, and cut within the same line as before.

On the other hand, an arch of a circle may always be described by the beam compass so as to pass through a given point, provided both points of the beam compass be conical. Let one point of the compass be set in the given point or conical hole in the brass plane; make the other point, whatever be it's distance, the central or still point; with the former point cut the arch, and it will be sure to pass through the given point in the brass plane, and the operation may be repeated safely, and the stroke be strengthened by degrees, as the moving point is not likely to be moved out of it's direction, nor the cutting point to be broken.

The visible divisions on a large quadrant are always the arches of a circle, though so short as not to be distinguished from strait lines; they should be perpendicular to the arch that bounds them, and therefore the still or central point of the beam compasses must be somewhere in the tangent to that arch; the bounding circle of the visible divisions, and the primitive circle, should be very near each other, that the arch forming the visible divisions may be as to sense perpendicular to both circles, and each visible division shew the original point from which it was cut.

Another maxim of Mr. Bird's, attributed to Mr. Graham, *That it is possible practically to bisect an arch or right line, but not to trisect, querquisect, &c.* The advantages to be obtained by bisection have been already seen; we have now to shew the objections against trisecting, &c.

1. That as the points of trisection in the primitive circle, must be made by pressing the point of the beam compass down into the metal, the least exuberance or hard particle will cause a deviation in the first impression of a taper point, and force the
point

point of the beam compass out of it's place; when a point is made by the pointing tool, the tool is kept turning round while it is pressed down, and therefore drills a conical hole.

2. Much less force is necessary to make a scratch or faint arch, than a hole by a pressure downwards of the point of the compass.

3. So much time must be spent in trials, that a partial expansion would probably take place, and perhaps many false marks or holes made, which might occasion considerable error.

Another maxim of Mr. Bird's was this, that *stepping* was liable to great uncertainties, and not to be trusted; that is, if the chord of 16° was assumed, and laid down five times in succession, by turning the compasses over upon the primitive circle, yet the arches so marked would not, in his opinion, be equal.

DESCRIPTION OF MR. BIRD'S SCALE OF EQUAL PARTS.

It consists of a scale of inches, each divided into tenths, and numbered at every inch from the left to the right, thus, 0, 1, 2, 3, &c. in the order of the natural numbers. The nonius scale is below this, but contiguous to it, so that one common line terminates the bottoms of the divisions on the scale of inches, and the tops of the divisions on the nonius; this nonius scale contains in length 101 tenths of an inch, this length is divided into 100 equal parts or visible divisions; the left hand division of this scale is set off from a point $\frac{1}{10}$ of an inch to the left of 0, on the scale of inches; therefore, the right hand end of the scale reaches to, and coincides with the 10th inch on the scale of inches. Every tenth division on this nonius scale
is

is figured from the right to the left, thus, 100. 90. 80. 70. 60. 50. 40. 30. 20. 10. 0. and thus 0 on the nonius coincides with 10 on the inches; and 100 on the nonius falls against the first subdivision (of tenths) to the left hand of 0 on the inches; and these two, viz. the first and last, are the only two strokes that do coincide in the two scales.

To take off any given number of inches, decimals, and millesimals of an inch; for example, 42,764, observe, that one point of the beam compass must stand in a (pointed) division on the nonius, and the other point of the compass in a pointed division on the scale of inches.

The left hand point of the beam compass must stand in that division on the nonius which expresses the number of millesimal parts; this, in our example, is 64.

To find where the other point must stand in the scale of inches and tenths, add 10 to the given number of inches and tenths (exclusive of the two millesimal figures); from this sum subtract the two millesimal figures, considered now as units and tenths, and the remainder will shew in what division, on the scale of inches, the other point of the compass must stand; thus, in our example, add 10 to 42,7, and the sum is 52,7; from this subtract 6,4 and the remainder is 46,3. Set then one point of the compass in the 64th division on the nonius, and the other point in 46,3 on the scale of inches, and the two points will comprehend between them 42,764 inches.

This will be plain, if we consider that the junction of the two scales is at the 10th inch on the scale of inches; therefore, the beam compass will comprehend 36,3 inches on the scale of inches; but it will likewise comprehend 64 divisions on the nonius

nonius scale. Each of these divisions is one tenth and one millesimal part of an inch ; therefore, 64 divisions is 64 tenths, and 64 millesimal parts, or 6,464 inches ; to 6,464 inches taken on the nonius, add 36,3 inches taken on the scale of inches, and the whole length is 42,764 inches ; and thus the whole is taken from two scales, viz. inches and the nonius ; each subdivision in the former is $\frac{1}{10}$ of an inch, each subdivision in the latter is $\frac{1}{10} + \frac{1}{1000}$ th of an inch. By taking a proper number of each sort of subdivisions (the lesser and the greater) the length sought is obtained.

The business of taking a given length will be expedited, and carried on with far less danger of injuring the scale, if the proposed length be first of all taken, *nearly*, on the scale of inches only, guessing the millesimal parts ; thus, in our case, we ought to take off from the scale of inches 42,7, and above half a tenth more ; for then if one point be set in the proper division on the nonius, the other point will fall so near the proper division on the scale of inches, as to point it out ; and the point of the beam compass may be brought, by the regulating screw, to fall exactly into the true division.

If, when the points of the compass are set, they do not comprehend an integral number of millesimal parts, they will not precisely fall into any two divisions, but will either exceed, or fall short ; let the exact distance of the points of the compass be 42,7645 ; if, as before, the left hand point be set in the 64th division of the nonius, then the right hand point will exceed 46,3 among the inches ; if the left hand point be carried one division more to the left, and stand in 65 of the nonius, then the right hand point will fall short of 46,2 in the scale
 of

of inches; the excess, in the former case, being equal to the defect in the latter. By observing whether the difference be equal, or as great again, in one case as the other, we may estimate to $\frac{1}{3}$ part of a millesimal. See Mr. Bird's tract, p. 2.

It may be asked, why should the nonius scale commence at the 10th inch; why not at 0, and so the nonius scale lie wholly on the left hand of the scale of inches? and, in this case, both scales might lie in one right line, and not one under the other; but, in such a case, a less distance than ten inches could not always be found upon the scale, as appears from the rule before given. The number 10 must not, in this case, be added to the inches and tenths, and then the subtraction before directed would not always be possible.

Yet, upon this principle, a scale, in one continued line, may be constructed for laying off inches, tenths, and hundredths of an inch, for any length above one inch; at the head of the scale of inches, to the left hand of 0, and in the same line, set off eleven tenths of an inch (or the multiple), which subdivide, in Mr. Bird's way, into ten equal parts. Such a compound scale would be far more exact than the common diagonal scale; for the divisions being pointed, you may feel far more nicely than you can see, when the points of the beam compass are set to the exact distance. But to return to Mr. Bird's tract,

The nature of Mr. Bird's scale being known, there will be no difficulty in understanding his directions how to divide it. A scale of this kind is vastly preferable to any diagonal scale; not only on account of the extreme difficulty of drawing the diagonals exactly, but also because there is no check upon the errors in that scale; here the uni-

form manner in which the strokes of one scale separate from those of the other, is some evidence of the truth of both; but Mr. Bird's method of assuming a much longer line than what is absolutely necessary for the scale, subdividing the whole by a continual bisection, and pointing the divisions as before explained, and guarding against partial expansions of the metal, is sure to render the divisions perfectly equal. The want of such a scale of equal parts (owing, perhaps, to their ignorance of constructing it) is one reason why Mr. Bird's method of dividing is not in so great estimation, among mathematical instrument makers, as it justly deserves.

AN OBSERVATION, OR METHOD OF GRADUATION,
OF MR. SMEATON'S.

As it is my intention to collect in this place whatever is valuable on this subject, I cannot refrain from inserting the following remark of Mr. Smeaton's, though it militates strongly against one of Mr. Bird's maxims. He advises us to compute from the measured radius the chord of 16 degrees only, and to take it from an excellent plain scale, and lay it off five times in succession from the primary point of 0 given, this would give 80 degrees; then to bisect each of these arches, and to lay off one of them beyond the 80th, which would give the 88th degree; then proceed by bisection, till you come to an arch of two degrees, which laid off from the 88th degree, will give the ninety degrees; then proceed again by bisection, till you have reduced the degrees into quarters, or every fifteen minutes. Here Mr. Smeaton would stop, being apprehensive, that divisions, when over close, cannot be accurately obtained even by bisection.

If

If it were necessary to have subdivisions upon the limb equivalent to 5 minutes, he advises us to compute the chord of $21^{\circ} 20'$ only, and to lay it off four times from the primary point; the last would give $85^{\circ} 20'$, and then to supply the remainder from the bisected divisions as they rise, *not from other computed chords*.

Mr. Bird asserts, that after he had proceeded by the bisections from the arc of $85^{\circ} 20'$, the several points of 30. 60. 75. 60, *fell in without sensible inequality*, and so indeed they might, though they were not equally true in their places; for whatever error was in them would be communicated to all connected with, or taking their departure from them. Every heterogeneous mixture should be avoided.

It is not the same thing whether you *twice* take a measure as *nearly* as you can, and lay them off separately, or lay off two openings of the compasses in succession *unaltered*; for though the same opening, carefully taken off from the same scale a second time, will doubtless fall into the points made by the first, without sensible error; yet as the sloping sides of the conical cavities made by the first point, will conduct the points themselves to the center, there may be an error, which, though insensible to the sight, would have been avoided by the more simple process of laying off the opening twice, without altering the compasses.

As the whole of the 90 arch may now be divided by bisection, it is equally unexceptionable with the 96 arc; and consequently, if another arc of 90, upon a different radius, was laid down, they would be real checks upon each other.

Mr. Ramsden, in laying down the original divisions on his dividing engine, divided his circle first into 5 parts, and each of these into 3; these
parts

parts were then bisected 4 times; but being apprehensive some error might arise from quinquisection and trisection, in order to examine the accuracy of the divisions, he described another circle $\frac{1}{16}$ inch, within the former by continual bisections, but found no sensible difference between the two sets of divisions. It appears, also, that Mr. Bird, notwithstanding all his objections to, and declamations against the practice of stepping, sometimes used it himself.

OF THE NONIUS DIVISIONS.

It will be necessary to give the young surveyor some account of the nature and use of that admirable contrivance commonly called a nonius, by which the divisions on the limbs of instruments are subdivided.

The nonius depends on this simple circumstance, that if any line be divided into equal parts, the length of each part will be greater, the fewer divisions there are in the original; on the contrary, the length of each division will be less in proportion, as the divisions are more numerous.

Thus, let us suppose the limb of Hadley's quadrant divided to every 20 minutes, which are the smallest divisions on the quadrant; the two extreme strokes on the nonius contain 7 degrees, or 21 of the afore-mentioned small divisions, but that it is divided only into 20 parts; each of these parts will be longer than those on the arch, in the proportion of 21 to 20; that is to say, they will be one-twentieth part, or one minute longer, than the divisions on the arch; consequently, if the first, or index division of the nonius, be set precisely opposite to any degree, the relative position of the nonius and the arch must be altered 1 minute before

fore the next division on the nonius will coincide with the next division on the arch, the second division will require a change of 2 minutes, the third of 3 minutes, and so on, till the 20th stroke on the nonius arrive at the next 20 minutes on the arch; the index division will then have moved exactly 20 minutes from the division whence it set out, and the intermediate divisions of each minute have been regularly pointed out by the divisions of the nonius.

To render this still plainer, we must observe, that the index, or counting division of the nonius, is distinguished by the mark 0, which is placed on the extreme right hand division, the numbers running regularly on thus, 20, 15, 10, 5, 0.

The index division points out the entire degrees and odd twenty minutes, subtended by the objects observed; but the intermediate divisions are shewn by the other strokes of the nonius; thus, look among the strokes of the nonius for one that stands directly opposite to, or perfectly coincident with some one division on the limb; this division reckoned on the nonius, shews the number of minutes to be added to what is pointed out by the index division.

To illustrate this subject, let us suppose two cases. The first, when the index division perfectly coincides with a division on the limb of the quadrant: here there is no difficulty, for at whatsoever division it is, that division indicates the required angle. If the index divisions stand at 40 degrees, 40 degrees is the measure of the required angle. If it coincides with the next division beyond 40 on the right hand, 40 deg. 20 min. is the angle. If with the second division beyond 40, then 40 deg.

40 min. is the angle, and so in every other instance.

The second case is, when the index line does not coincide with any divisions on the limb. We are, in this instance, to look for a division on the nonius, that shall stand directly opposite to one on the limb, and that division gives us the odd minutes, to be added to those pointed out by the index division: thus, suppose the index division does not coincide with 40 degrees, but that the next division to it is the first coincident division, then is the required angle 40 degrees 1 minute. If it had been the second division, the angle would have been 40 deg. 2 min. and so on to 20 min. when the index division coincides with the first 20 min. from 40 degrees. Again, let us suppose the index division to stand between 30 deg. and 30 deg. 20 min. and that the 16th division on the nonius coincides exactly with a division on the limb, then is the angle 30 deg. 16 min. Further, let the index division stand between 35 deg. 20 min. and 35 deg. 40 min. and at the same time the 12th division on the nonius stand directly opposite to a division on the arch, then will the angle be 35 deg. 32 min.

A GENERAL RULE FOR KNOWING THE VALUE OF EACH DIVISION, ON ANY NONIUS WHATSOEVER.

1. Find the value of each of the divisions, or subdivisions, of the limb to which the nonius is applied. 2. Divide the quantity of minutes or seconds thus found, by the number of divisions on the nonius, and the quotient will give the value of the nonius division.

Thus, suppose each subdivision of the limb be 30 minutes, and that the nonius has 15 divisions, then $\frac{30}{15}$ gives 2 minutes for the value of the nonius.

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If the nonius has 10 divisions, it would give 3 minutes; if the limb be divided to every 12 min. and the nonius to 24 parts, then 12 min. or 720 seconds divided by 24, gives 30 seconds for the required value.

OF INSTRUMENTS FOR DESCRIBING CIRCLES OF EVERY POSSIBLE MAGNITUDE.

As there are many cases, where arches are required to be drawn of a radius too large for any ordinary compasses, Mr. Heywood and myself contrived several instruments for this purpose; the most perfect of these is delineated at *fig. 5, plate 11*. It is an instrument that must give great satisfaction to every one who uses it, as it is so extensive in it's nature, being capable of describing arches from an infinite radius, or a strait line, to those of 2 or 3 inches diameter. When it was first contrived, both Mr. Heywood and myself were ignorant of what had been done by that ever to be celebrated mechanician, Dr. Hooke.

Since the invention thereof, I have received some very valuable communications from different gentlemen, who saw and admired the simplicity of it's construction; among others, from Mr. Nicholson, author of several very valuable works; Dr. Rotherham, Earl Stanhope, and J. Priestley, Esq. of Bradford, Yorkshire; the last gentleman has favoured me with so complete an investigation of the subject, and a description of so many admirable contrivances to answer the purpose of the artist, that any thing I could say, would be altogether superfluous; I shall, therefore, be very brief in my description of the instrument, represented *fig. 5, plate 11*, that I may not keep the reader from Mr. Priestley's valuable essay, sub-
joining

joining Dr. Hooke's account of his own contrivance to that of mine. Much is always to be gained from an attention to this great man; and I am sure my reader will think his time well employed, in perusing the short extract I shall here insert.

The branches A and B, *fig. 9, plate 11*, carry two independent equal wheels C, D. The pencil or point E, is in a line drawn between the center of the axis of the branches, and equidistant from each; a weight is to be placed over the pencil when in use. When all the wheels have their axes in one line, and the instrument is moved in rotation, it will describe an infinitely small circle; in this case the instrument will overset. When the two wheels C, D, have their horizontal axes parallel to each other, a right line, or infinitely large circle will be described; when these axes are inclined to each other, a circle of finite magnitude will be described.

The distance between one axis and the center, (or pencil,) being taken as unity, or the common radius, the numbers 1, 2, 3, 4, &c. being sought for in the natural tangents, will give arcs of inclination for setting the noniuses, and at which circles of the radii of the said numbers, multiplied into the common radius, will be described.

The common radius multiplied by	{	0.1	is the radius of a circle made by the rollers when inclined at these angles:	{	5.43
		0.2			11.19
		0.3			16.42
		0.4			21.48
		0.5			26.34
		0.6			30.58
		0.7			35.
		0.8			38.49
K 2				The	

The common radius multiplied by	0.9	is the radius of a circle made by the rollers when inclined at these angles :	41. 59
	1.0		45. 00
	2.0		63. 26
	3.0		71. 34
	4.0		75. 58
	5.0		78. 42
	6.0		80. 32
	7.0		81. 52
	8.0		82. 53
	9.0		83. 40
	10.0		84. 17
	20.0		87. 8
	30.0		88. 5
	40.0		88. 34

Extracts from Dr. Hooke, *on the difficulty, &c. of drawing arches of great circles.* "This thing, says he, is so difficult, that it is almost impossible, especially where exactness is required, as I was sufficiently satisfied by the difficulties that occurred in striking a part of the arch of a circle of 60 feet radius, for the gage of a tool for grinding telescope glasses of that length; whereby it was found, that a beam compass made with all care and circumspection imaginable, and used with as great care, would not perform the operation; nor by the way, an angular compass, such as described by Guido Ubaldu, by Clavius, and by Blagrove, &c.

"The Royal Society met; I discoursed of my instrument to draw a great circle, and produced an instrument I had provided for that purpose; and therewith, by the direction of a wire about 100 feet long, I shewed how to draw a circle of that radius, which gave great satisfaction, &c. Again, the last meeting I endeavoured to explain the difficulties there are in making considerable discoveries

ries either in nature or art; and yet, when they are discovered, they often seem so obvious and plain, that it seems more difficult to give a reason why they were not sooner discovered, than how they came to be detected now: how easy it was, we now think, to find out a method of printing letters, and yet, except what may have happened in *China*, there is no specimen or history of any thing of that kind done in this part of the world. How obvious was the vibration of pendulous bodies? and yet, we do not find that it was made use of to divide the spaces of time, till Galileo discovered it's isochronous motion, and thought of that proper motion for it, &c. And though it may be difficult enough to find a way before it be shewn, every one will be ready enough to say when done, that it is easy to do, and was obvious to be thought of and invented." To illustrate this, the Doctor produced an instrument somewhat similar to that described, *fig. 5, plate 11*, as appears from the journal of the Royal Society, where it is said, that Dr. Hooke produced an instrument capable of describing very large circles, by the help of two rolling circles, or truckles in the two ends of a rule, made so as to be turned in their sockets to any assigned angle. In another place he had extended his views relative to this instrument, that he had contrived it to draw the arch of a circle to a center at a considerable distance, where the center cannot be approached, as from the top of a pole set up in the midst of a wood, or from the spindle of a vane at the top of a tower, or from a point on the other side of a river; in all which cases the center cannot be conveniently approached, otherwise than by the sight. This he performed by two telescopes, so placed at the truckles, as thereby to see through both of them

the given center, and by thus directing them to the center, to set the truckles to their true inclination, so as to describe by their motion, any part of such a circle as shall be desired.

METHODS OF DESCRIBING ARCHES OF CIRCLES, OF LARGE MAGNITUDE. BY J. PRIESTLEY, ESQ, OF BRADFORD, YORKSHIRE.

In the projection of the sphere, perspective and architecture, as well as in many other branches of practical mathematics, it is often required to draw arches of circles whose radii are too great to admit the use of common or even beam compasses; and to draw lines tending to a given point, whose situation is too distant to be brought upon the plan. The following essay is intended to furnish some methods, and describe a few instruments that may assist the artist in the performance of both these problems.

OF FINDING POINTS IN, AND DESCRIBING ARCHES OF LARGE CIRCLES.

The methods and instruments I shall propose for this purpose, will chiefly depend on the following propositions, which I shall premise as principles.

Principle 1. The angles in the same segment of a circle, are equal one to another.

Let $ACDB$, *fig. 1, plate 10*, be the segment of a circle; the angles formed by lines drawn from the extremities A and B , of the base of the segment to any points C and D in it's arch, as the angles ACB , ADB , are equal.

This is the 31st proposition of Euclid's 3^d book of the Elements of Geometry.

Principle 2. If upon the ends AB , of a right line AB as an axis, two circles or rollers CD and EF
be

be firmly fixed, so that, *fig. 2, plate 10*, the said line shall pass through the centers, and at right angles to the planes of the circles; and the whole be suffered to roll upon a plane without sliding :

1. If the rollers *CD* and *EF* be equal in diameter, the lines described upon the plane by their circumferences, will be parallel right lines; and the axis *AB*, and every line *DF*, drawn between contemporary points of contact of the rollers and plane, will be parallel among themselves.

2. If the rollers *CD* and *EF* be unequal, then lines formed by their circumferences upon the plane will be concentric circles; and the axis *AB*, and also the lines *DF*, will, in every situation, tend to the common center of those circles.

Principle 3. If there be two equal circles or rollers *A* and *B*, *fig. 3, plate 10*, each separately fixed to it's own axis, moveable on pivots; and these axes placed in a proper frame, so as to lie in the same plane, and to maintain the situation given them with respect to each other; and if the apparatus be rolled upon a plane without sliding :

1. If the axes *CD* and *EF*, be placed in a parallel situation, the circumferences of the rollers *A* and *B* will trace upon the plane strait lines; which will be at right angles to the axes *CD* and *EF*.

2. If the axes *CD* and *EF*, continuing as before in the same plane, be inclined to each other, so as if produced to meet in some point *G*; the rollers *A* and *B* will describe in their motions upon the plane, arches of the same, or of concentric circles, whose center is a point *H*, in that plane perpendicularly under the point of intersection *G* of the two axes.

I shall not stop to demonstrate the truth of the

two last principles, it will easily appear on seeing the operations performed.

THE SIMPLE BEVEL.

In the performance of some of the following problems, an instrument not unlike *fig. 4, plate 10*, will be found useful. It consists of two rulers, moveable on a common center, like a carpenter's rule, with a contrivance to keep them fixed at any required angle. The center *C* must move on a very fine axis, so as to lie in a line with the fiducial edges *CB*, *CD* of the rulers, and project as little as possible before them. The fiducial edges of the legs represent the sides of any given angle, and their intersection or center *C* it's angular point.

A more complex instrument of this kind, adapted to various uses, will be described hereafter.

N. B. A pin fixed in the lower rule, passes through a semicircular groove in the upper, and has a nut *A* which screws upon it, in order to fix the rulers or legs, when placed at the desired angle.

PROBLEM I. *Given the three points A, B, and C, supposed to lie in the circumferences of a circle too large to be described by a pair of compasses; to find any number of other points in that circumference.*

This may be performed various ways. As for example, *fig. 5, plate 10*.

1. Join *AC*, which bisect with the line *FMG* at right angles; from *B*, draw *BD* parallel to *AC*, cutting *FG* in *E*; and making $ED = EB$, *D* will be a point in the same circumference in which *A*, *B* and *C* lie.

By joining *AB*, and bisecting it at right angles with *IK*; and from *C* drawing *Ca* parallel to *AB*, cutting *IK* in *L*, and making $La = LC$, *a* will be another of the required points.

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Continuing to draw from the point last found, lines alternately parallel to AC and AB ; those lines will be cut at right angles by FG and IK respectively; and by making the parts equal on each side of FG and IK , they become chords of the circle, in which the original points A, B and C lie, and, of consequence, determine a series of points on each side of the circumference.

It is plain from the construction, (which is too evident to require a formal demonstration,) that the arches $AD, Aa, Cc, \&c.$ intercepted between the points $A, D - A, a - C, c - \&c.$ are equal to the arch BC , and to one another.

In like manner, joining BC , and bisecting it at right angles with PQ ; drawing $A c'$ parallel to BC , and making $R c' = AR$, (c') is another of the required points; and, from (c') the point last found, drawing $c' a'$ parallel to CA , and making $a' N = N c'$, (a') is another point in the same circumference; and the arches comprehended between Cc' and Aa' are equal to that between AB . Hence, by means of the perpendiculars PQ and FG , any number of points in the circumference of the circle, passing through the given ones, A, B and C may be found, whose distance is equal to AB , in the same manner, as points at the distance of BC were found by the help of the perpendiculars IK and FG .

Again, if A, C and (c) or A, C and (c') be taken as the three given points, multiples of the arch AC may be found in the same manner as those of the arch AB were found as above described.

2. Another method of performing this problem, is as follows. *Fig. 6, plate 10.* Produce CB and CA ; and with a convenient radius on C , describe the arch DE ; on which set off the parts $FG, GE, \&c.$

&c. each equal to DF ; draw CG , CE , &c. continued out beyond G and E if necessary; take the distance AB , and with one foot of the compasses in A , strike an arch to cut CG produced in H ; and H is a point in the circumference of the circle, that passes through the given points ABC ; with the same opening AB , and center H , strike an arch to cut CE produced in I , which will be another of the required points, and the process may be continued as far as is necessary.

The reason of this construction is obvious; for since the $\angle BCA$, ACH , HCI , &c. are equal, they must intercept equal arches BA , AH , HI of the circumference.

If it were required to find a number of points K , L , &c. on the other side whose distances were equal to BC , lay down a number of angles CAK , KAL , &c. each equal to BAC , and make the distances CK , KL , &c. each equal to BC .

BY THE BEVEL.

This problem is much easier solved by the help of the bevel above described, as follows. See *Fig. 5*.

Bring the center of the bevel to the middle B , of the three given points A B and C , and holding it there, open or shut the instrument till the fiducial edges of the legs lie upon the other two points, and fix them there, by means of the screw A , (*fig. 4*); this is called *setting* the bevel to the given points. Then removing the center of the bevel, to any part between B and A or C , the legs of it being at the same time kept upon A and C , that center will describe (or be always found in) the arch which passes through the given points, and will, by that means, ascertain as many others as may be required within the limits of A and C .

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In order to find points without those limits, proceed thus: the bevel being *set* above described, bring the center to C, and mark the distance C B upon the *left* leg; remove the center to B, and mark the distance B A on the same leg; then placing the center on A, bring the right leg upon B, and the first mark will fall upon (a) a point in the circumference of the circle, passing through A, B and C, whose distance from A is equal to the distance B C. Removing the center of the bevel to the point (a) last found, and bringing the right leg to A, the second mark will find another point (a'') in the same circumference, whose distance a a'' is equal A B. Proceeding in this manner, any number of points may be found, whose distances on the circumference are alternately B C and B A.

In the same manner, making similar marks on the *right* leg, points on the other side, as at (c') and (c'') are found, whose distances C c', c' c'', are equal to B A, B C respectively.

It is almost unnecessary to add, that intermediate points between any of the above are given by the bevel, in the same manner as between the original points.

PROBLEM 2. *Fig. 7, plate 10. Three points A, B and C, being given, as in the last problem, to find a fourth point D, situated in the circumference of the circle passing through A, B and C, and at a given number of degrees distant from any of these points; A for instance.*

Make the angles A B D, and A C D, each equal to one half of the angle, which contains the given number of degrees, and the intersection of the lines B D, C D gives the point D required.

For an angle at the circumference being equal to half that at the center, the arch A D will contain

tain twice the number of degrees contained by either of the angles $A B D$ or $A C D$.

PROBLEM 3. *Fig. 8, plate 10. Given three points, as in the former problems, to draw a line from any of them, tending to the center of the circle, which passes through them all.*

Let A , B and C be the given points, and let it be required to draw $A D$, so as, if continued, it would pass through the center of the circle in which A , B and C lie.

Make the angle $B A D$ equal to the complement of the angle $B C A$, and $A D$ is the line required.

For, supposing $A E$ a tangent to the point A , then is $E A D$ a right angle, and $E A B = B C A$; whence, $B A D = \text{right angle, less the } \angle B C A$, or the complement of $B C A$.

Corollary 1. $A D$ being drawn, lines from B and C , or any other points in the same circle, are easily found; thus, make $A B G = B A D$, which gives $B G$; then make $B C F = C B G$, which gives $C F$; or $C F$ may be had without the intervention of $B G$, by making $A C F = C A D$.

Corollary 2. A tangent to the circle, at any of the points (A for instance), is thus found.

Make $B A E = B C A$, and the line $A E$ will touch the circle at A .

By the bevel. Set the bevel to the three given points A , B and C , (*fig. 8,*) lay the center on A , and the right leg to the point C ; and the other leg will give the tangent $A G'$. Draw $A D$ perpendicular to $A G'$ for the line required.

For $B A E$ being $= B C A$, the $\angle E A C$ is the supplement to $\angle A B C$, or that to which the bevel is set; hence, when one leg is applied to C , and the center brought to A , the direction of the other leg must be in that of the tangent $G E$.

PROBLEM

PROBLEM 4. *Fig. 9, plate 10. Three points being given, as in the former problems, to draw from a given fourth point, a line tending to the center of a circle passing through the three first points.*

Let the three points, through which the circle is supposed to pass, be A, B and C, and the given fourth point D; it is required to draw through D, a line Dd tending to the center of the said circle.

From A and B, the two points nearest D, draw by the last problem, the lines Aa, Bb, tending to the said center; join AB, and from any point E, taking in Bb, (the farther from B the better) draw EF parallel to Aa, cutting AB in F; join AD and BD, and draw FG parallel to AD, cutting DB in G; join GE, and through D parallel thereto, draw Dd for the line required.

For (continuing Dd and Bb till they meet in O,) since Aa and Bb, if produced, would meet in the center, and FE is parallel to Aa, we have $BF:BA::BE:\text{radius}$; also, since AD and FG are parallel, $BF:BA::BG:BD$; therefore $BG:BD::BE:\text{radius}$; but from the parallel lines Dd and GE, we have $BG:BD::BE:BO$; hence, BO is the radius of the circle passing through A, B and C.

By the bevel. On D with radius DA describe an arch AK; set the bevel to the three given points A, B and C, and bring it's center (always keeping the legs on A and C) to fall on the arch AK, as at H; on A and H severally, with any convenient radius, strike two arches crossing each other at I; and the required line Dd, will pass through the points I and D.

For a line drawn from A to H, will be a common chord to the circles AHK and ABC; and the

the line ID bisecting it at right angles, must pass through both their centers.

PROBLEM 5. *Fig. 9, plate 10. Three points being given, as before, together with a fourth point, to find two other points, such, that a circle passing through them and the fourth point, shall be concentric to that passing through three given points,*

Let A , B and C be the three given points, and D the fourth point; it is required to find two other points, as N and P , such that a circle passing through N , D and P , shall have the same center with that passing through A , B and C ,

The geometrical construction being performed as directed by the last problem, continue EG to L , making $EL = EB$; and through B and L , draw BLM , cutting Dd produced in M ; make AN and BP severally equal to MD , and N and P are the points required.

For, since LE is parallel to MO , we have $BE : LE :: BO : MO$; but $BE = LE$ by the construction; therefore, $MO = BO =$ radius of the circle passing through A , B and C , and M is in the circumference of that circle. Also, N , D and P being points of the radii, equally distant from A , M and B respectively, they will be in the circumference of a circle, concentric to that passing through A , M and B , or A , B and C .

By the bevel. Draw Aa and Cc tending to the center, by problem 3; set the bevel to the three given points A , B and C ; bring the center of the bevel to D , and move it upon that point till it's legs cut off equal parts AN , CQ of the lines Aa and Cc ; and N and Q will be the points required.

For, supposing lines drawn from A to C , and from N to Q ; the segments ABC and NDQ will be

be similar ones; and, consequently, the angles contained in them will be equal.

PROBLEM 6. *Fig. 10, plate 10. Three points A, B and C, lying in the circumference of a circle, being given as before; and a fourth point D, to find another point F, such, that a circle passing through F and D, shall touch the other passing through A, B and C, at any of these points, B for instance.*

Draw B E a tangent to the arch A B C, by problem 3, corollary 2; and join B D; draw B F, making the angle $B D F = E B D$, with the distance B D; on D strike an arch to cut B F in F; and F is the point sought.

Since $D F = D B$, the $\angle D F B = D B F$; but $D B F = D B E$ by construction; therefore, $D F B = D B E$, and E B is a tangent to the arch B D F at B; but E B is also a tangent to the arch A B C (by construction) at the same point; hence, the arch B D F touches A B C as required.

PROBLEM 7. *Fig. 13, plate 10. Two lines tending to a distant point being given, and also a point in one of them; to find two other points, (one of which must be in the other given line,) such, that a circle passing through those three points, may have it's center at the point of intersection of the given lines.*

Let the given lines be A B and C D, and E the given point in one of them; it is required to find two other points, as I and H, one of which (I) shall be in the other line, such that a circle H I E passing through the three points, shall have it's center at O, where the given lines, if produced, would meet.

From E the given point, draw E H, crossing A B at right angles in F; make $F H = F E$, and H is one of the required points. From any point D in C D, the farther from E the better, draw G D parallel

parallel to AB , and make the angle HEI , equal to half the angle GDE ; and EI will cut AB in I , the other required point.

For, since EH crosses AB at right angles, and HF is equal to FE , IH will be equal to IE , and the $\angle HEI = \angle EHI$; also, since GD is parallel to AB , the $\angle GDE = \angle FOE =$ double $\angle IHE =$ double $\angle HEI$; but $HEI =$ half $\angle GDE$ by construction; hence, the points E , I and H are in the circle whose center is O .

By the bevel. Draw EH at right angles to AB , and make $FH = FE$ as before; set the bevel to the angle GDO , and keeping it's legs on the points H and E , bring it's center to the line AB , which will give the point I .

PROBLEM 8. *Fig. 13, plate 10. Two lines tending to a distant point, being given, to find the distance of that point.*

Let AB and CD be the two given lines, tending to a distant point O ; and let it be required to find the distance of that point, from any point (E for instance) in either of the given lines.

From E draw EF , perpendicular to AB ; and from D (a point taken any where in CD , the farther from E the better) draw DG parallel to AB . On a scale of inches and parts, measure the lengths of GE , ED and EF separately; then say, as the length of GE is to ED , so is EF to EO , the distance sought.

For the triangles EGD and EFO are similar, and from thence the rule is manifest.

OF INSTRUMENTS FOR DRAWING ARCHES OF LARGE CIRCLES, AND LINES TENDING TO A DISTANT POINT.

I shall now proceed to give some idea of a few instru-

instruments for these purposes, whose *rationale* depends on the principles laid down in the beginning of this essay.

AN IMPROVED BEVEL.

Fig. 12, plate 10, is a sketch of an instrument, grounded upon principle 1, by which the arches of circles of any radius, without the limits attainable by a common pair of compasses, may be described.

It consists of a ruler A B, composed of two pieces rivetted together near C, the center or axis, and of a triangular part C F E D. The axis is a hollow socket, fixed to the triangular part, about which another socket fixed to the arm C B, of the ruler A B, turns. These sockets are open in the front, for part of their length upwards, as represented in the section at L, in order that the point of a tracer or pen, fitted to slide in the socket, may be more easily seen.

The triangular part is furnished with a graduated arch D E, by which, and the vernier at B, the angle D C B may be determined to minutes. A groove is made in this arch, by which, and by the nut and screw at B, or some similar contrivance, the ruler A B may be fixed in any required position.

A scale of radii is put on the arm C B, by which the instrument may be *set* to describe arches of given circles, not less than 20 inches in diameter. In order to *set* the instrument for any given radius, the number expressing it in inches on C B, is brought to cut a fine line drawn on C D, parallel and near to the fiducial edge of it, and the arms fastened in that position by the screw at B.

Two heavy pieces of lead or brass, G, G, made
L
in

in form of the sector of a circle, the angular parts being steeled and wrought to a true upright edge, as shewn at H, are used with this instrument, whose arms are made to bear against those edges when the arches are drawn. The under sides of these sectors are armed with fine short points, to prevent them from sliding.

The fiducial edges of the arms C A and C D are each divided from the center C, into 200 equal parts.

The instrument might be furnished with small castors like the pentagraph, but little buttons fixed on it's underside, near A, E, and D, will enable it to slide with sufficient ease.

SOME INSTANCES OF IT'S USE.

1. To describe an arch, which shall pass through three given points.

Place the sectors G, G, with their angular edges over the two extreme points; apply the arms of the bevel to them, and bring at the same time it's center C (that is, the point of the tracer or pen, put in the socket,) to the third point, and there fix the arm C B; then bringing the tracer to the left hand sector, slide the bevel, keeping the arms constantly bearing against the two sectors, till it comes to the right-hand sector, by which the required arch will be described, by the motion of it's center C.

If the arch be wanted in some part of the drawing *without* the given points, find, by problem 1, other points in those parts where the arch is required. By this means a given arch, may be lengthened as far as is requisite.

2. To describe an arch of a given radius, not less than 10 inches.

Fix

Fix the arm CB , so that the part of its edge, corresponding to the given radius, always reckoned in inches, may lie over the fine line drawn on CD for that purpose; bring the center to the point through which the arch is required to pass, and dispose the bevel in the direction it is intended to be drawn; place the sectors G, G exactly to the divisions 100 on each arm, and strike the arch, as above described.

3. The bevel being set to strike arches of a given radius, as directed in the last paragraph, to draw other arches whose radii shall have a given proportion to that of the first arch.

Suppose the bevel to be set for describing arches of 50 inches radius, and it be required to draw arches of 60 inches radius, with the bevel so set.

Say, as 50, the radius to which the bevel is set,

Is to 60, the radius of the arches required;

So is the constant Now 100, Place the sectors

To 120, the number on the arms CA and CD

to which the sectors must be placed, in order to

describe arches of 60 inches radius.

$N.B.$ When it is said that the bevel is set to

draw arches of a particular radius, it is always un-

derstood, that the sectors G, G are to be placed

at 100 on CA and CD , when those arches are

drawn.

4. An arch ACB of a given plate being

given, to draw other arches concentric thereto,

which shall pass through given points, (as P for

instance) in those parts where the arch

Through the extremities A and B of the given

arch, draw lines A, p, B, p tending to its center

by problem 3. Take the nearest distance of the

given point P from the arch, and set it from A to

p , and from B to p . Hold the center of the level

on C, (any point near the middle of the given arch,) and bring it's arms to pass through A and B at the same time, and there fix them. Place the sectors to the points p and p', and with the bevel, set as above directed, draw an arch, which will pass through P the given point, and be concentric to the given arch A C B.

5. Through a point A, (*fig. 14, plate 10,*) in the given line A B, to strike an arch of a given radius, and whose center shall lie in that line, (produced if necessary).

Set the bevel to the given radius, as above described, (*Use 2*).

Through A, at right angles to A B, draw C D; lay the center of the bevel (*set as above*) on A, and the arm C A, on the line A C; and draw a line A E along the edge C D of the other arm. Divide the angle D A E into two equal parts by the line A F; place the bevel so, that it's center being at A, the arm C D shall lie on A F; while in this situation, place the sectors at No. 100 on each arm, and then strike the arch.

6. An arch being given, to find the length of it's radius.

Place the center of the bevel on the middle of the arch, and open or shut the arms, till No. 100 on C A and C D fall upon the arch on each side the center; the radius will be found on C B (in inches) at that point of it, where it is cut by the line drawn on C D.

If the extent of the arch be not equal to that between the two Nos. 100, make use of the Nos. 50, in which case the radius found on C B, will be double of that sought; or the arch may be lengthened, (by problem 1,) till it be of an extent sufficient to admit the Nos. 100.

Many more instances of the use of this instrument might be given; but from what has been already done, and an attentive perusal of the foregoing problems, the principal of them may be easily conceived.

2. THE OBLIQUE RULER.

An instrument for drawing lines that are parallel, is called a *parallel ruler*; one for drawing lines tending to a point, as such lines are oblique to each other, may, by analogy, be called an *oblique ruler*.

Fig. 17, plate 10, represents a simple contrivance for this purpose; it consists of a cylindrical or prismatical tube A B, to one end of which is fixed the roller A; into this tube, there slides another C B of six or eight flat sides. The tubes slide stiffly, so as to remain in the position in which they are placed. Upon the end C, I screw different rollers, all of them something smaller than A.

In order to describe arches, a drawing pen E, and a tracer, may be put on the pin D, and are retained there by a screw G; the pen is furnished with a moveable arm E F, having a small ball of brass F at the end, whose use is to cause the pen to press with due force upon the paper, the degree of which can be regulated by placing the arm in different positions.

The ruler A B being set to any given line, by rolling it along other lines may be drawn, all of which will tend to some *one* point in the given line, or a continuation of it, whose distance will be greater, as the distance between the rollers A and C is increased; and as the diameter of C approaches that of A; all which is evident from principle 2.

It also appears from the said principle, that

during the motion of the bruler, any point in its axis will accurately describe the arch of a circle, having the said point of intersection for its center; and, consequently, the pen or tracer, put on the pin D, will describe such arches.

The rollers, as C, which screw upon the end of the inner tube are numbered, 1, 2, 3, &c. and as many scales are drawn on that tube as there are rollers, one belonging to each, and numbered accordingly. These scales show the distance in inches, of the center or point of intersection, reckoned from the middle of the pin D, (agreeing to the point of the pen or tracer; thus,

No. 1, will describe circles, or serve for drawing lines tending to a point, whose radius or distance from D, is from 1200 inches to 600 inches, according as the tube is drawn out.

No.	from 600 inches to 300 inches
3	300
4	150
5	80
6	40
7	20

If it should be required to extend the radius or distance farther than 1200 inches, by using another ruler, it might be carried to 2400 inches; but lines in any common sized drawing, which tend to a point above 300 feet distance, may be esteemed as parallel.

3. ANOTHER RULER OF THE SAME KIND.

Fig. 18, plate 10. This is nothing more than the last instrument applied to a flat ruler, in the manner the rolling parallel rulers are made. CD is a hexagonal axis, moveable on pivots in the

the heads A and F fixed upon a flat ruler; on this axis the smaller roller B, is made to slide through one half of it's length; the larger roller A, is screwed on the other end of the axis, and can be changed occasionally, for others of different diameters. Scales adapted to each of the rollers at A, are either put on the flat sides of the axis from C to E, or drawn on the corresponding part of the flat ruler; and the scales and rulers distinguished by the same number: at F is a screw to raise or lower the end C of the axis, till the ruler swims parallel to the paper on which the drawing is made; and at G there is a socket, to which a drawing pen and tracer is adapted for describing arches.

In using these instruments, the fingers should be placed about the middle part, between the rollers; and the ruler drawn, or pushed at right angles to it's length. The tube A B, *fig. 14*, and one or both of the edges of the flat ruler, *fig. 18*, are divided into inches and tenths.

4. THE CYCLOGRAPH.

This instrument is constructed upon the 3d principle mentioned in the beginning of this essay.

Fig. 16, plate 10, is composed of five rulers; four of them, D E, D F, G E and G F, forming a trapezium, are moveable on the joints D, E, F and G; the fifth ruler, D I, passes under the joint D, and through a socket carrying the opposite joint G. The distances from the center of the joint D, to that of the joints E and F, are exactly equal, as are the distances from G to the same joints. The rulers D E and D F pass beyond the joints E and F, where a roller is fixed to each; the rollers are fixed upon their axes, which move freely, but steadily on pivots, so as to admit of no

L 4

shake

shake by which the inclination of the axes can be varied. The ruler *DI* passing beyond the joint *D*, carries a third roller *A* like the others, whose axis lies precisely in the direction of that ruler; the axes of *B* and *C*, extend to *K* and *L*.

A scale is put on the ruler *DI*, from *H* to *G*, shewing by the position of the socket *G* thereon, the length of the radius of the arch in inches, that would be described by the end *I*, in that position of the trapezium. When the socket *G* is brought to the end of the scale near *I*, the axes of the two rollers *B* and *C*, the ruler *DI*, and the axis of the roller *A*, are precisely parallel; and in this position, the end *I*, or any other point in *DI*, will describe strait lines at right angles to *DI*; but on sliding the socket *G* towards *H*, an inclination is given to the axes of *B* and *C*, so as to tend to some point in the line *ID*, continued beyond *D*, whose distance from *I* is shewn by the scale.

A proper socket for holding a pen or tracer, is made to put on the end *I*, for the purpose of describing arches; and another is made for fixing on any part of the ruler *DI*, for the more convenient description of concentric arches, where a number are wanted.

It is plain from this description, that the middle ruler *DI* in this instrument, is a true oblique ruler, by which lines may be drawn tending to a point, whose distance from *I* is shewn by the position of the socket *G* on the scale; and the instrument is made sufficiently large, so as to answer this purpose as well as the other.

5. A DIFFERENT CONSTRUCTION OF THE SAME INSTRUMENT.

In *Fig. 16*, plate 10, the part intended to be used

used in drawing lines, lies within the trapezium, which is made large on that account, but this is not necessary: and fig. 15, plate 10, will give an idea of a like instrument, where the trapezium may be made much smaller, and consequently less cumbersome.

D B E C represents such a trapezium, rollers, socket, and scale as above described, but much smaller. Here the ruler E D is continued a sufficient length beyond D, as to A, where the third roller is fixed; a pen or tracer may be fitted to the end E, or made to slide between D and A, for the purpose of drawing arches.

METHODS OF DESCRIBING AN ELLIPSE, AND SOME OTHER CURVES.

To describe an ellipse, the transverse and conjugate axes being given.

Let A B be the given transverse, and C D the conjugate axis, fig. 13, plate 13.

Method 1. By the line of lines on the sector, open the sector with the extent A G of the semi-transverse axis in the terms of 90 and 90, take out the transverse distance of 70 and 70, 60 and 60, and so for every tenth line, and set them off from G to A, and from G to B, then draw lines through these points perpendicular to A B. Make G C a transverse distance between 90 and 90, and set off each tenth line from G towards C, and from G towards D, and through these points draw lines parallel to A B, which will intersect the perpendiculars to A B in the points A, a, b, c, d, e, f, g, h, i, k, l, m, n, o, p, q, B, for half the ellipse, through which points and the intersections of the other half, a curve being drawn with a steady hand, will complete the ellipse.

Method

Method. 2. With the elliptical compasses, *fig. 3*, *plate 11*, apply the transverse axis of the elliptical compass to the line *A B*, and discharge the screws of both the sliders; set the beam over the transverse axis *A B*, and slide it backwards and forwards until the pencil or ink point coincides with the point *A*, and tighten the screw of that slider which moves on the conjugate axis; now turn the beam, so as to lay over the given conjugate axis *C D*, and make the pencil or ink point coincide with the point *C*, and then fix the screw, which is over the slider of the transverse axis of the compass; the compasses being thus adjusted, move the ink point gently from *A*, through *C* to *B*, and it will describe the semi-ellipse *A C B*; reverse the elliptical compass, and describe the other semi-ellipse *B D A*. These compasses were contrived by my father in 1748; they are superior to the trammel which describes the whole ellipse, as these will describe an ellipse of any excentricity, which the others will not.

Through any given point F to describe an ellipse, the transverse axis A B being given.

Apply the transverse axis of the elliptical compass to the given line *A B*, and adjust it to the point *A*; fix the conjugate screw, and turn the beam to *F*, sliding it till it coincide therewith, and proceed as in the preceding problem.

Fig. 2, *plate 13*, represents another kind of elliptical compasses, acting upon the principle of the oval lathes; the paper is fixed upon the board *A B*, the pencil *C* is set to the transverse diameter by sliding it on the bar *D E*, and is adjusted to the conjugate diameter by the screw *G*; by turning the board *A B*, an ellipse will be described by the pencil. *Fig. 2*, *A*, *plate 11*, is the trammel, in which the

the pins on the under side of the board AB , move for the description of the ellipse.

Ellipses are described in a very pleasing manner by the geometria pen, fig. 1, plate 1. this part of that instrument may be had separate, if required.

To describe a parabola, whose parameter shall be equal to a given line. Fig. 17, plate 13.

Draw a line to represent the axis, in which make AB equal to half the given parameter. Open the sector, so that AB may be the transverse distance between 90 and 90 on the line of sines, and set off every tenth line from A towards B ; and through the points thus found, draw lines at right angles to the axis AB . Make the lines Aa , $10b$, $20c$, $30d$, $40e$, &c. respectively equal to the chords of 90° , 80° , 70° , 60° , 50° , &c. to the radius AB , and the points a b c d e , &c. will be in the parabolic curve: for greater exactness, intermediate points may be obtained from the intermediate degrees; and a curve drawn through these points and the vertex B , will be the parabola required: if the whole curve be wanted, the same operation must be performed on the other side of the axis.

As the chords on the sector run no further than 60° , those of 70 , 80 and 90 , may be found by taking the transverse distance of the sines of 35° , 40° , 45° , to the radius AB , and applying those distances twice along the lines $20c$, $10b$, &c.

Fig. 4, plate 11, is an instrument for describing a parabola: the figure will render its use sufficiently evident to every geometrician. $ABCD$ is a wooden frame, whose sides AC , BD are parallel to each other. $EFGH$ is a square frame of brass on wood, sliding against the sides AC , BD of the exterior frame. H a socket sliding on the bar EF of the interior frame, and carrying the pencil

pencil I; K, a fixed point in the board, (the situation of which, may be varied occasionally); E a K is a thread equal in length to E, one end thereof is fixed at E, the other to the piece K, going over the pencil at a. Bring the frame, so that the pencil may be in a line with the point K; then slide it in the exterior frame, and the pencil will describe one part of a parabola. If the frame E F G H be turned about, so that E F may be on the other side of the point K, the remaining part of the parabola may be completed.

To describe an hyperbola, the vertex A, and asymptotes B H, B I being given. Fig. 18, plate 13.

Draw A I, I C, parallel to the asymptotes. Make A C a transverse distance to 45, and 45 on the upper tangents of the sector, and apply from B as many of these tangents taken transversely as may be thought convenient; as B D 50° , D E 55° , and so on; and through these points draw D d, E c, &c, parallel to A C.

Make A C a transverse distance between 45 and 45 of the lower tangents, and take the transverse distance of the cotangents before used, and lay them on those parallel lines; thus, make D d equal 40° , E c to 35° , E F to 80° , &c. and these points will be in the hyperbolic curve, and a line drawn through them will be the hyperbola required.

To assist the hand in drawing curves through a number of points, artists make use of what is termed the bow, consisting of a spring of hard wood, or steel, so adapted to a firm strait rule, that it may be bent more or less by 3 screws passing through the strait rule.

A set of spirals cut out in brass, are extremely convenient for the same purpose; for there are

few

few curve lines of a short extent, to which some part of these will not apply.

Fig. 6, plate 11, represents an instrument for drawing spirals; A the foot by which it is affixed to the paper, B the pencil, *a, b, c, d* a running line going over the cone G and cylinder H, the ends being fastened to the pin *e*. On turning the frame K N O, the thread carries the pencil progressively from the cone to the cylinder, and thus describes a spiral. The size of the spiral may be varied, by placing the thread in different grooves, by putting it on the furthestmost cone, or by putting on a larger cone.

OF THE GEOMETRIC PEN.

The geometric pen is an instrument in which, by a circular motion, a right line, a circle, an ellipse, and a great variety of geometrical figures, may be described.

This curious instrument was invented and described by John Baptist Suardi, in a work entitled *Nouvo Istromenti per la Descrizione di diverse Curve Antichi e Moderne, &c.*

Though several writers have taken notice of the curves arising from the compound motion of two circles, one moving round the other, yet no one seems to have realized the principle, and reduced it to practice, before J. B. Suardi. It has lately been happily introduced into the steam engine, by Messieurs Watt and Bolton; a proof, among many others, not only of the use of these speculations, but of the advantages to be derived from the higher parts of the mathematics, in the hands of an ingenious mechanic. There never was, perhaps, any instrument which delineates so many different curves as the geometric pen; the author
enumerates

enumerates 1273, as possible to be described by it in the simple form, and with the few wheels appropriated to it for the present work.

Fig. 1, plate II, represents the geometric pen, A, B, C, the stand by which it is supported; the legs A, B, C, are contrived to fold one within the other, for the convenience of packing.

A strong axis D is fitted to the top of the frame, to the lower part of this axis, any of the wheels (as i) may be adapted; when screwed to it they are immoveable.

E G is an arm contrived to turn round upon the main axis D; two sliding boxes are fitted to this arm; to these boxes any of the wheels belonging to the geometric pen may be fixed, and then slid so that the wheels may take into each other, and the immoveable wheel i; it is evident, that by making the arm E G revolve round the axis D, these wheels will be made to revolve also, and that the number of their revolutions will depend on the proportion between the teeth.

F g is an arm carrying the pencil; this arm slides backwards and forwards in the box c d, in order that the distance of the pencil, from the center of the wheel h, may be easily varied; the box c d is fitted to the axis of the wheel h, and turns round with it, carrying the arm f g along with it; it is evident, therefore, that the revolutions will be fewer or greater, in proportion to the difference between the numbers of the teeth in the wheels h and i; this bar and socket are easily removed for changing the wheels.

When two wheels only are used, the bar f g moves in the same direction with the bar E G; but if another wheel is introduced between them, they move in contrary directions.

The

The number of teeth in the wheels, and consequently, the relative velocity of the epicycle or arm *fg*, may be varied in infinitum.

The numbers we have used, are 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96.

The construction and application of this instrument is so evident from the figure, that nothing more need be pointed out than the combinations, by which the figures here delineated may be produced.

To render the description as concise as possible, I shall in future describe the arm *E G* by the letter *A*, and *fg* by the letter *B*.

To describe *fig. 1*, *plate 12*. The radius of *A* must be to that of *B*, as 10 to 5 nearly, their velocities, or the numbers of teeth in the wheels to be equal, the motion to be in the same direction.

If the length of *B* be varied, the looped figure delineated at *fig. 12*, will be produced.

A circle may be described by equal wheels, and any radius but the bars, must move in contrary directions.

To describe the two leaved figures, seen *plate 12*, *fig. 11*. Let the radius of *A* to *B*, be as 10 to 3, the velocities as 1 to 2, the motion in the same direction.

To describe by this CIRCULAR MOTION, A STRAIT LINE AND AN ELLIPSE. For a strait line, equal radii, the velocity as 1 to 2, the motion in a contrary direction; the same data will give a variety of ellipses, only the radii must be unequal; the ellipses may be described in any direction; see *fig. 10*, *plate 13*.

Fig. 14, *plate 12*, with seven leaves, is to be formed when the radii are as 7 to 2, velocity as 2 to 3, motion in contrary directions.

The

The six triangular figures, seen at *fig. 2, 4, 6, 8, 9, 10*, are all produced by the same wheels, by only varying the length of the arm B, the velocity should be as 1 to 3, the arms are to move in contrary directions.

Fig. 3, plate 12, with eight leaves, is formed by equal radii, velocities as 5 to 8, A and B to move the same way; if an intermediate wheel is added, and thus a motion produced in a contrary direction, the pencil will delineate *fig. 16, plate 12*.

The ten-leaved figure, *plate 12, fig. 15*, is produced by equal radii, velocity as 3 to 10, directions of the motions contrary to each other.

Hitherto the velocity of the epicycle has been the greatest; in the three following figures, the curves are produced when the velocity of the epicycle is less than that of the primum mobile.

For *fig. 7*, the radius of A to B, to be as 2 to 1, the velocity as 3 to 2; to be moved the same way.

For *fig. 14*, the radius of A, somewhat less than the diameter given to B, the velocity as 3 to 1; to be moved in a contrary direction.

For *fig. 5*, equal radii, velocity as 3 to 1; moved the same way. These instances are sufficient to shew how much may be performed by this instrument; with a few additional pieces, it may be made to describe a cycloid, with a circular base, spirals, and particularly the spiral of Archimedes, &c.

OF THE DIVISION OF LAND.

To know how to divide land into any number of equal or unequal parts, according to any assigned proportion, and to make proper allowances for the different qualities of the land to be divided, forms a material and useful branch of surveying.

In

In dividing of land, numerous cases arise; in some it is to be divided by lines parallel to each other, and to a given fence or road; sometimes, they are to intersect a given line; the division is often to be made according to the particular direction of the parties concerned. In a subject which has been treated so often, novelty is perhaps not to be desired, and scarcely expected. No considerable improvement has been made in this branch of surveying, since the time of Speidell. Mr. Talbot, whom we shall chiefly follow, has arranged the subject better than those who preceded him, and added thereto two or three problems; his work is well worth the surveyor's perusal. Some problems also in the foregoing part of this work, should be considered in this place.

PROBLEM I. *To divide a triangle in a given ratio, by right lines drawn from any angle to the opposite side thereof.*

1. Divide the opposite side in the proposed ratio.
2. Draw lines from the several points of division to the given angle, and then divide the triangle as required.

Thus, to divide the triangle ABC , *fig. 13, plate 8*, containing $26\frac{1}{2}$ acres, into three parts, in proportion to the numbers 40, 20, 10, the lines of division to proceed from the angle C to AB , whose length is 28 chains; now as the ratio of 40, 20, 10, is the same as 4, 2, 1, whose sum is 7, divide AB into 7 equal parts; draw Aa at four of these parts, Ab at 6 of them, and the triangle is divided as required.

Arithmetically. As 7, the sum of the ratios, is to AB 28 chains; so is 4, 2, 1, to 16, 8, and 4 chains respectively; therefore $Aa = 16$, $ab = 8$, and $bB = 4$ chains.

M

To

To know how many acres in each part, say as the sum of the ratios, is to the whole quantity of land, so is each ratio to the quantity of acres:

$$\text{as } \left\{ \begin{array}{l} 7:26,5 :: 4:15,142857 \\ 7:26,5 :: 2:7,571428 \\ 7:26,5 :: 1:3,785714 \end{array} \right\} \begin{array}{l} = \text{triangle A C a} \\ = \text{triangle a C b} \\ = \text{triangle b C B.}^* \end{array}$$

PROBLEM 2. *To divide a triangular field into any number of parts, and in any given proportion from a given point in one of the sides.*

1. Divide the triangle into the given proportion, from the angle opposite the given point. 2. Reduce this triangle by problem 51, so as to pass through the given point.

Thus, to divide the field A B C, *fig. 14, plate 8*, of seven acres into two parts, in the proportion of 2 to 5, for two different tenants, from a pond b, in B C, but so that both may have the benefit of the pond.

1. Divide B C into 7 equal parts, make B a = 5, then C A = 2, draw A a, and the field is divided in the given ratio. 2. To reduce this to the point b, draw A b and a c parallel thereto, join c b, and it will be the required dividing line.

Example 2. To divide A B C, *fig. 15, plate 8*, into three equal parts from the pond C. 1. Divide A B into 3 equal parts, A a, a b, b B, and C a C b will divide it, as required, from the angle A; reduce these as above directed to c d and c e, and they will be the true dividing lines. *In the field*, divide B C in the ratio required, and set up a mark at the point a, and also at the pond b; measure the angle b A a; at a lay off the same angle A a c, which will give the point c in the side A a c, from whence the fence must go to the pond.

PROBLEM

* Talbot's Complete Art of Land Measuring.

PROBLEM 3. *To divide a triangular field in any required ratio, by lines drawn parallel to one side, and cutting the other.*

Let $A B C$, *fig. 16, plate 8*, be the given triangle to be divided into 3 equal parts, by lines parallel to $A B$, and cutting $A C$, $B C$.

Rule 1. Divide one of the sides that is to be cut by the parallel lines, into the given ratio. 2. Find a mean proportional between this side, and the first division next the parallel side. 3. Draw a line parallel to the given side through the mean proportional. 4. Proceed in the same manner with the remaining triangle.

Example 1. Divide $B C$ into three equal parts $B D$, $D P$, $P C$. 2. Find a mean proportional between $B C$ and $D C$. 3. Make $C G$ equal to this mean proportional, and draw $G H$ parallel to $A B$. Proceed in the same manner with the remaining triangle $C H G$, dividing $G C$ into 2 equal parts at I , finding a mean proportional between $C G$, and $C I$; and then making $C L$ equal to this mean proportional, and drawing $L M$ parallel to $A B$, the triangle will be divided as required.

A square, or rectangle, a rhombus, or rhomboides, may be divided into any given ratio, by lines cutting two opposite parallel sides, by dividing the sides into the proposed ratio, and joining the points of division.

PROBLEM 4. *To divide a right-lined figure into any proposed ratio, by lines proceeding from one angle.*

PROBLEM 5. *To divide a right-lined figure into any proposed ratio, by lines proceeding from a given point in one of the sides.*

PROBLEM 6. *To divide a right-lined figure into any proposed ratio, by right lines proceeding from a given point, within the said figure or field.*

It would be needless to enter into a detail of the

mode, of the performing the three foregoing problems, as the subject has been already sufficiently handled in pages 84, 85, &c.

PROBLEM 7.* *It is required to divide any given quantity of ground into any given number of parts, and in proportion as any given numbers."*

Rule. "Divide the given piece after the rule of fellowship, by dividing the whole content by the sum of the numbers expressing the proportions of the several shares, and multiplying the quotient severally by the said proportional numbers for the respective shares required."

Example. It is required to divide 300 acres of land among A, B, C, and D, whose claims upon it are respectively in proportion as the numbers 1, 3, 6, 10, or whose estates may be supposed 100l. 300l. 600l. and 1000l. per annum.

The sum of these proportional numbers is 20, by which dividing 300 acres, the quotient is 15 acres, which being multiplied by each of the numbers 1, 3, 6, 10, we obtain for the several shares as follows:

	<i>a.</i>	<i>r.</i>	<i>p.</i>
A's share =	15	: 0	: 00
B's share =	45	: 0	: 00
C's share =	90	: 0	: 00
D's share =	150	: 0	: 00

Sum = 300 : 0 : 00 the proof.

But this is upon supposition that the land is all of an equal value.

Now let us suppose the land to be laid off for each

* Problem 7, an erroneous rule given by a late writer, introduced here to prevent the practitioner being led into error.—Problem 8, from Talbot's Complete Art of Land Measuring.

each person's share, is of the following different values per acre, viz. A's=5s. B's=8s. C's=12s. D's=15s. an acre; whose sum 40s. divided by 4, their number, quotes 10s. for the mean value per acre. And, according to a late author, we must augment or diminish each share as follows:

	s.	s.	a.	a.	
as	5	10	15	30	A's share
	8	10	45	56,3	B's share
	12	10	90	75	C's share
	15	10	150	100	D's share

sum of shares = 261,3 acres,

which is less than 300, by more than 38 acres, and shews the rule to be absolutely false; for when each person's share is laid out as above, there remain 38 acres unapplied; I suppose for the use of the surveyor,

But suppose we change the value of each person's land, and call A's 15s. B's 12s. C's 8s. and D's 5s. then we shall have

	s.	s.	a.	a.	
as	15	10	15	10	A's share
	12	10	45	37,5	B's share
	8	10	90	112,5	C's share
	5	10	150	300	D's share

sum of the shares 460 acres :

how must the surveyor manage here? as he must make 460 acres of 300, for here D's share only takes the whole 300, where is he to find the 160 acres for the other three shares? But enough of this; see it truly and methodically performed in the next problem.

M 3

PROBLEM

PROBLEM 8. *It is required to divide any given quantity of land, among any given number of persons, in proportion to their several estates, and the value of the land that falls to each person's share.*

Rule. Divide the yearly value of each person's estate by the value per acre, of the land that is allotted for his share, and take the sum of the quotients, by which divide the whole given quantity of land, and this quotient will be a common multiplier, by which multiply each particular quotient, and the product will be each particular share of the land.

Or say, as the sum of all the quotients, is to the whole quantity of land; so is each particular quotient, to it's proportional share of the land.

Example. Let 300 acres of land be divided among A, B, C, D, whose estates are 100l. 300l. 600l. and 1000l. respectively per annum; and the value of the land allotted to each is 5, 8, 12, and 15 shillings an acre, as in the example to the last problem.

Then $\frac{100}{5} = 20$, $\frac{300}{8} = 37.5$, $\frac{600}{12} = 50$, and $\frac{1000}{15} = 66.666$, and the sum of these quotients is 174.166. $\therefore \frac{300}{174.166} = 1.72248$, the common multiplier:

			acres.
then	{	1.72248×20	$= 34.45$ A's share
		1.72248×37.5	$= 64.593$ B's share
		1.72248×50	$= 86.124$ C's share
		1.72248×66.666	$= 114.832$ D's share

sum of the shares $= 299.999$ acres, or 300 very nearly, and is a proof of the whole.

Let

Let us now change the values of land as in the last problem, and see what each will have for his share. Suppose A's 15, B's 12, C's 8, and D's 5, shillings an acre; then $\frac{100}{15} = 6,666$, $\frac{300}{12} = 25$, $\frac{600}{8}$

$= 75$, $\frac{1000}{5} = 200$, and the sum of these quo-

tients is 306,666, therefore $\frac{300}{306,666} = 0,97826$

the common multiplier:

		<i>acres.</i>
then	$\left\{ \begin{array}{l} 0,97826 \times 6,666 = \\ 0,97826 \times 25, = \\ 0,97826 \times 75, = \\ 0,97826 \times 200, = \end{array} \right.$	$\left\{ \begin{array}{l} 6,5217 \text{ A's share} \\ 24,4565 \text{ B's share} \\ 73,3695 \text{ C's share} \\ 195,6520 \text{ D's share} \end{array} \right.$

the sum of the shares = 229,9997 acres,
which proves the whole to be right.

Example 2. Let 500 acres be divided among six persons, whose estates are as follow, viz. A's 40l. B's 20l. C's 10l. D's 100l. E's 400l. and F's 1000l. per annum, and the value of the land most convenient for each is A's, B's, and C's, each 7s. D's 10s. E's 15s. and F's 12s. an acre; now each estate divided by the value of his share of the land, will stand thus:

	s.	l.		
A	7)	40	(	5,71428
B	7)	20	(	2,85714
C	7)	10	(	1,42857
D	10)	100	(	10,
E	15)	400	(	26,66666
F	12)	1000	(	83,33333

} quotients ;

sum of the quotients = 129,99998, or 130.

M 4

Now

Now $\frac{500}{130} = 3,846\frac{1}{3}$, the common multiplier, by which multiplying each quotient, we shall have for each share as follows, viz.

	acres.
A =	21,9778
B =	10,9918
C =	5,4941
D =	38,4615
E =	102,5641
F =	320,5127

their sum is = 500,0020, and proves the whole.

N. B. If any single share should contain land of several different values, then use the mean to divide his estate by.

Also if there be different quantities as well as values, find what each quantity is worth at it's value, and add their sums together; then say, as the sum of the quantities, is to this sum; so is one acre, to it's mean value, to be made use of.

Now having found each person's share, they may be laid out in any form required, by the directions and problems given in the next section. But when several shares contain land of the same value, it is best to lay out their sum in the most convenient form, and then subdivide it.

For dividing of commons, &c. Surveyors generally measure the land of different value in separate parcels, and find the separate value thereof, which added in different sums, gives the whole content, and whole value.

By problem 1, they find each man's proportional share of the whole value, and then lay out for

for each person, a quantity of land equal in value to his share; this they effect by first laying out a quantity by guess, and then casting it up and finding it's value; and if such value be equal to his share of the whole value, the dividing line is right; if otherwise, they shift the dividing line a little, till by trial they find a quantity just equal in value, to the value of the required share.

If any single share contains land of several different values, each is measured separately, and their several values found; and if the sum of them be equal to the value sought, the division is right; if not, it must be altered till it is so.

OF LAYING OUT ANY GIVEN QUANTITY OF LAND.

As the quantity of land is generally given in acres, roods, and perches, it is necessary, first, to reduce them to square links, which may be performed by the following rule.

To reduce acres, roods, and perches, into square links,

Rule. 1. To the acres annex five cyphers on the right hand, and the whole will be links. *2.* Place five cyphers to the right of the roods, and divide this by 4, the quotient will be links. *3.* Place 4 cyphers on the right hand of the perches, divide this by 16, the quotient will be links. *4.* These sums added together, give the sum of square links in the given quantity.

PROBLEM I. To lay out a piece of land, containing any desired number of acres in form of a square.

This is no other than to determine the side of a square, that shall contain any desired number of acres; reduce therefore the given number of acres to square links, and the square root thereof will be the side of the square required.

PROBLEM

PROBLEM 2. *To lay out any desired quantity of land in form of a parallelogram, having either it's base or altitude given.*

Divide the content or area by the given base, and the quotient is the altitude; if divided by the altitude, the quotient is the base; by the same rule a rectangle may be laid out.

PROBLEM 3. *To lay out any desired quantity of land, in form of a parallelogram, whose base shall be 2, 3, 4, &c. times greater than it's altitude.*

Divide the area by the number of times the base is to be greater than the altitude, and extract the square root of the quotient; this square root will be the required altitude, which being multiplied by the number of times that the base is to be greater than the altitude, will give the length of the required base.

PROBLEM 4. *To lay out a given quantity of land in form of a triangle, having either the base or the perpendicular given.*

Divide the area by half the given base, if the base be given; or by half the given perpendicular, if the perpendicular be given; and the quotient will be the perpendicular or base required.

PROBLEM 5. *To lay out any given quantity of land in a regular polygon.*

1. Find in the following table, the area of a polygon of the same name with that required, the side of which is 1. 2. Divide the proposed area by that found in the table. 3. Extract the square root of the quotient, and the root is the side of the polygon required.

No. sides	Names.	Areas.
3	Triangle	0,433
4	Square	1,
5	Pentagon	1,72
6	Hexagon	2,598
7	Heptagon	3,634
8	Octagon	4,828
9	Nonagon	6,182
10	Decagon	7,694
11	Undecagon	9,365
12	Duodecagon	11,196

PROBLEM 6. *To lay out any quantity of land in a circle.*

1. Divide the area by 7854. 2. Extract the square root of the quotient, for the diameter required.

OF PLANE TRIGONOMETRY.

Plane Trigonometry is the art of measuring and computing the sides of plane triangles, or of such whose sides are right lines.

As this work is not intended to teach the elements of the mathematics, it will be sufficient for me just to point out a few of the principles, and give the rules of plain trigonometry, for those cases that occur in surveying. In most of these cases, it is required to find lines or angles, whose actual admeasurement is difficult or impracticable; they are discovered by the relation they bear to other given lines or angles, a calculation being instituted for that purpose; and as the comparison of one right line with another right line, is more convenient and easy, than the comparison of a right

right line to a curve; it has been found advantageous to measure the quantities of angles, not by the arc itself, which is described on the angular point, but by certain lines described about that arc.

If any three parts of a plane triangle be given, any required part may be found both by construction and calculation.

If two angles of a plane triangle are known in degrees, minutes, &c. the third angle is found by subtracting their sum from 180 degrees,

In a right-angled plane triangle, if either acute angle (in degrees) be taken from 90 degrees, the remainder will express the other acute angle.

When the sine of an obtuse angle is required, subtract such obtuse angle from 180 degrees, and take the sine of the remainder or supplement.

If two sides of a triangle are equal, a line bisecting the contained angle, will be perpendicular to the remaining side, and divide it equally.

Before the required side of a triangle can be found by calculation, it's opposite angle must first be given or found,

The required part of a triangle must be the last term of four proportionals, written in order under one another, whereof the three first are given or known.

In four proportional quantities, either of them may be made the last term; thus, let A, B, C, D, be proportional quantities.

As first to second, so is third to fourth, $A : B :: C : D$,

As second to first, so is fourth to third, $B : A :: D : C$,

As third to fourth, so is first to second, $C : D :: A : B$,

As fourth to third, so is second to first, $D : C :: B : A$.

Against the three first terms of every proportion, or stating, must be written their respective values taken from the proper tables.

If

If the value of the first term be taken from the sum of the second and third, the remainder will be the value of the fourth term or thing required; because the addition and subtraction of logarithms corresponds with the multiplication and division of natural numbers.

If to the complement of the first value, be added the second and third values, the sum, rejecting the borrowed index, will be the tabular number expressing the thing required: this method is generally used when radius is not one of the proportionals.

The complement of any logarithm, sine, or tangent, in the common tables, is it's difference from the radius 10.000.000, or it's double 20.000.000.

CANONS FOR TRIGONOMETRICAL CALCULATION.

1. The following proportion is to be used, when two angles of a triangle, and a side opposite to one of them is given to find the other side.

As the sine of the angle opposite the given side,
Is to the sine of the angle opposite the required
side;

So is the given side,
To the required side.

2. When two sides, and an angle opposite to one of them, is given to find another angle, use the following rule:

As the side opposite the given angle,
Is to the side opposite the required angle;
So is the sine of the given angle,
To the sine of the required angle.

The memory will be assisted in the foregoing cases, by observing that when a *side* is wanted, the proportion must begin with an *angle*; and when an *angle* is wanted, it must begin with a *side*.

3. When

3. When two sides of a triangle, and the included angle, are given, to find the other angles and side.

As the sum of the two given sides,

Is to their difference;

So is the tangent of half the sum of the two unknown angles,

To the tangent of half their difference.

Half the difference thus found, added to half their sum, gives the greater of the two angles, which is the angle opposite the greatest side. If the third side is wanted, it may be found by solution 1.

4. The following steps and proportions are to be used when the three sides of a triangle are given, and the angles required.

Let ABC , *plate 9, fig. 30*, be the triangle; make the longest side AB the base; from C , the angle opposite to the base, let fall the perpendicular CD on AB , this will divide the base into two segments AD , BD .

The difference between the two segments is found by the following proportion:

As the base AB ,

Is to the sum of the other sides ($AC + BC$);

So is the difference of the sides ($AC - BC$),

To the difference of the segments of the base ($AD - DB$).

Half the difference of the segments thus found, added to the half of AB , gives the greater segment AD , or subtracted, leaves the least DB .

In the triangle ADC find the angle ACD by solution 2; for the two sides AD and AC are known, and the right angle at D is opposite to one of them.

The complement of ACD gives the angle A .

Then in the triangle ABC , you have two sides AB , BC , and the angle at A opposite to one of them, to find the angles C and B .

OF THE LOGARITHMIC SCALES ON THE SECTOR.

There are three of these lines usually put on the sector, they are often termed the Gunter's lines, and are made use of for readily working proportions; when used, the sector is to be quite opened like a strait rule.

If the 1 at the beginning of the scale, or at the left-hand of the first interval, be taken for unity, then the 1 in the middle, or that which is at the end of the first interval, and beginning of the second, will express the number 10; and the 10 at the end of the right-hand of the second interval or end of the scale, will represent the number 100. If the first is 10, the middle is 100, and the last 1000; the primary and intermediate divisions in each interval, are to be estimated according to the value set on their extremities.

In working proportions with these lines, attention must be paid to the terms, whether arithmetical or trigonometrical, that the first and third term may be of the same name, and the second and fourth of the same name. To work a proportion, take the extent on their proper line from the first term to the third in your compasses, and applying one point of the compasses to the second, the other applied to the right or left, according as the fourth term is to be more or less than the second, will reach to the fourth.

Example 1. If 4 yards of cloth cost 18 shillings, what will 32 yards cost? This is solved by the line of numbers; take in your compasses the distance between 4 and 32, then apply one foot thereof on the

the same line at 18, and the other will reach 144, the shillings required.

Example 2. As radius to the hypotenuse 120, so is the sine of the angle opposite the base $30^{\circ} 17'$ to the base. In this example, radius or the sine of 90° , and the sine of $30^{\circ} 17'$ taken from the line of sines, and one foot being then applied to 120, on the line of numbers, and the other foot to the left will reach to $60\frac{1}{2}$ the length of the required base. The foot was applied to the left, because the legs of a right-angled triangle are less than the hypotenuse.

Example 3. As the cosine of the latitude $51^{\circ} 30'$, (equal the sine of $38^{\circ} 30'$) is to radius; so is the sine of the sun's declination $20^{\circ} 14'$, to the sine of the sun's amplitude. Take the distance between the sines of $38^{\circ} 30'$ and $20^{\circ} 14'$ in your compasses; set one foot on the radius or sine of 90° , and the other will reach to $33\frac{1}{4}^{\circ}$, the sun's amplitude required.

CURIOUS AND USEFUL TRIGONOMETRICAL PROBLEMS.

The following problems, though of the greatest use, and sometimes of absolute necessity to the surveyor, are not to be found in any of the common treatises on surveying. The maritime surveyor can scarce proceed without the knowledge of them; nor can a kingdom, province, or county be accurately surveyed, unless the surveyor is well acquainted with the use and application of them. Indeed, no man should attempt to survey a county, or a sea-coast, who is not master of these problems. The second problem, which is peculiarly useful for determining the exact situation of sands, or rocks, within sight of three places upon land,

land, whose distances are well known, was first proposed by Mr. Townly, and solved by Mr. Collins, Phil. Trans. N^o 69. There is "no problem more useful in surveying, than that by which we find a *station*, by observed angles of three or more objects, whose reciprocal distances are known, but distance and bearing from the place of observation are unknown.

"Previous to the resolution of these problems, another problem for the easy finding the segment of a circle, capable of containing a given angle, is necessary, as will be clear from the following observation.

"Two objects can *only* be seen under the *same* angle, from some part of a circle passing through those objects, and the place of observation.

"If the angle under which those objects appear, be less than 90° , the place of observation will be somewhere in the greater segment, and those objects will be seen under the same angle from every part of the segment.

"If the angle, under which those objects are seen, be more than 90° , the place of observation will be somewhere in the lesser segment, and those objects will be seen under the same angle from every part of that segment." * Hence from the situation of three known objects, we are able to determine the station point with accuracy.

PROBLEM. To describe on a given line *BC*, fig. 29, plate 9, a segment of a circle capable of containing a given angle.

Method 1. Bisect *BC* in *A*. 2. Through the point of bisection, draw the indefinite right line *DE* perpendicular to *BC*. 3. Upon *BC*, at the point

* Dalrymple's Essay on Nautical Surveying, in which another mode of solving this problem is given.

point C, constitute the angles DCB , FCB , GCB , HCB , respectively equal to the difference of the angles of the intended segments and 90 degrees: the angle to be formed on the same side with the segment, if the angle be less than 90; but on the opposite, if the angle is to be greater than 90 degrees. 4. The points D, F, G, H, where the angular lines CD, CF, CG, CH, &c. intersect the line DE, will be the centers of the intended segments.

Thus, if the intended segment is to contain an angle of 120° , constitute on BC, at C, (on the opposite side to which you intend the segment to be described,) the angle DCB equal to 30° , the difference between 90° and 120° ; then on center D and radius DC, describe the segment C, 120, B, in every part of which, the two points C and B will subtend an angle of 120 degrees.

If I want the segment to contain 80 degrees at C, on BC make an angle BCG , equal 10 degrees, and on the same side of BC as the intended segment; then on G, with radius GC, describe segment C 80 B, in every part of which, C and B will subtend an angle of 80 degrees.

Method 2. By the sector. Bisect BC as before, and draw the indefinite line DE, make AC radius, and with that extent open the sector at 45 on the line of tangents, and set off on the line DE, the tangent of the difference between the observed angle and 90 degrees, on the same side as the intended segment, if the observed angle is less than 90; on the contrary side if more than 90 degrees.

If the angle is of 90 degrees, A is the center of the circle, and AC the radius.

The intersection of the line DE with any circle, is the center of a segment, corresponding to

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half the angle of the segment in the first circle. Hence, if the difference between the observed angle and 90° , be more than the scale of tangents contains, find the center to double the angle observed, and the point where the circle cuts the indefinite line, will be the required center.

PROBLEM I. *To determine the position of a point, from whence three points or a triangle can be discovered, whose distances are known.*

The point is either without, or within the given triangle, or in the direction of two points of the triangle.

CASE I. *When the three given objects form a triangle, and the point or station whose position is required, is without the triangle.*

Example. Suppose I want to determine the position of a rock D, fig. 27, plate 9; from the shore; the distances of the three points A, C, B, or rather the three sides AC, CB, AB, of the triangle A, B, C being given.

In the first place, the angles ADC, CDB, must be measured by an Hadley's sextant or theodolite; then the situation of the point D may be readily found either by calculation or construction.

By construction. Method 1. On AC, fig. 28, plate 9, describe, by problem 1, a circle capable of containing an angle equal to the angle ADC; on CB, a segment containing an angle equal to the angle CDB; and the point of intersection D is the place required.

Another method. Make the angle EBA, fig. 39, plate 9, equal to the angle ADE, and the angle BAE equal to the angle EDB. Through A, B, and the intersection E, describe a circle AEBDA; through E, C, draw EC, and produce it to intersect

sect the circle at D; join A D, B D, and the distances A D, C D, B D, will be the required distances.

Calculation. In the triangle A B C, are given the three sides, to find the angle B A C. In the triangle A E B are given the angle B A E, the angles A B E, A E B, and the side A B, to find A E and B E.

In the triangle A E D, we have the side A E, and the angles A E D, A D E, and consequently D A F, to find the sides A D.

The angle A D E added to the angle A E C, and then taken from 180° , gives the angle D A E. The angle C A E taken from the angle D A E, gives the angle C A D, and hence D C. Lastly, the angle A E C taken from A E B, gives D E B, and consequently, in the triangle D E B, we have E B, the angle D E B, and the angle E D B, to find B D.

In this method, when the angle B D C is less than that of B A C, the point C will be above the point E; but the calculation is so similar to the foregoing, as to require no particular explanation.

When the points E and C fall too near together to produce E C towards D with certainty, the first method of construction is the most accurate.

CASE 2. When the given place or station D, fig. 38, plate 9, is without the triangle made by the three given objects A B C, but in a line with one of the sides produced.

Measure the angle A D B, then the problem may be easily resolved either by construction or calculation.

By construction. Subtract the measured angle A D B from the angle C A B, and you obtain the angle A B D; then at B, on the side B A, draw the angle

angle $A B D$, and it will meet the produced side CA at D ; and DA , DC , DB , will be the required distances.

By calculation. In the triangle ABD , the angle D is obtained by observation, the $\angle B A D$ is the supplement of the angle $C A B$ to 180° : two angles of the triangle being thus known, the third is also known; we have consequently, in the triangle ABD , three angles and one side given to find the length of the other two sides, which are readily obtained by the preceding canons.

CASE 3. When the station point is in one of the sides of the given triangle, fig. 1, plate 13.

By construction. 1. Measure the angle $B D C$. 2. Make the angle $B A E$ equal to the observed angle. 3. Draw $C D$ parallel to $E A$, and D is the station point required.

By calculation. Find the angle B in the triangle ABC , then the angles B and $B D C$ being known, we obtain $D C B$; and consequently, as $\sin.$ angle $B D C$ to $B C$, so is $\sin.$ angle $D C B$ to $B D$.

CASE 4. When the three given places are in a strait line.

Example. Being at sea near a strait shore, I observed three objects A , B , C , fig 2, plate 13, which were truly laid down on my chart; I wished to lay down the place of a sunken rock D ; for this purpose, the angles $A D B$, $B D C$, were observed with Hadley's quadrant.

By construction. Method 1. On $A B$, fig. 3, plate 13, describe the segment of a circle, capable of containing the observed angle $A D B$. On $B C$ describe the segment of a circle capable of containing the angle $B D C$; the point D will be at the intersection of the arches, and by joining DA , DB , DC , you obtain the required distances.

Method 2. Make the angle ACE , *fig. 4, plate 13*, equal to ADB , and the angle EAC equal to BDC ; and from the point of intersection E , through B , draw a line to ED , to intersect the arc ADC ; join A, D ; and D, C , and DA, DB, DC , are the required distances.

By calculation. 1. In the triangle CAE , *fig. 4, plate 13*, we have all the angles, and the side AC , to find AE . 2. In the triangle ABE , AB, AE , and the included angle are given, to find the angles ABE, ABE . 3. In the triangle BDC , the angle BDC and $DCB (= ABE)$ are given, and consequently, the angle DCB , and the side BC ; hence it is easy to obtain DB .

CASE 5. When the station falls within the triangle, formed by the three given objects.

Let ABC , *fig. 19, plate 13*, represent three towers, whose distance from each other is known; to find the distance from the tower D , measure the angles ADC, BDC, ADB .

Construction. On two of the given sides AC, AB , *fig. 20, plate 13*, describe segments of circles capable of containing the given angles, and the point D of their intersection, will be the required place.

Another method. Or, we may proceed as in some of the foregoing cases; making the angle ABE , *fig. 21, plate 13*, equal to the angle ADE , and BAE equal BDE ; and describe a circle through the three points A, B, E , and join E, C , by the line EC ; and the point D , where BC intersects the circle E, A, D, B, E , will be the required station.

*CASE 6. When the station point D , *fig. 7, plate 13*, falls without the triangle ABC , but the point C falls towards D .*

Thus, let ABC , *fig. 7, plate 13*, represent three towers, whose respective distances from each other are known; required their distance from the point D .

D. Measure the angles $A D C$, $B D C$, and to prove the truth of the observations, measure also $A D B$.

Then, *by construction, method 1.* On $A C$, *fig. 8, plate 13*, describe a circle capable of containing the angle $B D C$, and on $A B$, one capable of containing the angle $A D B$, and the point of intersection will be the place required.

Or, it may be constructed by method 2, case 1, which gives us the point D , *fig. 9, plate 13*, compared with *fig. 39, plate 13*. The calculation is upon principles so exactly like those in method 2, case 1, that a further detail would be superfluous.

The point of station D found instrumentally.

The point D may be readily laid down on a draught, by drawing on a loose transparent paper indefinite right lines $D A$, $D B$, $D C$, at angles equal to those observed; which being placed on the draught so as each line may pass over, or coincide with, it's respective object, the angular point D will then coincide with the place of observation.

Or,
Provide a graduated semicircle of brass, about 6 inches in diameter, having three radii with chamfered edges, each about 20 inches long, (or as long as it may be judged the distance of the stations from the three given objects may require) one of which radii to be a continuation of the diameter that passes through the beginning of the degrees on the semicircle, but immoveably fixed to it, the other two moveable round the center, so as to be set and screwed fast to the semicircle at any angle. In the center let there be a small socket, or hole, to admit a pin for marking the central point on the draught. When the sloped edges of the two moveable radii are set and screwed fast to the semicircle, at the respective degrees and minutes of

the two observed angles, and the whole instrument moved on the draught until the edges of the three radii are made to lie along the three stasimetric points, each touching it's respective point, the center of the semicircle will then be in the point of station D; which may be marked on the draught, through the socket, with a pin. Such an instrument as this may be called a *station-pointer*; and would be found convenient for finding the point of station readily and accurately, except when the given objects were near; when the breadth of the arch, and of the radii, and of the brass about the center of the semicircle might hinder the points to be seen, or the radii to be placed so as to comprehend a very small angle between them.

The three succeeding problems may occur at sea, in finding the distances and position of the rocks, sands, &c. from the shore; in many cases of maritime surveying, they are also very serviceable in making a map of a country, from a series of triangles, derived from one or more measured bases.

PROBLEM I. *Given the distance of two objects A B, fig. 5, plate 9, and the angles A D B, B D C, B C A, to find the distance of the two stations D, C, from the objects A, B.*

By construction. Assume d c any number at pleasure, and make the angles b d c, a d c, &c. respectively equal to the angles B D C, A D C, &c. and join a b; it is plain that this figure must be similar to that required; therefore, draw A B, fig. 4, plate 9, equal the given distance, and make A B C equal to a b c, B A C to b a c, and so on respectively; join the points, and you have the distances required.

By calculation. In the triangle a d c, we have d c, a d c, and a c d, to find a d, a c; in b c d, we have
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in like manner the three angles, and $d c$, to find $d b$, $b c$.

In the triangle $a d b$, we have $a d$, $b d$, and the angle $a d b$, to find $a b$. Hence by the nature of similar figures, as $a b$ to $A B :: d c$ to $D C :: a d$ to $A D :: b d$ to $B D :: b c$ to $B C$.

PROBLEM 2. *The distances of three objects A, B, C, from each other, and the angles $A D C$, $C D E$, $C E D$, $C E B$, fig. 6, plate 9, being given, to find the sides $A D$, $D C$, $D E$, $E C$, and $E B$.*

Assume any line $d c$ at pleasure, make the angle $c d e$ equal the angle $C D E$, and angle $c e d$ equal to the angle $C E D$; also the angle $c d a$, equal to the angle $C D A$, and the angle $c e b$ equal to the angle $C E B$; produce $a d$, $b c$, to intersect each other at f , and join $c f$.

It is evident that the figures $c d f e$, $C D F E$, are similar; therefore, on $A C$, fig. 7, plate 9, describe a segment of a circle, containing an angle $A F C$ equal to $a f c$; and on $C B$ a segment capable of containing an angle $C F B$, equal the angle $c f b$; from the point of intersection F , draw $F A$, $F B$, $F C$; make the angle $F C D$ equal the angle $f c d$, and $F C E$ equal the angle $f c e$, which completes the construction; then by assuming $d c$ equal any number, the rest may be found as before.

This method fails when $A D$ is parallel to $B E$, fig. 8, plate 9; therefore, having described the segments $A D C$, $B C$, draw $C F$, to cut off a segment equal to the angle $C D F$, and the right line $C G$, to cut off a segment equal to the angle $C E G$; G , F will be in the right line $D E$; therefore, join G , F , and produce the line each way, till it intersects the segments, and the points D , E will be the stations required.

PROBLEM

PROBLEM 3. *Four points B, C, D, F, fig. 9, plate 9, or the four sides of a quadrilateral figure, with it's angles being given, and the angles BAC, BAE, AED, DEF, known by observation; to find the station point A, E, and consequently, the length of the lines AB, AC, ED, EF.*

By construction. 1. On BC, describe the segment of a circle, to contain an angle equal to BAC. 2. From C, draw the chord CM, so that the angle BCM may be equal to the supplement of the angle BAE. 3. On DF, describe the segment of a circle capable of containing an angle equal to DEF; join MN cutting the two circles at A, and E, the required points.

By calculation. In the triangle BCM, the angle BCM, (the supplement of BAE,) and the angle BMC, (equal BAC,) and the side BC are given, whence it is easy to find MC. In the same manner, DN in the triangle DNF may be found; but the angle MCD (equal angle BCD, less angle BCM) is known with the legs MC, CD; consequently MD, and the angle MDC, will be readily found.

The angle MDN (equal angle CDF, less CDM, less FDN,) and MD, DN, are known; whence we find MN, and the angles DMN, DN M.

The angle CMA, (equal DMC added to DMN,) the angle MAC, (equal MAB added to BAC,) and the side MC are given; therefore, by calculation, MA, and AC, will also be known.

In the triangle EDN, the side DN, and the angles E and N are given; whence we find EN, ED, and consequently AE equal MN, less MA, less EN.

And in the triangle ABC, the angle A, with
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it's sides BC , AC , are known; hence AB , and angle BCA , are found.

In the triangle EFD , the angle E , with the sides ED , DF , being known, EF and the angle EDF will be found.

Lastly, in the triangle ACD , the angle ACD , (equal BCD , less BCA ,) and AC , CD , are given; hence AD is found, as in the same manner EC in the triangle ECD .

Note. That in this problem, and in problems 1, and 2, if the two stations fall in a right line with either of the given objects, the problem is indeterminate. As to the other cases of this problem, they fall in with what has already been said.

The solution of this problem is general, and may be used for the two preceding ones: for suppose CD the same point in the last figure, it gives the solution of problem 2; but if B , C be supposed the same points DF , you obtain the solution of problem 1.

PROBLEM 4. *Having the distance and magnetic bearings of two points A and B , fig. 10, plate 9, protracted, at any station S , not very oblique to AB , to find it's distance from these points by the needle.*

At S , with a good magnetic needle, take the bearings of A and B in degrees, and parts of a degree; then from these points, draw out their respective bearings in the opposite direction towards S ; that is, if A bears exactly north, draw a line from the point A exactly south; if it bears east, 10 or 20 degrees southward, draw the line west, 10 or 20 degrees northward; and so for any other bearing, draw the opposite bearing of B in the same manner, and S , the intersection of these two points, will be the point of station, and SA , SB , the distances required.

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This is an easy method of finding the distance of any station from two places, whose distances have been accurately determined before, and will be found very convenient in the course of a survey, and on many occasions sufficiently exact, provided the places are not too remote from the station, nor the intersection of the bearings too oblique. If the needle be good, a distance of 20 miles is not too far, when the angle subtended by the two places, is not less than 50 degrees, or more than 140.

PROBLEM 5. *To reduce angles to the center of the station.*

In surveys of kingdoms, provinces, counties, &c. where signals, churches, &c. at a distance are used for points of observation, it very often happens that the instrument cannot be placed exactly at the center of the signal or mark of observation; consequently, the angle observed, will be either greater, less, or equal to that which would have been found at the center. This problem shews how to reduce them to the center; the correction seldom amounts to more than a few seconds, and is therefore seldom considered, unless where great accuracy is required.

The observer may be considered in three different positions with respect to the center, and the objects; for he is either in a line with the center, and one of these objects, or in an intermediate one, that is, a line from this center to the observer produced, would pass between the objects; or he is, lastly, in an oblique direction, so that a line from the center to him would pass without the objects.

In the first position, *fig. 11, plate 9*, where the observer is at *o*, between the center and one of the objects, the exterior angle *m o n* is greater than the

the angle $m e n$, at the center, by the angle $e m o$; therefore, taking $e m o$ from the observed angle, you have that at the center.

If the observer is at a , *fig. 11, plate 9*, the exterior angle $m a n$ is greater than that of $m e n$ at the center, by the value of n ; therefore, taking this from the observed angle, you obtain the angle at the center. But if the observer is further from the objects than the center, as at i , the observed angle $m i n$, is less than that at the center $m e n$, by the angle m ; therefore, by adding m to the observed angle, you obtain the angle $m e n$ at the center. In the same manner, if the observer is at u , we should add the angle n to the observed $m u n$, in order to have the angle $m e n$ at the center.

Case 2. When the observer is at o , *fig. 12, plate 9*, draw $a o$, and the exterior angle d exceeds the angle u at the center by the angle m , and the exterior angle c exceeds the angle at the center a , by the angle n ; therefore, $m o n$ exceeds the angle at the center, by the value of the two angles m and n : these, therefore, must be subtracted from it to obtain the central angle. On the contrary, if the observer is at a , the two angles m and n must be added to the observed angle.

Case 3. *Fig. 13, plate 9.* When the observer is at o , having measured the angles $m o n$, $m o e$, the angle i is exterior to the two triangles $m o i$, $n e i$; therefore, to render $m e n$, equal to $m i n$, we must add the angle n ; and to render the exterior angle $m i n$, equal to the observed angle $m o n$, we must take away the angle m ; therefore, adding m to the observed angle, and subtracting n from the total, we obtain the central angle $m e n$.

From what has been said it is clear, that in the first case, you are to add or subtract from the observed

served angles, that of the angles m or n , which is not in the direction of the observer.

In the second position, you have either to subtract or add the two angles m e n .

In the third position, you add to the observed angle, that of the two m or n , which is of the same side with the observer, and subtract the other.

To know the position of the observer, care must be taken to measure the distance of the instrument from the center, and the angle this center makes with the objects.

An inspection of the figures is sufficient to shew how the value of the angles m n may be obtained. Thus, in the triangle m o e , we have the angle at o , the distance o e , and the distances e m , o m , (which are considered as equal,) given.

OF THE REDUCTION OF TRIANGLES, FROM ONE PLANE TO ANOTHER.*

After the reduction of the observed angles to the center of each respective station, it is generally necessary to reduce the parts of one, or of several triangles to the same level.

Case 1. Let us suppose the three points A , P , E , *fig. 15, plate 9*, to be equally distant from the center of the earth, and that the point R is higher than these points by the distance or quantity R E ; now it is required to reduce the triangle A P R to that of A P E .

By the following rule, you may reduce the angles R A P , R P A , which have their summits in the plane of reduction, to the angle E P A , E A P .

Rule.

* *Traité de Trigonometrie, par Cagnoli.*

Rule. The cosine of the reduced angle is equal to the cosine of the observed angle, divided by the cosine of the angle of elevation.

These two angles being known, the third E is consequently known; we shall, however, give a rule for finding $A E P$, independent of the other two.

Rule. The cosine of the reduced angle is equal to the cosine of the observed angle, lessened by the rectangle of the sines of the angles of elevation, divided by the rectangle of the cosine of the same angles.

The reduction of the sides can be no difficulty.

Case 2. Fig. 17, plate 9. Let $A R r$ be the triangle to be reduced to the plane $A E e$, the points E, e , of the vertical lines $R E, r e$, being supposed equally distant from the center of the earth.

Prolong the plane $A E e$ to P , that is, till it meets the line $R r$ produced to P ; and the value of $E A e$ will be found by the following formulæ.

$$1. \text{Tang. } \frac{1}{2} (P A R + P A r) = \text{tang. } \frac{1}{2} R A r \times \frac{\text{tang. } \frac{1}{2} (R A E + r A e)}{\text{tang. } \frac{1}{2} (R A E - r A e)}.$$

Knowing the half sum and half difference of $P A R$ and $P A r$, we obtain the value of each of the angles; the value of $P A E$ and $P A e$, may be then obtained by the first of the two preceding rules, and the difference between them is the angle sought.

Let C , *fig. 16, plate 9*, be the center of the earth, let $A B$ be the side of a triangle reduced to a common horizon by the preceding methods; if it be required to reduce this to the plane $D E$, as these planes are parallel, the angles will remain the same; therefore, the sides only are to be reduced, the mode of performing which is evident from the figure.

Method

Method of referring a series of triangles to a meridian line, and another line perpendicular to it.

This method will be found somewhat similar to Mr. Gale's, described at full in the article of surveying; it is a mode that should be adopted wherever extreme accuracy is required, for whatever care is taken to protract a series of triangles, the protractor, the points of the compasses, the thickness of the line, the inequality of the paper, &c. will produce in the fixing of the points of a triangle an error, which though small at first, will have it's influence on those that succeed, and become very sensible in proportion as the number of triangles is augmented. This multiplication of errors is avoided by the following problem.

Let A B, *fig. 14, plate 9*, be the meridian, C D the perpendicular, and the triangles o a d, d a e, d e g, e g i, g i l, those that have been observed; from the point o, (which is always supposed to be on a meridian, or whose relation to a meridian is known) observe the angle B o a, to know how much the point a declines from the meridian.

In the right-angled triangle o B a, we have the angle B o a, and the right angle, and consequently the angle o B, a b, together with the side o a, to find O B, and B a.

For the point d, add the angle B o a to the observed angle a o d, for the $\angle$ d o b, or it's equal o d m, and the complement is the angle m o d, whence as before, to find o m and m d.

For the point g, add the angles m d o, o d a, a d e, and e d g, which subtract from 360, to obtain the angle g d r, of the right-angled triangle g d r; hence we also readily, as in the preceding triangles, obtain $r g = m t$, which added to m o, gives t o the distance from the meridian. Then we obtain

obtain rd , from which taking dm , you obtain rm , equal gt , the distance from the perpendicular.

For the point e , take the right angle rdf , from the two angles rdg , and the remainder is the angle fdg of the right-angled triangle dfe ; hence we obtain fed and df , which added to db , gives bf , equal to xe , the distance from the meridian; from the same right-angled triangle we obtain fe , which added to fn , equal to dm , gives en , the distance from the perpendicular.

For the point i , add together the angles rgd , dge , egi , and from the sum subtract the right angle rgh , and you obtain the angle g , of the right-angled triangle ghi , and consequently the angle i ; hence also we get hi , equal tp , which added to to , gives op distance from the meridian, and gh , from which subtracting gt , we obtain th , equal pi distance from the perpendicular.

For the point l , the angle hge added to the angle lgi , gives angle lgk of the right-angled triangle gkl , and of course the angle glk , whence we obtain kl or ty , which added to to , gives oy distance from the meridian; hence we also obtain gk , which taken from gt , gives Kt , equal ly distance from the perpendicular.

If, before the operation, no fixed meridian was given, one may be assumed as near as possible from the point o ; for the error in it's position will not at all influence the respective position of the triangles.



OF SURVEYING.

THE practice of surveying may be considered as consisting of four parts. 1. Measuring strait lines. 2. Finding the position of strait lines with respect to each other. 3. Laying down or planning upon paper these positions and measures. 4. Obtaining the superficial measure of the land to be surveyed.

We may therefore define *land surveying* to be the art which teaches us to find how many times any customary measure is contained in a given piece of ground, and to exhibit the true boundaries thereof, in a plan or map.

A *station line* is a strait line, whose length is accurately ascertained by a chain, and the bearing determined by some graduated instrument.

An *offset* is the distance of any angular point, in the boundary from the station line, measured by a line perpendicular thereto.

The curvature of the earth within the limits of an ordinary survey, is so inconsiderable, that it's surface may be safely considered by the land surveyor as a *plane*. In a large extent, as a province, or a kingdom, the curvature of the earth's surface becomes very considerable, and due allowance must be made for it.

All planes, how many sides soever they consist of, may be reduced into triangles, and may therefore be considered as composed thereof; and consequently what is required in surveying, are such instruments as will measure the length of a side, and the quantity of an angle of a triangle.

GENERAL RULES.

A few general observations, or hints, can only be expected in this place ; for after all that we can say, the surveyor must depend on his own judgment for contriving his work, and his own skill in discriminating, among various methods, that which is best.

The first business of the surveyor is, to take such a general view of the ground to be surveyed, as will fix a map thereof in his mind, and thence determine the situations for his station lines, and the places where his instruments may be used to the greatest advantage.

Having settled the plan of operations, his next business is to examine his instruments, and see that they are all in proper order, and accurately adjusted. He should measure carefully his chain, and if there be any errors therein, correct them ; prepare staves, marks, &c. for distinguishing the several stations.

The fewer stations that can be made use of, the less will be the labour of the survey ; it will also be more accurate, less liable to mistakes while in the field, or errors when plotting the work at home.

The station lines should always be as long as possible, where it can be done without rendering the offsets too large ; where great accuracy is required, these lines should be repeatedly measured, the first great point being the careful mensuration of your station lines, the second to determine the situation of places adjoining to them. For every station line is the basis of the succeeding operations, and fixes the situation of the different parts.

The surveyor should so contrive his plan, as to avoid the multiplication of small errors, and particularly

ticularly those that by communication will extend themselves through the whole operation. If the estate be large, or if it be subdivided into a great number of fields, it would be improper to survey the fields singly, and then put them together, nor could a survey be accurately made by taking all the angles and boundaries that inclose it. If possible, fix upon for station points two or more eminent situations in the estate, from whence the principal parts may be seen; let these be as far distant from each other as possible; the work will be more accurate when but few of these stations are made use of, and the station lines will be more convenient, if they are situated near the boundaries of the estate.

Marks should be erected at the intersection of all hedges with the station line, in order to know where to measure from, when the fields are surveyed. All necessary angles between the main stations should be taken, carefully measuring in a right line the distance from each station, noting down, while measuring, those distances where the lines meet a hedge, a ditch, &c. If any remarkable object be situated near the station line, its perpendicular distance therefrom should be ascertained; in the same manner, all offsets from the ends of hedges, ponds, houses, &c. from the main station line should be obtained, care being taken that all observations from the station line, as the measure of angles, &c. be always made from points in the station line.

When the main stations, and every thing adjoining to them, have been found, then the estate may be subdivided into 2 or 3 parts by new station lines, fixing the stations where the best views can be obtained; these station lines must be accurately measured,

measured, and the places where they intersect hedges be exactly ascertained, and all the necessary offsets determined.

This effected, proceed to survey the adjoining fields, by observing the angles that the sides make with the station lines at their intersections therewith; the distances of each corner of the field from these intersections, and that of all necessary offsets.

Every thing that could be determined from these stations being found, assume more internal stations, and thus continue to divide and subdivide, till at last you obtain single fields, repeating the same operations as well for the inner as exterior work, till all be finished.

Every operation performed, and every observation made, is to be carefully noted down as the data for fixing the situations upon the plan. The work should be closed as often as convenient, and in as few lines as possible; what is performed in one day should be carefully laid down every night, in order not only to discover the regular process of the work, but to find whether any circumstance has been neglected, or any error committed, noticing in the field-book how one field lies by another, that they may not be displaced in the draft.

If an estate be so situated, that the whole cannot be surveyed together, because one part cannot be seen from the other, divide it into 3 or 4 parts, and survey them separately, as if they were lands belonging to different persons, and at last join them together.

As it is thus necessary to lay down the work as you proceed, it will be proper to find the most convenient scale for this purpose: to obtain this,

measure the whole length of the estate in chains, then consider how many inches long your plan is to be, and from these conditions you will ascertain how many chains you must have in an inch, and thence choose your scale.

In order that the surveyor may prove his work, and see daily that it goes on right, let him choose some conspicuous object that may be seen from all or greater part of the estate he is surveying, and then measure with accuracy the angle this object makes, with two of the most convenient stations in the first round, entering them in the field book or sketch were they were taken; when you plot your first round, you will find the true situation of this object by the intersection of the angles. Measure the angle this object makes, with one of your station lines in the second, third, &c. rounds: these angles, when plotted one day after another, will intersect each other in the place of the object, if the work be right; otherwise some mistake has been committed, which must be corrected before the work is carried any further.

Fields plotted from measured lines only, are always plotted nearest the truth, when those lines form at their meeting, angles, nearly approaching to right angles.

OF THE MOST ADVANTAGEOUS CIRCUMSTANCES FOR A SERIES OF TRIANGLES.

The three angles of every triangle should always, if possible, be measured.

As it is impossible to avoid some degree of error in taking of angles, we should be careful so to order our operations, that this error may have the least possible influence on the sides, the exact measure of which is the end of the operations.

Now

Now in a right-lined triangle, it is necessary to have at least one side measured mediately or immediately; the choice of the base is therefore the fundamental operation; the determinations will be most accurate to find one side, when the base is equal to the side required; to find two sides, an equilateral triangle is most advantageous.

In general, when the base cannot be equal to the side or sides sought, it should be as long as possible, and the angles at the base should be equal.

In any particular case, where only two angles of a triangle can be actually observed, they should be each of them as near as possible to 45° ; at any rate their sum should not differ much from 90° , for the less the computed angle differs from 90° ; the less chance there will be of any considerable error in the intersection.

DESCRIPTION OF THE VARIOUS INSTRUMENTS USED IN SURVEYING, AND THE METHOD OF APPLYING THEM TO PRACTICE, AND EXAMIN- ING THEIR ADJUSTMENTS.

The variety of instruments that are now made use of in surveying is so great, and the improvements that have been made within these few years are so numerous, that a particular description of each is become necessary, that by seeing their respective merits or defects, the purchaser may be enabled to avail himself of the one, and avoid the other, and be also enabled to select those that are best adapted to his purposes.

The accuracy of geometrical and trigonometrical mensuration, depends in a great degree on the exactness and perfection of the instruments made use of; if these are defective in construction, or difficult in use, the surveyor will either be subject to

error, or embarrassed with continual obstacles. If the adjustments, by which they are to be rendered fit for observation, be troublesome and inconvenient, they will be taken upon trust, and the instrument will be used without examination, and thus subject the surveyor to errors that he can neither account for, nor correct.

In the present state of science, it may be laid down as a maxim, that every instrument should be so contrived, that the observer may easily examine and rectify the principal parts; for however careful the instrument-maker may be, however perfect the execution thereof, it is not possible that any instrument should long remain accurately fixed in the position in which it came out of the maker's hands, and therefore the principal parts should be moveable, to be rectified occasionally by the observer.

AN ENUMERATION OF INSTRUMENTS, NECESSARY
FOR A SURVEYOR;

Fewer or more of which will be wanted, according to the extent of his work, and the accuracy required.

A case of good pocket instruments.

A pair of beam compasses.

A set of plotting scales.

Three or four parallel rules, either those of *fig. C, plate 2*, or *fig. F GH, plate 2*.

A pair of proportionable compasses.

A pair of triangular ditto.

A pantographer.

A cross staff.

A circumferenter.

Hadley's sextant.

A theodolite.

A surveying compass.

Measuring

Measuring chains.

King's surveying quadrant.

A measuring wheel.

A spirit level.

Station staves for the level.

A protractor, with or without a nonius.

To these should be added for county and marine surveying :

An astronomical quadrant.

A good refracting and reflecting telescope.

A copying glass.

For marine surveying.

A station pointer.

An azimuth compass.

One or two boat compasses.

Besides these, a number of measuring rods, iron pins, or arrows, &c. will be found very convenient, and two or three offset staves, which are strait pieces of wood, six feet seven inches long, and about an inch and a quarter square; they should be accurately divided into ten equal parts, each of which will be equal to one link. These are used for measuring offsets, and to examine and adjust the chain.

Five or 6 staves of about 5 feet in length, and $1\frac{1}{2}$ inch in diameter, the upper part painted white, the lower end shod with iron, to be struck in the ground as marks.

Twenty or more arrows, ten of which are always wanted to use with the chain, to count the number of links, and preserve the direction of the chain, so that the distance measured may be really in a strait line.

Or

OF THE INSTRUMENTS USED IN MEASURING
STRAIT LINES.

OF THE CHAIN.

The length of a strait line must be found mechanically by the chain, previous to ascertaining any distance by trigonometry: on the exactness of this mensuration the truth of the operations will depend. The surveyor, therefore, cannot be too careful in guarding against rectifying, or making allowances for every possible error; and the chain should be examined previous and subsequent to every operation.

For the chain, however useful and necessary, is not infallible, it is liable to many errors. 1. In itself. 2. In the method of using it. 3. In the uncertainty of pitching the arrows; so that the surveyor who wishes to obtain an accurate survey, will depend as little as possible upon it, using it only where absolutely necessary as a basis, and then with every possible precaution.

If the chain be stretched too tight, the rings will give, the arrow incline, and the measured base will appear shorter than it really is; on the other hand, if it be not sufficiently stretched, the measure obtained is too long. I have been informed, by an accurate and very intelligent surveyor, that when the chain has been much used, he has generally found it necessary to shorten it every second or third day.

Gunter's chain is the measure universally adopted in this kingdom for the purposes of land-surveying, being exceeding well adapted for the mensuration of land, and affording very expeditious methods of casting up what is measured. It is 66 feet, or 4 poles in length, and is divided into 100 links, each link with the rings between them is 7.92 inches

ches long, every tenth link is pointed out by pieces of brass of different shapes, for the more ready counting of the odd links.

The English acre is 4840 square yards, and Gunter's chain is 22 yards in length, and divided into 100 links; and the square chain, or 22 multiplied by 22, gives 484, exactly the tenth part of an acre; and ten chains squared are equal to one acre; consequently, as the chain is divided into 100 links, every superficial chain contains 100 multiplied by 100, that is, 10.000 square links; and 10 superficial chains, or one acre, contains 100.000 square links.

If, therefore, the content of a field cast up in square links be divided by 100.000, or (which is the same thing) if from the content we cut off the five last figures, the remaining figure towards the left hand, gives the content in acres, and consequently the number of acres at first sight; the remaining decimal fraction, multiplied by 4, gives the roods, and the decimal part of this last product multiplied by 40, gives the poles or perches.

Thus, if a field contains 16,54321 square links, we see immediately that it contains 16 acres. 54321 multiplied by 4, gives 2,17284, or 2 roods and 17284 parts; these multiplied by 40, produce 6,91360, or 6 poles, 91360 parts.

Directions for using the chain. Marks are first to be set up at the places whose distance is to be obtained; the place where you begin may be called your first station; and the station to which you measure the second station: two persons are to hold the chain, one at each end; the foremost, or chain leader, must be provided with nine arrows, one of which is to be put down (perpendicularly) at the end of the chain when stretched out, and to

to be afterwards taken up by the follower, by way of keeping an account of the number of chains. When the arrows have been all put down, the leader must wait till the follower brings him the arrows, then proceeding onwards as before, but without leaving an arrow at the tenth extension of the chain. In order to keep an account of the number of the times which the arrows are thus exchanged, they should each tie a knot on a string carried for that purpose (and which may be fastened to the button or button-hole of the coat): they should also call out the number of those exchanges, that the surveyor may have a check on them.

It is very necessary, that the chain-bearers should proceed in a strait line; to this end, the second, and all the succeeding arrows, should always be so placed, that the next foregoing one may be in a line with it, the place measured from, and that to which you are advancing; it is a very good method to set up a staff at every ten chains, as well for the purpose of a guide to preserve the rectilinear direction, as to prevent mistakes.

All distances of offsets from the chain line to any boundary which are less than a chain, are most conveniently measured by the offset staff; the measure must always be obtained in a direction perpendicular to the chain.

The several problems that may be solved by the chain alone, will be found in that part of the work, which treats of practical geometry on the ground.

DESCRIPTION AND USE OF THE SURVEYING
QUADRANT, FOR ADJUSTING AND REGULATING
THE MEASURES OBTAINED BY THE CHAIN
WHEN

WHEN USED ON HILLY GROUND, INVENTED BY
R. KING, SURVEYOR.

There are two circumstances to be considered in the measuring of lines in an inclined situation: the *first* regards the plotting or laying down the measures on paper; the *second*, the area or superficial content of the land. With respect to the first, it is evident that the oblique lines will be longer than the horizontal ones, or base; if therefore the plan be laid down according to such measures, all the other parts thereof would be thereby pushed out of their true situations; hence it becomes necessary to reduce the hypotenusal lines to horizontal, which is easily effected by Mr. King's quadrant.

With respect to the area, there is a difference among surveyors; some contending that it should be made according to the hypotenusal, others according to the horizontal lines: but as all are agreed to the necessity of the reduction for the first purpose, we need not enter minutely into their reasons here; for even if we admit, that in some cases more may be grown on the hypotenusal plane than the horizontal, even then the area should be given according to both suppositions, as the hilly and uneven ground requires more labour in the working.

The quadrant A B, *fig. 1, plate 14*, is fitted to a wooden square, which slides upon an offset staff, and may be fixed at any height by means of the screw C, which draws in the diagonal of the staff, thus embracing the four sides, and keeping the limb of the square perpendicular to the staff; the staff should be pointed with iron to prevent wear; when the staff is fixed in the ground on the station

station line, the square answers the purpose of a cross staff, and may, if desired, have sights fitted to it.

The quadrant is furnished with a spirit level, and is fastened to the limb D E of the square, by the screw G.

When the several lines on the limb of the quadrant, have their first division coincident with their respective index divisions, the axis of the level is parallel to the staff.

The first line next the edge of the quadrant is numbered from right to left, and is divided into 100 parts, which shews the number of links in the horizontal line, which are completed in 100 links on the hypotenusal line, and in proportion for any lesser number.

The second, or middlemost line, shews the number of links the chain is to be drawn forward, to render the hypotenusal measure the same as the horizontal.

The third, or uppermost line, gives the perpendicular height, when the horizontal line is equal to 100.

To use the quadrant. Lay the staff along the chain line on the ground, so that the plane of the quadrant may be upright, then move the quadrant till the bubble stands in the middle, and on the several lines you will have, 1. The horizontal length gone forward in that chain. 2. The links to be drawn forward to complete the horizontal chain. 3. The perpendicular height or descent made in going forward one horizontal chain.

The two first lines are of the utmost importance in surveying land, which cannot possibly be planned with any degree of accuracy without having the horizontal line, and this is not to be obtained by

by any instrument in use, without much loss of time to the surveyor. For with this, he has only to lay his staff on the ground, and set the quadrant till the bubble is in the middle of the space, which is very soon performed, and he saves by it more time in plotting his survey, than he can lose in the field; for as he completes the horizontal chain as he goes forwards, *the offsets are always in their right places*, and the field book being kept by horizontal measure, his lines are always sure to close.

If the superficial content by the hypotenusal measure be required for any particular purpose, he has that likewise by entering in the margin of his field-book the links drawn forward in each chain, having thus the hypotenusal and horizontal length of every line.

The third line, which is the perpendicular height, may be used with success in finding the heights of timber; thus measure with a tape of 100 feet, the surface of the ground from the root of the tree, and find, by the second line, how much the tape is to be drawn forward to complete the distance of 100 horizontal feet; and the line of perpendiculars shews how many feet the foot of the tree is above or below the place where the 100 feet distance is completed.

Then inverting the quadrant by means of sights fixed on the staff, place the staff in such a position as to point to that part of the tree whose height you want; and slide the quadrant till the bubble stands level, you will have on the line of perpendiculars on the quadrant, the height of that part of the tree above the level of the place where you are; to which add or subtract the perpendicular height of the place from the foot of the tree, and you obtain the height required.

OF THE IMPROVED MEASURING WHEEL.

A measuring wheel consists of a wheel of wood, shod or lined with iron to prevent the wear; a short axis is fixed to this wheel, which communicates motion to the wheel-work, included in the body of the instrument.

In this instrument, the circumference of the wheel is 8 feet 3 inches, or half a pole; one revolution of this wheel turns a single-threaded worm once round; the worm takes into a wheel of 80 teeth, and turns it once round in 80 revolutions; on the socket of this wheel is fixed an index, which makes one revolution in 40 poles, or one furlong; on the axis of this worm is fixed another worm with a single thread, that takes into a wheel of 40 teeth; on the axis of this wheel is another worm with a single thread, turning about a wheel of 160 teeth, whose socket carries an index that makes one revolution in 80 furlongs, or ten miles. On the dial-plate there are three graduated circles, the outermost is divided in 220 parts, or the yards in a furlong; the next into 40 parts, the number of poles in a furlong; the third into 80 parts, the number of furlongs in ten miles; every mile being distinguished by it's proper Roman figure.

This wheel is much *superior* to those hitherto made, because the worms and wheels act without shake, and as they have only very light indexes to carry, move with little or no friction, and are therefore not liable to wear or be soon out of order, which is not the case with the general number of those that are made, in which there is a long train of wheels and pinions, and consequently a great deal of shake and friction. 2. The divisions on the graduated circles are at a much greater distance

tance, and may therefore be subdivided into feet if required. 3. The measure shewn by the indexes is far more accurate, as there is no shake nor any loss of time in the action of one part or another.

This instrument is very useful for measuring roads, commons, and every thing where expedition is required; one objection is however made to it, namely, that it gives a measure somewhat too long by entering into hollows, and going over small hills; this is certainly the case; the measuring wheel is not an infallible mode of ascertaining the horizontal distance between any two places; but then it may with propriety be asked whether any other method is less fallible? whether upon the whole and in the circumstances to which the measuring wheel is usually appropriated, the chain is not equally uncertain, and the measure obtained from it as liable to error, as that from the wheel.

OF THE SURVEYING CROSS, *fig. 2, plate 14.*

The cross consists of two pair of sights, placed at right angles to each other: these sights are sometimes pierced out in the circumference of a thick tube of brass about 2 inches diameter, as in *fig. 3, plate 14.* Sometimes it consists of four sights strongly fixed upon a brass cross; this is, when in use, screwed on a staff having a sharp point at the bottom to stick in the ground; one of this kind is represented at *fig. 2, plate 14.*

The surveying cross is a very useful instrument for placing of offsets, or even for measuring small pieces of ground; its accuracy entirely depends on the placing the sights, which are to be exactly at right angles to each other. It may be proved by looking at one object through two of the sights,
P
and

and observing at the same time, without moving the instrument, another object through the other two sights; then turning the cross upon the staff, look at the same objects through the opposite sights; if they are accurately in the direction of the sights, the instrument is just.

It is usual, in order to ascertain a crooked line by offsets, first to measure a base or station line in the longest direction of the piece of ground, and while measuring, to find by the cross the places where perpendiculars would fall from the several corners and bends of the boundary; this is done by trials, fixing the instrument so that by one pair of sights both ends of the line may be seen; and by the other pair, the corresponding bend or corner; then measuring the length of the said perpendicular. To be more particular, let A, h, i, k, l, m, *fig. 35, plate 9*, be a crooked hedge or river; measure a strait line as A B along the side of the foregoing line, and while measuring, observe when you are opposite to any bend or corner of the hedge, as at c, d, e; from thence measure the perpendicular offsets, as at c h, d i, &c. with the offset staff, if they are not too long; if so, with the chain. The situation of the offsets are readily found, as above directed, by the cross, or King's surveying quadrant; they are to be registered in the field book.

Of surveying with the chain and cross. What has been denominated by many writers, surveying by the *chain only*, is in fact surveying by the cross and chain; for it is necessary to use the cross, or optical square, for determining their perpendicular lines, so that the whole that has been said, even by these men in favour of the *chain alone*, is founded in fallacy. To survey the triangular field A B C, *fig. 22, plate 9*, by the chain and cross: 1. Set up marks
at

at the corners of the field. 2. Beginning (suppose) at A, measure on in a right line till you are arrived near the point D, where a perpendicular will fall from the angle, let the chain lie in the direction or line AB. 3. Fix the cross over A B, so as to see through one pair of sights the mark at A or B, and through the other the mark at c; if it does not coincide at c, with the mark, the cross must be moved backwards or forwards, till by trials one pair of the sights exactly coincide with the mark at C, and the other with A or B. 4. Observe how many chains and links the point D is from A, suppose 3. 0. 3. which must be entered in the field-book. 5. Measure the perpendicular DC, 6.43. 7. finish the measure of the base line, and the work is done. *This mode is used at present by many surveyors, probably because there is no check whereby to discover their errors, which must be very great if the survey is of any extent.*

To plot this, make AB equal 11.41. A D equal to 3. 0. 3. on the point D erect the perpendicular DC, and make it equal 6.43. then draw AC, BC, and the triangle is formed.

OF THE OPTICAL SQUARE, *fig. 4, plate 14.*

This instrument, which is constructed on the principles of Hadley's quadrant, was contrived by my father; it is in most, if not in all respects, superior to the common surveying cross, because it requires no staff, may be used in the hand, and is of course of great use to a military officer. It consists of two plane mirrors, so disposed, that an object seen by reflexion from them both, will appear to coincide with another object seen by *direct vision* whenever the two objects subtend a right angle from the center of the instrument, and serves therefore to

raise or let fall perpendiculars on the ground, as a square does on paper, of which we shall give some examples. It's application to the purposes of surveying will be evident from these, and what has been already said concerning the cross.

N. B. *Fig. 4, plate 14*, represents the instrument without it's cover, in order to render the construction more evident. There is a cover with a slit or sight for viewing the objects; the object seen directly always coincides with the object seen by reflexion, when they are at right angles to each other.

From a given point in a given line, to raise a perpendicular. 1. The observer is to stand with this instrument over the given point, causing a person to stand with a mark, or fixing one at some convenient distance on the given line. 2. An assistant must be placed at a convenient distance, with a mark somewhere near the line in which it is supposed the perpendicular will fall; then if on looking at one of the objects, the other be seen in a line with it, the place where the mark of the assistant is fixed, is the required point.

From a given point over a given line to let fall a perpendicular. Every strait line is limited and determined by two points, through which it is supposed to pass; so in a field the line is determined by two fixed objects, as steeples, trees, marks erected for the purpose, &c. For the present operation, the two objects that determine must be on one side the point where the perpendicular falls; or in other words, the observer must not be between the objects, he must place himself over the line, in which he will always be when the two objects coincide; he must move himself backwards on this line, till the mark (from whence the perpendicular is to be let fall) seen by direct vision coin-

coincides with one of the objects which determine the given line seen by reflexion, and the instrument will be over the required point.

To measure inaccessible distances by the optical square.

Required the distance from the steeple A, *fig. 20, plate 9*, to B. Let the observer stand with his instrument at B, and direct an assistant to move about C with a staff as a mark, until he sees it coincide by direct vision with the object at A; let him fix the staff there; then let the observer walk along the line until A and B coincide in the instrument, and B D will be perpendicular to A C; measure the three lines B C, D C, B D, and then the following proportion will give the required distance, for as D C is to D B, so is B C to A B.

Second method. 1. Make B C, *fig. 21, plate 9*, perpendicular to B A. 2. Divide B C into four equal parts. 3. Make C D perpendicular to C B. 4. Bring F E A into one line, and the distance from C to F, will be $\frac{1}{3}$ of the distance from B to A.

OF THE CIRCUMFERENTER, *fig. 1, plate 15.*

This instrument consists of a compass box, a magnetic needle, and two plain sights, perpendicular to the meridian line in the box, by which the bearings of objects are taken from one station to another. It is not much used in England where land is valuable; but in America where land is not so dear, and where it is necessary to survey large tracts of ground (overstocked with wood,) in a little time, and where the surveyor must take a multitude of angles, in which the sight of the two lines forming the angle may be hin-

dered by underwood, the circumferenter is chiefly used.

The circumferenter, see *fig. 1, plate 15*, consists of a brass arm about 14 or 15 inches long, with sights at each end, and in the middle thereof a circular box (with a glass cover) of about $5\frac{1}{2}$ inches diameter; within the box is a brass graduated circle, the upper surface divided into 360 degrees, and numbered 10. 20. 30. to 360; every tenth degree is cut down on the inner edge of the circle. The bottom of the box is divided into four parts or quadrants, each of which is subdivided into 90 degrees numbered from the meridian, or north and south points, each way to the east and west points; in the middle of the box is placed a steel pin finely pointed, called the center pin, on which is placed a magnetic needle, the quality of which is such, that (allowing for the difference between the astronomic and magnetic meridians) however the instrument may be moved about, the bearing or angle which any line makes with the magnetic meridian, is at once shewn by the needle.

At each end of the brass rule, and perpendicular thereto, sights are fixed; in each sight there is a large and a small aperture, or slit, one over the other, these are alternate; that is, if the aperture be uppermost in one sight, it will be lowest in the other, and so of the small ones; a fine piece of sewing silk, or a horse hair runs through the middle of the large slit. Under the compass box, is a socket to fit on the pin of the staff; the instrument may be turned round on this pin, or fixed in any situation by the milled nut; it may also be readily fixed in an horizontal direction by the ball and socket of the staff, moving for this purpose the box, till the ends of the needle are equidistant

from the bottom, and traverse or play with freedom.

Occasional variations in the construction of this instrument, are, 1. In the sights, which are sometimes made to turn down upon an hinge, in order to lessen the bulk of the instrument, and render it more convenient for carriage; sometimes they are made to slide on and off with a dovetail, sometimes to fit on with a screw and two steady pins. 2. In the box; which in some instruments has a brass cover, and very often a spring is placed within the box, to throw the needle off the center pin, and press the cap close against the glass, to preserve the point of the center pin from being blunted by the continual friction of the cap of the needle. 3. In the needle itself, which is made of different forms. 4. A further variation will be noticed under the account of the improved circumferenter.

The surveying compass represented at *fig. 3, plate 15*, is a species of circumferenter, which has hitherto been only applied to military purposes; it consists of a square box, within which there is a brass circle divided into 360 degrees; in the center of the box is a pin to carry a magnetic needle; a telescope is fixed to one side of the box in such a manner, as to be parallel to the north and south line; the telescope has a vertical motion for viewing objects in an inclined plane; at the bottom of the box, is a socket to receive a stick or staff for supporting the instrument.

To use the circumferenter. Let *A B C, fig. 19, plate 9*, be the angle to be measured, 1. The instrument being fixed on the staff, place it's center over the point *B*. 2. Set it horizontal, by moving the ball in it's socket till the needle is parallel to

the bottom of the compass box. 3. Turn that end of the compass box, on which the flower de lys is engraved, next the eye. 4. Look along BA, and observe at what degree the needle stands, suppose 30. 5. Turn the instrument round upon the pin of the ball and socket, till you can see the object C, and suppose the needle now to stand at 125. 6. Take the former number of observed degrees from the latter, and the remainder is the required angle.

If in two observations to find the measure of an angle, the needle points in the first on one side 360°, and in the second on the other, add what one wants of 360°, to what the other is past 360°, and the sum is the required angle.

This general idea of the use of the circumferenter, it is presumed, will be sufficient for the present; it will be more particularly treated of hereafter.

When in the use of the circumferenter, you look through the upper sights from the ending of the station to the beginning, it is called a *back sight*; but when you look through the lower slit from the beginning of the station towards the end, it is termed the *fore sight*. A theodolite, or any instrument which is not set by the needle, must be fixed in its place, by taking back and fore sights at every station, for it is by the foregoing station that it is set parallel; but as the needle preserves its parallelism throughout the whole survey, whosoever works by the circumferenter, need take no more than one sight at every station.

There is indeed a difference between the magnetic and astronomic or true meridian, which is called *the variation of the needle*. This variation is different at different places, and is also different at different times; this difference in the variation is called

called the variation of the variation ; but the increase and decrease thereof, both with respect to time and place, proceeds by such very small increments or decrements, as to be altogether insignificant and insensible, within the small limits of an ordinary survey, and the short time required for the performance thereof.

The causes of the variation of the needle have long been the subject of philosophical inquiry, but still remain undeveloped.

OF THE IMPROVED CIRCUMFERENTER.

The excellency and defects of the preceding instrument both originate in the needle ; from the regular direction thereof, arise all it's advantages ; the unsteadiness of the needle, the difficulty of ascertaining with exactness the point at which it settles, are some of it's principal defects. In this improved construction, these are obviated, as will be evident from the following description. One of these instruments is represented at *fig. 2, plate 15.*

A pin of about three quarters of an inch diameter goes through the middle of the box, and forms as it were a vertical *axis*, on which the instrument may be turned round horizontally ; on this axis an *index* A B is fastened, moving in the inside of the box, having a nonius on the outer end to cut, and subdivide the degrees on the graduated circle. By the help of this index, angles may be taken with much greater accuracy than by the needle alone ; and as an angle may be ascertained by the index with or without the needle, it of course removes the difficulties, which would otherwise arise if the needle should at any time happen to be acted upon, or drawn out of it's ordinary position by extraneous matter ; there is a pin beneath, whereby

whereby the index may be fastened temporarily to the bottom of the box, and a screw as usual, to fix the whole occasionally to the pin of the ball and socket, so that the body of the instrument, and the index, may be either turned round together, or the one turned round, and the other remain fixed, as occasion shall require. A further improvement is that of *preventing all horizontal motion of the ball in the socket*; the ball has a motion in the socket every possible way, and every one of these possible motions is necessary, *except the horizontal one*, which is here totally destroyed, and every other possible motion left perfectly free.

GENERAL IDEA OF THE USE OF THESE INSTRUMENTS.

For this purpose, let *fig. 1, plate 18*, represent a field to be surveyed. 1. Set up the circumferenter at any corner as at A, and therewith take the course or bearing* of the side A B, and measure the length thereof with the chain.

If the circumferenter be a common one, having no index in the box, the course or bearing is taken by simply turning the sights in a direct line from A to B, and when the needle settles, the needle will point out on the graduated limb, the course or number of degrees which the line bears from the magnetic meridian.

But if the circumferenter has an index in the box, it is thus used. 1. Bring the index to the north point on the graduated limb, and fix it there, by fastening the body of the instrument, and the under part together, by the pin for that purpose, and

* By the course or bearing of a line, is here meant the angle which such line makes with the magnetic meridian as shown by the needle,

and turn the instrument about, so that the needle shall settle at the same point; then fasten the under part of the instrument to the ball and socket, and taking the pin out, turn the sights in a direct line from A towards B, so will the course and bearing be pointed out on the graduated circle, both by the needle and by the index. This done, fasten the body of the instrument to the under part again, and having set the instrument up at B, turn the sights in a direct line back from B to A, and there fasten the under part of the instrument to the ball and socket; then take out the pin which fastens the body of the instrument to the under part, and turn the sights in a direct line towards C, and proceed in the same manner all round the survey; so will the courses or bearings of the several lines be pointed out both by the needle and the index, unless the needle should happen to be drawn out of it's course by extraneous matter; but in this case the index will not only shew the course or bearing, but will likewise shew how much the needle is so drawn aside. After this long parenthesis to explain more minutely the use of the instruments, we may proceed. 2. Set the circumferenter up at B, take the course and bearing of B C, and measure the length thereof, and so proceed with the sides C D, D E, E F, F G, G A, all the way round to the place of beginning, noting the several courses or bearings, and the lengths of the several sides in a field-book, which let us suppose to be as following:

1.	A B	North	7	West	21. 00.
2.	B C	North	55 15	East	18. 20.
3.	C D	South	62 30	East	14. 40.
4.	D E	South	40	West	11.
					5. E F

5.	E F	South	4	15	East	14.
6.	F G	North	73	45	West	12. 40.
7.	G A	South	52		West	9. 17.

N. B. By north 7° west, is meant seven degrees to the westward (or left hand) of the north, as shewn by the needle; by north $55^{\circ} 15'$ east, fifty-five degrees fifteen minutes to the eastward, or right hand of the north, as shewn by the needle.

In like manner by south $62^{\circ} 30'$ east, is meant sixty-two degrees and thirty minutes to the eastward, or left hand of the south; and by south 40° west, forty degrees to the westward, or right hand of the south.

The 21 chains, 18 chains 20 links, &c. are the lengths or distances of the respective sides, as measured by the chain.

Fig. 4, plate 15, represents a small circumferenter, or theodolite; it is a kind that was much used by General Roy, for delineating the smaller parts of a survey.

OF THE COMMON THEODOLET.

The error to which an instrument is liable, where the whole dependance is placed on the needle, soon rendered some other invention necessary to measure angles with accuracy; among these, the common theodolite, with four plain sights, took the lead, being simple in construction, and easy in use.

The common theodolite is represented, *fig. 5, plate 14*; it consists of a brass graduated circle, a moveable index A B; on the top of the index is a compass with a magnetic needle, the compass box is covered with a glass, two sights C, D, are fixed to the index, one at each end, perpendicular to the plane

plane of the instrument ; there are two more sights EF, which are fitted to the graduated circle at the points of 360° and 180° ; they all take on and off for the conveniency of packing. In each sight, there is, as in the circumferenter, a large and a small aperture placed alternately, the large aperture in one sight, being always opposed to the narrow aperture in the other ; underneath the brass circle, and in the center thereof, is a sprang to fit on the pin of the ball and socket, which fixes on a three-legged staff.

The circle is divided into degrees, which are all numbered one way to 360° , usually from the left to the right, supposing yourself at the center of the instrument ; on the end of the index is a nonius division, by which the degrees on the limb are subdivided to five minutes ; the divisions on the ring of the compass box are numbered in a contrary direction to those of the limb.

As much of geometrical mensuration depends on the accuracy of the instrument, it behoves every surveyor to examine them carefully ; different methods will be pointed out in this work, according to the nature of the respective instruments. In that under consideration, the index should move regularly when in use ; the theodolite should always be placed truly horizontal, otherwise the angles measured by it will not be true ; of this position you may judge with sufficient accuracy by the needle, for if this be originally well ballanced, it will not be parallel to the compass-plate unless the instrument be horizontal ; two bubbles, or spirit levels, are sometimes placed in a compass box at right angles to each other, in order to level the instrument, but it appears to me much better to depend on the needle: 1. Because the bubbles, from their

their size, are seldom accurate. 2. Because the operator cannot readily adjust them, or ascertain when they indicate a true level.

To examine the instrument, on an extensive plain set three marks to form a triangle; with your theodolite take the three angles of this triangle, and if these, when added together make 180° , you may be certain of the justness of your instrument.

To examine the needle, observe accurately where the needle settles, and then remove it from that situation, by placing a piece of steel or magnet near it; if it afterwards settles at the same point, it is so far right, and you may judge it to be perfectly so if it settles properly in all situations of the box. If in any situation of the box a deviation is observed, the error is most probably occasioned by some particles of steel in the metal, of which the compass box is made.

To examine the graduations, set the index division of the nonius to the beginning of each degree of the theodolite, and if the last division of the nonius always terminates precisely (at each application) with it's respective degree, then the divisions are accurate.

Cautions in the use of the instrument. 1. Spread the legs that support the theodolite pretty wide, and thrust them firmly in the ground, that they may neither yield nor give unequally during the observation. 2. Set the instrument horizontal. 3. Screw the ball firm in it's socket, that in turning the index, the theodolite may not vary from the objects to which it is directed. 4. Where accuracy is required, the angles should always be taken twice over, oftener where great accuracy is required,

required, and the mean of the observation must be taken for the true angle.

To measure an angle with the theodolite. Let A B, B C, fig. 19, plate 9, represent two station lines; place the theodolite over the angular point, and direct the fixed sights along one of the lines, till you see through the sights the mark; at these screw the instrument fast; then turn the moveable index till through it's sights you see the other mark C; then the degrees cut by the index upon the graduated limb or ring of the instrument, shews the quantity of the angle.

The fixed sights are always to be directed to the last station, and those on the index to the next. If the beginning of the degrees are towards the surveyor when the fixed sights are directed to an object, and the flower-de-lys towards him in directing the index, then that end of the index towards the surveyor will point out the angle, and the south end of the needle the bearing; the application of the instrument to various cases that may occur in surveying, will be evident from what we shall say on that subject in the course of this work.

OF THE COMMON PLANE TABLE, fig. 1, plate 17.

The tabular part of this instrument is usually made of two well seasoned boards, forming a parallelogram of about 15 inches long, and 12 inches broad; the size is occasionally varied to suit the intentions of the operator.

The aforesaid parallelogram is framed with a ledge on each side to support a box frame, which frame confines the paper on the table, and keeps it close thereto; the frame is therefore so contrived, that it may be taken off and put on at pleasure,
either

either side upwards; each side of the frame is graduated; one side is usually divided into scales of equal parts, for drawing lines parallel or perpendicular to the edges of the table, and also for more conveniently shifting the paper; the other face or side of the frame is divided into 360° , from a brass center in the middle of the table, in order that angles may be measured as with a theodolet; on the same face of the frame, and on two of the edges, are graduated 180° , the center of these degrees is exactly in the middle between the two ends, and about $\frac{1}{4}$ part of the breadth from one of the sides.

A magnetic needle and compass box, covered with a glass and spring ring, slides in a dovetail on the under side of the table, and is fixed there by a finger screw; it serves to point out the direction, and be a check upon the sights.

There is also a brass index somewhat larger than the diagonal of the table, at each end of which a sight is fixed; the vertical hair, and the middle of the edge of the index, are in the same plane; this edge is chamfered, and is usually called the fiducial edge of the index. Scales, of different kinds, are usually laid down on the edge of the index.

Under the table is a sprang to fit on the pin of the ball and socket, by which it is placed upon a three-legged staff.

To place the paper on the table, take a sheet of paper that will cover it, and wet it to make it expand, then spread it flat upon the table, pressing down the frame upon the edges to stretch it, and keep it in a fixed situation; when the paper is dry it will by contracting become smooth and flat.

Of

OF SHIFTING THE PAPER ON THE PLANE TABLE.

When the paper on the table is full, and there is occasion for more, draw a line in any manner through the farthest point of the last station line, to which the work can be conveniently laid down; then take the sheet of paper off, and fix another on the table; draw a line upon it in a part most convenient for the rest of the work; then fold or cut the old sheet of paper by the line drawn on it, apply the edge to the line on the new sheet, and, as they lie in that position, continue the last station line upon the new paper, placing upon it the rest of the measures, beginning where the old sheet left off, and so on from sheet to sheet.

To fasten all the sheets of paper together, and thus form one rough plan, join the aforesaid lines accurately together, in the same manner as when the lines were transferred from the old sheets to the new one. But if the joining lines upon the old and new sheet have not the same inclination to the side of the table, the needle will not point to the original degree when the table is rectified. If the needle therefore should respect the same degree of the compass, the easiest way of drawing the line in the same position, is to draw them both parallel to the same sides of the table, by means of the scales of equal parts on the two sides.

To use the plane table. Fix it at a convenient part of the ground, and make a point on the paper to represent that part of the ground.

Run a fine steel pin or needle through this point into the table, against which you must apply the fiducial edge of the index, moving it round till you perceive some remarkable object, or mark

set up for that purpose. Then draw a line from the station point, along the fiducial edge of the index.

Now set the sights to another mark, or object, and draw it's line, and so proceed till you have obtained as many angular lines as are necessary from this station.

The next requisite is the measure or distance from the station to as many objects, as may be necessary, taking at the same time the off-sets to the required corners or crooked parts of the hedges; setting off all the measures upon their respective lines upon the table.

Now remove the table to some other station, whose distance from the foregoing was previously measured; then lay down the objects which appear from thence, and continue these operations till your work is finished, measuring such lines as are necessary, and determining as many as you can by intersecting lines of direction, drawn from different stations.

It seems to be the universal opinion of the best surveyors, that the plane table is not an instrument to be trusted to in large surveys, or on hilly situations; that it can only be used to advantage, in planning the ground plot of buildings, gardens, or a few small parcels of land nearly on a level.

Mr. Gardiner, whose authority as a surveyor is inferior to no one, asserts that the plane table surveyors, when they find their work not to close right, do often close it wrong, not only to save time and labour, but the acknowledgment of an error; which they are not sure they can amend.

In uneven ground, where the table cannot in all stations be set horizontal, or uniformly in any one place

place, it is impossible the work should be true in all parts.

The contraction and expansion of the paper according to the state of moisture in the air, is a source of many errors in plotting; for between a dewy morning, and the heat of the sun at noon, there is a great difference, which may in some degree be allowed for in small work, but cannot be remedied in surveys of considerable extent.

OF THE IMPROVED PLANE TABLE, *fig. 2. plate 17.*

To remedy some of the inconveniences, and correct some of the errors to which the common plane table is liable, that which we are now going to describe has been constructed. It is usually called *Beighton's plane table*, though differing in many respects from that described by him in the *Philosophical Transactions*.

It is a plain board, 16 inches square, with a frame of box or brass round the edge, for the purpose of being graduated. On the sides AB, CD, are two grooves and holdfasts for confining firmly, or easily removing the paper; they are disengaged by turning the screws under the table from the right towards the left, and drawn down and made to press on the paper by turning the screws the contrary way. When the holdfasts are screwed down, their surface is even with that of the table. There are two pincers to hold that part of the paper, which in some cases lies beyond the table, and prevent it's flapping about with the wind.

The compass box is made to fit either side of the table, and is fixed by two screws, and does not when fixed project above one inch and a half from the side of the table.

Q 2

There

There is an index with telescopic sights, EFGH; it is sometimes so constructed, as to answer the purpose of a parallel rule.

The figure renders the whole so evident, that a greater detail would be multiplying words to no purpose.

The papers or charts for this table, are to be either a fine thin pasteboard, fine paper pasted on cartridge paper, or two papers pasted together, cut as exactly square as possible, and of such a length that they may slide in easily, between the upright parts, and under the flat part of the holders.

Any one of these charts may be put into the table any of the four ways, be fixed, taken out, and changed at pleasure; any two of them may be joined together on the table, by making each of them meet exactly at the middle, whilst near one half of each will hang over the sides of the table; or by doubling them both ways through the middle, four of them may be put on at one time, meeting in the center of the table. For this purpose, each chart is always to be crossed quite through the middle; by these means the great trouble and inaccuracy in shifting the papers is removed.

The charts thus used, are readily laid together by corresponding numbers on their edges, and thus make up the whole map, in one view; and being in squares are portable, easily copied, enlarged, or contracted.

The line of sight in viewing objects may, if that method be preferred, be always over the center of the table, and the station lines drawn parallel to those measured on the land. Underneath the table

is

is a sprang to fit on the socket of a staff, with parallel plates and adjusting screws.

SEARLE'S PLANE TABLE.

This instrument, whose size (which renders it convenient, while it multiplies every error,) is only five inches square, consists of two parts, the table and the frame. The frame, as usual, to tighten the paper of observation. In the center of the table is a screw, on which the index sight turns; this screw is tightened after taking an observation.

THEODOLETS WITH TELESCOPIC SIGHTS.

In proportion as science advances, we find ourselves standing upon higher ground, are enabled to see further and distinguish objects better than those that went before us: thus the great advances in dividing of instruments, have rendered observers more accurate, and more attentive to the necessary adjustments of their instruments. Instruments are not now considered as perfect, unless they are so constructed that the person who uses them, may either correct or allow for the errors to which they are liable.

Theodolites with telescopic sights are without doubt the most accurate, commodious, and universal instruments for the purposes of surveying, and have been recommended as such, by the most expert practitioners, and best writers on the subject, as Gardiner, Hammond, Cunn, Stone, Wild, Waddington, &c.

The leading requisites in a good theodolite are,
1. That the parts be firmly connected, so that they may always preserve the same figure. 2. The circles must be truly centered and accurately graduated.

duated. 3. The extremity of the line of sight should describe a true circle.

Fig. 1. plate 16, represents a theodolite of the second best kind; the principal parts are, 1. A telescope and it's level. 2. The vertical arch. 3. The horizontal limb and compass. 4. The staff with it's parallel plates.

An attentive view of the instrument or drawing compared with what has been said before, will shew that it's perfect adjustment consists in the following particulars.

1. The horizontal circle A A, must be truly level. 2. The plane of the vertical circle B B, must be truly perpendicular to the horizon. 3. The line of sight, or line of collimation, must be exactly in the center of the circles on which the telescope turns. 4. The level must be parallel to the line of collimation.

OF THE TELESCOPE C C.

Telescopic sights not only enable the operator to distinguish objects better, but direct the sight with much greater accuracy than is attainable with plain sights; hence also we can make use of much finer subdivisions. The telescope generally applied to the best instruments, is of the acromatic kind, in order to obtain a larger field, and greater degree of magnifying power. In the focus of the eye glass, are two very fine hairs at right angles to each other, whose intersection is in the plane of the vertical arch. The object glass may be moved to different distances from the eye glasses, by turning the milled nut *a*, and may by this means be accommodated to the eye of the observer, and the distance of the object. The screws for moving and adjusting the cross hairs are sunk a little within the eye tube,

and

and at about one inch from the eye end: there are four of these screws, two of which are exactly opposite to each other, and at right angles to the other two. By easing one of the screws, and tightening the opposite one, the wire connected with it may be moved one way or the other. On the outside of the telescope are two metal rings, which are ground perfectly true; these rings are to lay on the supporters or Y's, which are fixed to the vertical arch.

OF THE VERTICAL ARCH BB.

This arch is firmly fixed to a long axis which is at right angles to the plane of the arch. This axis is sustained by, and moveable on, the two supports which are fixed firmly to the horizontal plate; on the upper part of the vertical arch, are the two Y's, for holding the telescope; the inner sides of these Y's are so framed as to be tangents to the cylindric rings of the telescope, and therefore bear only on one part. The telescope is confined to the Y's by two loops, which turn on a joint, and may therefore be readily opened and turned back, when the two pins are taken out.

One side of the vertical arch is graduated to every half degree, which are subdivided to every minute of a degree by the nonius. It is numbered each way, from 0 to 90°, towards the eye end, for angles of altitude; from 0 to 50°, towards the object end, for angles of depression. *On the other side* of the vertical arch, are two ranges of divisions, the lowermost for taking the upright height of timber in 100th parts of the distance the instrument is placed at from the tree at the time of observation. The uppermost circle is for reducing hypotenusal lines to horizontal, or to shew the difference between the hypotenuse and base of a

right-angled triangle, always supposing the hypothenuse to consist of 100 equal parts; consequently it gives by inspection the number of links to be deducted from each chain's length in measuring up or down any ascent, or descent, in order to reduce it to a true horizontal distance.

The vertical arch may be moved regularly and with ease, by turning the milled nut *b*; there is a pin by which it may be fixed when at *o*.

OF THE COMPASS.

The compass is fixed to the upper horizontal plate, the ring of the compass is divided into 360° , which are numbered in a direction contrary to those on the horizontal limb. The bottom of the box is divided into four parts, or quadrants; each of which is subdivided to every 10 degrees, numbered from the meridian or north and south points each way to the east and west points. In the middle of the box is a steel pin finely pointed, on which is placed the magnetic needle; there is a contrivance for throwing the needle off it's point, when not in use.

OF THE HORIZONTAL LIMB, A A.

This limb consists of two plates, one moveable on the other; the outside edge of the upper plate is chamfered, to serve as an index to the degrees on the limbs. The upper plate, together with the compass, vertical arch, and telescope, are easily turned round by a pinion fixed to the screw, *c*; *d* is a nut for fixing the index to any part of the limb, and thereby making it so secure, that there is no danger of it's being moved out of it's place, while the instrument is removed from one station to another. The horizontal limb is divided to
half

half degrees, and numbered from the right hand towards the left, 10, 20, 30, &c. to 360; the divisions are subdivided by the nonius scale to every minute of a degree.

On the upper plate, opposite to the nonius, are a few divisions for measuring the diameter of trees.

Underneath the horizontal limb, is a second telescope which has a vertical and horizontal motion, and may thereby be brought accurately to the same object as the upper telescope, namely, the commencing point, or that from which angles are measured. The whole instrument fits on the conical ferril of a strong brass headed staff, with three substantial wooden legs; the top or head of the staff consists of two brass plates parallel to each other; four screws pass through the upper plate, and rest on the lower plate; by the action of these screws, the situation of the plate may be varied, so as to set the horizontal limb truly level, or in a plane parallel to the horizon; for this purpose, a strong pin is fixed to the under side of the plate, this pin is connected with a ball that fits into a socket in the lower plate; the axis of the pin and ball are so framed, as to be always perpendicular to the plate, and consequently to the horizontal limb.

TO ADJUST THE THEODOLET.

As so much of surveying depends on the accuracy of the instruments, it is absolutely necessary that the surveyor should be very expert in their adjustments, without which he cannot expect the instruments will properly answer the purposes they were designed for, or that his surveys will have the requisite exactness.

The necessary adjustment to the theodolite,
which

which we have just described, are, 1. That the line of sight or collimation* be exactly in the center of the cylindric rings round the telescope, and which lie in the Y's. 2. That the level be parallel to this line, or the axis of the above-mentioned rings. 3. The horizontal limb must be so set, that when the vertical arch is at zero, and the upper part moved round, the bubble of the level will remain in the middle of the open space.

Previous to the adjustments, place the instrument upon the staff, and set the legs thereof firmly on the ground, and at about 3 feet from each other, so that the telescope may be at a proper height for the eye, and that two of the screws on the staff that are opposite to each other, may be nearly in the direction of some conspicuous and distant object.

TO ADJUST THE LINE OF COLLIMATION.

Having set up the theodolite agreeable to the foregoing direction, direct the telescope to some distant object, placing it so, that the horizontal hair or wire may exactly coincide with some well defined part of the object; reverse the telescope, that is, so that the tube of the spirit level may be uppermost, and observe whether the horizontal hair still coincides with the object; if it does, the hair is in it's right position; if not, correct half the difference by moving the hair or wire, which motion is effected by easing one of the screws in the eye tube, and tightening the other; then turn the telescope round to it's former position with the tube

* The line of collimation is the line of vision, cut by the intersecting point of the cross hairs in the telescope, answering to the visual line, by which we take aim at objects with plane sights.

tube of the spirit level lowermost, and make the hair coincide with the object, by moving the vertical arch; reverse the telescope again, and if the hair does not coincide with the same part of the object, you must repeat the foregoing operation, till in both positions it perfectly coincides with the same part of the object.

The precise situation of the horizontal hair being thus ascertained, adjust the vertical hair in the same manner, laying it for this purpose in an horizontal position: the spirit tube will, during the adjustment of the vertical hair, be at right angles to it's former position. When the two wires are thus adjusted, their intersection will coincide exactly with the same point of the object, while the telescope is turned quite round; and the hairs are not properly adjusted, till this is effected.

ADJUSTMENT OF THE LEVEL.

To render the level parallel to the line of collimation, place the vertical arch over one pair of the staff screws, then raise one of the screws and depress the other, till the bubble of the level is stationary in the middle of the glass; now take the telescope out of the Y's, and turn it end for end, that is, let the eye end lay where the object end was placed; and if, when in this situation, the bubble remains in the middle as before, the level is well adjusted; if it does not, that end to which the bubble runs is too high; the position thereof must be corrected by turning one or both of the screws, which pass through the end of the tube, till the bubble has moved half the distance it ought to come to reach the middle, and cause it to move the other half by turning the staff screws. Return the telescope to it's former position, and if the adjustments have been

been well made, the bubble will remain in the middle; if otherwise, the process of altering the level and the staff screws, with the reversing, must be repeated till it bears this proof of it's accuracy. In some instruments there is a provision for raising or lowering the Y's a small degree, in order more conveniently to make the bubble continue in it's place when the vertical arch is at o, and the horizontal limb turn round.

TO ADJUST THE LEVEL OF THE HORIZONTAL LIMB.

Place the level so, that it may be in a line with two of the staff screws, then adjust it, or cause the bubble to become stationary in the middle of the open space by means of these screws. Turn the horizontal limb half round, and if the bubble remains in the middle as before, the level is well adjusted; if not, correct half the error by the screws at the end of the level, and the other half by the staff screws. Now return the horizontal limb to it's former position, and if it remains in the middle, the errors are corrected; if not, the process of altering must be pursued till the error is annihilated. See this adjustment in the description of Ramsden's theodolet.

When the bubble is adjusted, the horizontal limb may always be levelled by means of the staff screws.

OF THE THEODOLET, AS IMPROVED BY RAMSDEN, *fig. 2, plate 16.*

Among the improvements the instruments of science have received from Mr. Ramsden, we are to rank those of the theodolet; in the present instance, he has happily combined elegance and

neatness of form, with accuracy of construction; and the surveyor will contemplate with pleasure this instrument, and the various methods by which the parts concur to give the most accurate results.

The principal parts of this instrument are however so similar to the foregoing, that a description thereof must, in many respects, be a repetition of what has been already described with some variation only of language.

FF represents the horizontal limb, so called, because when in use it ought always to be placed parallel to the horizon. It consists like the former of two plates, the edges of these two are chamfered, so that the divisions and the nonius are in the same plane, which is oblique to the plane of the instrument. The limb is divided into half degrees, and subdivided by the nonius to every minute; it is numbered to 360 from the north towards the east: besides these, the tangents to 100 of the radius are laid down thereon.

The upper plate is moved by turning the pinion; on this plate are placed, at right angles to each other, two spirit levels for adjusting more accurately the horizontal limb.

NOP is a solid piece fitted on the upper horizontal plate, by means of three capstan headed screws, passing through three similar screws. By the action of these, the vertical arch may be set perpendicular to the horizontal limb, or be made to move in a vertical plane. On this solid piece, are fixed two stout supports to carry the axis of the vertical arch, which arch is moveable by the pinion E. On the upper part of the vertical arch, are the Y's and loops to support and confine the telescope; the Y's are tangents to the cylindric rings

rings of the telescope, which rings are turned and then ground as true as possible, and are prevented from moving backwards or forwards, by means of two shoulders. The telescope is acromatic, and may be adjusted to the eye of the observer, or the distance of the object, by turning the milled nut B. The hairs are adjusted by the screws in the eye tube. Under the telescope is fixed a spirit level C, the distance of whose ends from the telescope may be regulated by the screws c c.

Beneath the horizontal limb there is a second telescope, which has both an horizontal and vertical motion: it is moved horizontally by the milled screw H, and when directed to any object, is fixed in it's situation by another milled screw; it moves vertically on the axis; there is an adjustment to this axis, to make the line of collimation move in a vertical plane. By the horizontal motion, this telescope is easily set to what is called the backset stations; the under telescope keeping in view the back object, while the upper one is directed to the fore object. Underneath the lower telescope is a clip to fasten occasionally the main axis; this clip is tightened by the finger screw L, and when tightened, a small motion of the adjusting screw K, will move the telescope a few degrees, in order to set it with great accuracy. Beneath these is the staff, the nature of which will be sufficiently evident from what was said thereon in the description of the last theodolite, or by inspection of the figure.

TO ADJUST THE LEVELS OF THE HORIZONTAL PLATE.

1. Place the instrument on it's staff, with the legs thereof at such a distance from each other, as will

give the instrument a firm footing on the ground. 2. Set the nonius to 360, and move the instrument round, till one of the levels is either in a right line with two of the screws of the parallel plates, or else parallel to such a line. 3. By means of the two last mentioned screws, cause the bubble in the level to become stationary in the middle of the glass. 4. Turn the horizontal limb by the milled nut half round, or till the nonius is at 180, and if the bubble remains in the middle as before, the level is adjusted; if it does not, correct the position of the level, by turning one or both the screws which pass through it's ends, till the bubble has moved half the distance it ought to come to reach the middle, and cause it to move the other half by turning the screws of the parallel plates. 5. Return the horizontal limb to it's former position, and if the adjustments have been well made, the bubble will remain in the middle; if otherwise, the process of altering must be repeated till it bears this proof of accuracy. 6. Now regulate the screws of the staff head, so that the bubble remain in the middle while the limb is turned quite round. 7. Adjust the other level by it's own proper screws to agree with that already adjusted.

TO ADJUST THE LEVEL UNDER THE TELESCOPE.

1. The horizontal plate being levelled, set the index of the nonius of the vertical arch to 0, pull out the two pins, and open the loops which confine the telescope. 2. Adjust the bubble by it's own screws. 3. Reverse the level, so that it's right hand end may now be placed to the left; if the bubble continues to occupy the middle of the glass, it is in it's right position; if not, correct one half of the error by the capstan screws under the

the plate, and the other half by the screws under the level. 4. Reverse the level, and correct if there is any occasion, continuing the operations till the error vanishes, and the bubble stands in the middle in both positions.

TO ADJUST THE LINE OF COLLIMATION.

1. Direct the telescope, so that the horizontal wire may coincide with some well defined parts of a remote object. 2. Turn the telescope so that the bubble may be uppermost; if the wire does not coincide with the same part of the object as before, correct half the difference by moving the vertical circle, and the other half by moving the wire, which is effected by the screws in the eye tube of the telescope; and so on repeatedly, till the difference wholly disappears. Lastly, adjust the vertical wire in the same manner; when the two wires are properly adjusted, their intersection will coincide exactly with the same point of an object, while the telescope is turned quite round.

VARIATIONS IN THE CONSTRUCTION OF THEODOLITES WITH TELESCOPIC SIGHTS.

To accommodate those who cannot afford to go to the price of the foregoing instruments, others have been made with less work, in order to be afforded at a lower price; one of these is represented at *fig. 5, plate 15*. It is clear from the figure, that the difference consists principally in the solidity of the parts, and in there being no rack work to give motion to the vertical arch and horizontal limb. The mode of using it is the same with the other, and the adjustment for the line of collimation, and the level under the telescope, is perfectly similar to the same adjustments in the instruments already

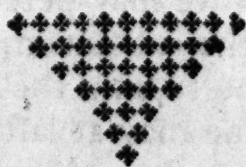
already described, a further description is therefore unnecessary.

A larger kind is also made without rack work, like *fig. 1, plate 16*.

A small one was made by the late Mr. B. Martin, in which the telescope has no vertical motion, but the horizontal motion is given by a pinion.

I have contrived a theodolet of the best sort, whose adjustments are I think more perfect than those of any other theodolet; it is represented in the frontispiece of this work, but I could not get it finished in time, to have the description inserted in this place.

For the prices of these instruments, see catalogue at the end of the book.



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DESCRIP-

DESCRIPTION, USE,
AND METHOD OF ADJUSTING
HADLEY'S QUADRANT.*

“AT the appointed time, when it pleased the Supreme Dispenser of every good gift to restore light to a bewildered world, and more particularly to manifest his wisdom in the simplicity, as well as in the grandeur, of his works, he opened the glorious scene with the revival of sound astronomy.” This observation of an excellent philosopher and physician † is verified in every instance of the progress of science; in each of which we may trace some of the steps of that vast plan of Divine Providence, to which all things are converging, namely, the bringing all his creatures to a state of truth, goodness, and consequent happiness; an end worthy of the best and wisest of beings, and which we may perceive to be gradually effecting, by the advancement of knowledge, the diffusion of liberty, and the removal of error, that truth and virtue may at last shine forth in all the beauty of their native colours.

It is thus that the discovery of the compass gave rise to the present art of navigation; and when this art grew of more importance to mankind, Divine Providence blessed them with the invention of *Hadley's quadrant*, and in our own day and our
own

* This account of Hadley's Quadrant, &c. is extracted from a small tract I published thereon some time since.

† Sir John Pringle's Six Discourses to the Royal Society.

own time has further improved both it and the art of navigation, by the present method of finding the longitude, which enables the mariner to ascertain with certainty his situation on the unvaried face of the ocean.

Hadley's quadrant or sextant is the only known instrument on which the mariner can depend, for determining with accuracy and precision his latitude and longitude. It is to the use of this instrument that navigation is indebted for the very great and rapid advances it has made within these few years. It is easy to manage, and of extensive use, requiring no peculiar steadiness of hand, nor any such fixed basis as is necessary to other astronomical instruments. It is not the science of navigation only, which is indebted to this instrument, but it's uses are so extensive in astronomy, that it may with propriety be called a *portable observatory*, and in this work we shall exemplify it's application to surveying.

Mankind are ever desirous of knowing to whom they are indebted for any peculiar or useful discovery ; it is the tribute of gratitude, and a reward to merit. In the present instance there is no difficulty in giving the information, the respective claims of the inventors are easily decided. The first thought originated with the celebrated Dr. Hooke, it was completed by Sir Isaac Newton, and published by Mr. Hadley.

Notwithstanding, however, the manifest superiority of this instrument over those that were in use at the time of it's publication, it was many years before the sailors could be persuaded to adopt it, and lay aside their imperfect and inaccurate instruments : so great is the difficulty to

remove prejudice, and emancipate the mind from the slavery of opinion.

No instrument has undergone, since the original invention, more changes than the quadrant of Hadley; of the various alterations, many have had no better foundation than the caprice of the makers, who by these attempts have often rendered the instrument more complicated in construction, and more difficult in use, than it was in it's original state.

ESSENTIAL PROPERTIES OF HADLEY'S QUADRANT.

It is not my intention under this head to enumerate all the advantages of this instrument; but barely to point out one or two of those essential properties which distinguish it from every other instrument of the kind, and rank it among one of the greatest improvements in the practice of navigation.

It is an essential property of this instrument, derived from the laws of reflection, that half degrees on the arch answer to whole ones in the angles measured: hence an octant, or the eighth part of a circle, or 45 degrees on the arch, serves to measure 90 degrees; and sextants will measure an angular distance of 120 degrees, though the arch of the instrument is no more than 60 degrees. It is from this property that foreigners term that instrument an octant, which we usually call a quadrant, and which in effect it is. This property reduces indeed considerably the bulk of the instrument; but at the same time it calls for the utmost accuracy in the divisions, as every error on the arch is doubled in the observation.

Another essential, and indeed an invaluable property of this instrument, whereby it is rendered
pecu-

peculiarly advantageous in marine observations, is, that it does not require any peculiar steadiness of the hand, nor is liable to be disturbed by the ship's motion; for provided the mariner can see distinctly the two objects in the field of his instrument, no motion nor vacillation of the ship will injure his observation.

Thirdly, the errors to which it is liable are easily discovered, and readily rectified, while the application and use of it is facile and plain.

The principal requisites in a good sextant or quadrant, are, 1. That it be strong, and so constructed as not to bend across the plane. 2. That it be accurately divided. 3. That the surfaces of the glasses be perfectly plane and parallel to each other. 4. That the index turn upon a long axis. 5. That the motion be free and easy in every part, and yet without the least shake or jerk.

DESCRIPTION OF HADLEY'S QUADRANT.

Fig. 1, plate 19, represents a quadrant, or octant, of the common construction. The following parts are those which require the particular attention of the observer.

- I. B C the arch.
- II. D the index, a b the nonius scale.
- III. E the index-glass.
- IV. F the fore horizon-glass.
- V. G the back horizon-glass.
- VI. K the dark glasses or screens.
- VII. H I the vanes or sights.
- VIII. The arc a b is called the *limb* or *quadrantal arch*; the arc c d lying from o towards the right is called the *arc of excess*.

OF THE QUADRANT.

The quadrant consists of an arch BC , firmly attached to two radii, or bars, AB , AC , which are strengthened and bound together by the two braces LM .

OF THE INDEX.

The index D is a flat bar of brass, that turns on the center of the octant; at the lower end of the index there is an oblong opening, to one side of this opening a nonius scale is fixed, to subdivide the divisions of the arch; at the bottom or end of the index there is a piece of brass, which bends under the arch, carrying a spring to make the nonius scale lie close to the divisions; it is also furnished with a screw to fix the index in any desired position.

The best instruments have an adjusting screw fitted to the index, that it may be moved more slowly, and with greater regularity and accuracy than by the hand. It is proper, however, to observe, that the index must be previously fixed near it's right position by the above-mentioned screw, before the adjusting screw is put in motion.

The circular arches on the arch of the quadrant are drawn from the center on which the index turns: the smallest excentricity in the axis of the index would be productive of considerable errors,

It is the position of the index on the arch, after an observation, which points out the number of degrees and minutes contained in the observed angle.

OF THE INDEX-GLASS E.

Upon the index, and near it's axis, is fixed a plane

plane speculum, or mirror of glass quicksilvered. It is set in a brass frame, and is placed so that the face of it is perpendicular to the plane of the instrument; this mirror being fixed to the index, moves along with it, and has it's direction changed by the motion thereof.

This glass is designed to receive the image of the sun, or any other object, and reflect it upon either of the two horizon glasses, according to the nature of the observation.

The brass frame with the glass is fixed to the index by the screw c; the other screw serves to re-place it in a perpendicular position, if by any accident it has been deranged, as will be seen hereafter.

The index glass is often divided into two parts, the one silvered, the other black with a small screen in front. A single black surface has indeed some advantages; but if the glasses be well selected, there is little danger to be apprehended of error from a want of parallelism; more is to be feared from the surfaces not being flat.

OF THE HORIZON-GLASSES F, G.

On the radius A B of the octant are two small speculums. The surface of the upper one is parallel to the index-glass, when the counting division of the index is at 0 on the arch; but the surface of the lower one is perpendicular to the index-glass, when the index is at 0 degrees on the arch: these mirrors receive the reflected rays from the object, and transmit them to the observer.

The horizon-glasses are not entirely quicksilvered; the upper one, F, is only silvered on it's lower part, or that half next the quadrant, the other half being transparent, and the back part of

the frame is cut away, that nothing may impede the sight through the unsilvered part of the glass. The edge of the foil of this glass is nearly parallel to the plane of the instrument, and ought to be very sharp, and without a flaw.

The other horizon-glass is silvered at both ends; in the middle there is a transparent slit, through which the horizon may be seen.

Each of these glasses is set in a brass frame, to which there is an axis; this axis passes through the wood-work, and is fitted to a lever on the under side of the quadrant; by this lever the glass may be turned a few degrees on it's axis, in order to set it parallel to the index-glass. The lever has a contrivance to turn it slowly, and a button to fix it. To set the glasses perpendicular to the plane of the quadrant, there are two sunk screws, one before and one behind each glass; these screws pass through the plate on which the frame is fixed, into another plate, so that by loosening one, and tightening the other of these screws, the direction of the frame, with it's mirror, may be altered, and thus be set perpendicular to the plane of the instrument.

OF THE SHADES, OR DARK GLASSES, K.

There are two red or dark glasses, and one green one; they are used to prevent the bright rays of the sun, or the glare of the moon, from hurting the eye at the time of observation. They are each of them set in a brass frame, which turns on a center, so that they may be used separately, or together, as the brightness of the sun may require. The green glass may be used also alone, if the sun be very faint; it is also used for taking the altitude

of

of the moon, and in ascertaining her distance from a fixed star.

When these glasses are used for the fore observation, they are fixed as in *fig. 1*; when used for the back observation, they are removed to N.

OF THE TWO SIGHT VANES H, I.

Each of these vanes is a perforated piece of brass, designed to direct the sight parallel to the plane of the quadrant. That which is fixed at I is used for the fore, the other for the back observation.

The vane I has two holes, one exactly at the height of the quicksilver edge of the horizon glass, the other somewhat higher, to direct the sight to the middle of the transparent part of the mirror, for those objects which are bright enough to be reflected from the unsilvered part of the mirror.

OF THE DIVISIONS ON THE LIMB OF THE QUADRANT, AND OF THE NONIUS ON THE INDEX.

For a description of these divisions see page 127.

DIRECTIONS TO HOLD THE INSTRUMENT.

It is recommended to support the weight of the instrument by the right hand, and reserve the left to govern the index. Place the thumb of the right hand against the edge of the quadrant, under the swelling part on which the fore sight (I) stands, extending the fingers across the back of the quadrant, so as to lay hold on the opposite edge, placing the fore finger above, and the other fingers below

the swelling part, or near the fore horizon glass: thus you may support the instrument conveniently, in a vertical position, by the right hand only; by resting the thumb of the left hand against the side, or the fingers against the middle bar, you may move the index gradually either way.

In the back observation, the instrument should be supported by the left hand, and the index be governed by the right.

OF THE AXIS OF VISION, OR LINE OF SIGHT.

Of the two objects which are made to coincide by this instrument, the one is seen directly by a ray passing through, the other by a ray reflected from the same point of the horizon glass to the eye. This ray is called the visual ray; but when it is considered merely as a line drawn from the middle of the horizon glass to the eye hole of the sight vane, it is called the axis of vision.

The axis of the tube, or telescope, used to direct the sight, is also called the axis of vision.

The quadrant, if it be held as before directed, may be easily turned round between the fingers and thumb, and thus nearly on a line parallel to the axis of vision; thus the plane of the quadrant will pass through the two objects when an observation is made, a circumstance absolutely necessary, and which is more readily affected when the instrument is furnished with a telescope; within the telescope are two parallel wires, which by turning the eye glass tube may be brought parallel to the plane of the quadrant, so that by bringing the object to the middle between them, you are certain of having the axis of vision parallel to the plane of the quadrant.

OF

OF THE NECESSARY ADJUSTMENTS.

It is a peculiar excellence of Hadley's quadrant, that the errors to which it is liable are easily detected, and soon rectified; the observer may, therefore, if he will be attentive, always put his instrument in a state fit for accurate observation. The importance of this instrument to navigation is self-evident; yet much of this importance depends on the accuracy with which it is made, and the necessary attention of the observer; and one would hardly think it possible that any observer would, to save the trifling sum of one or two guineas, prefer an imperfect instrument to one that was rightly constructed and accurately made; or that any consideration should induce him to neglect the adjustments of an instrument, on whose truth he is so highly interested. But such is the nature of man! he is too apt to be lavish on baubles, and penurious in matters of consequence; active about trifles, indolent where his welfare and happiness are concerned. The adjustments for the fore observation are, 1. To set the fore horizon glass parallel to the index glass; this adjustment is of the utmost importance, and should always be made previous to actual observation. The second is, to see that the plane of this glass is perpendicular to the plane of the quadrant. 3. To see that the index glass is perpendicular to the plane of the instrument.

TO ADJUST THE FORE HORIZON GLASS.

This rectification is deemed of such importance, that it is usual to speak of it as if it included all the rest, and to call it ADJUSTING THE INSTRUMENT. It is, so to place the horizon glass, that the index may shew upon the arch the true angle between
between

between the objects: for this purpose, set the index line of the nonius exactly against 0 on the limb, and fix it there by the screw at the under side. Now look through the sight I at the edge of the sea, or some very distant well-defined small object. The edge of the sea will be seen directly through the unsilvered part of the glass, but by reflection in the silvered part. If the horizon in the silvered part exactly meets, and forms one continued line with that seen through the unsilvered part, then is the instrument said to be adjusted, and the horizon glass to be parallel to the index glass. But if the horizons do not coincide, then loosen the milled nut on the under side of the quadrant, and turn the horizon glass on it's axis, by means of the adjusting lever, till you have made them perfectly coincide; then fix the lever firmly in the situation thus obtained, by tightening the milled nut. This adjustment ought to be repeated before and after every astronomical observation.

So important is this rectification, that experienced observers, and those who are desirous of being very accurate, will not be content with the preceding mode of adjustment, but adopt another method which is usually called finding the index error; a method preferable to the foregoing, both for ease and accuracy.

TO FIND THE INDEX ERROR,

Instead of fixing the index at (0), and moving the horizon glass, till the image of a distant object coincides with the same object seen directly; *let the horizon glass remain fixed*, and move the index till the image and object coincide; then observe whether

whether the index division on the nonius agrees with the 0 line on the arch; if it do not, the number by which they differ is a quantity to be added to the observation, if the index line is beyond the 0 on the limb; but if the index line of the nonius stands between 0 and 90 degrees, then this error is to be subtracted from the observation.*

We have already observed, that that part of the arch beyond 0, towards the right hand, is called the arch of excess; and that the nonius when at that part must be read the contrary way, or which is the same thing, you may read them off in the usual way, and then their complement to 20 min. will be the real number of degrees and minutes to be added to the observation.

TO MAKE THE INDEX GLASS AND FORE HORIZON GLASS PERPENDICULAR TO THE PLANE OF THE INSTRUMENT.

Though these adjustments are neither so necessary nor important as the preceding one, yet as after being once performed they do not require to be repeated for a considerable time, and as they add to the accuracy of observation, they ought not to be neglected; and further, a knowledge of them enables the mariner to examine and form a proper judgment of his instrument.

To

* This adjustment may be made more accurately, or the error better found, by using the sun instead of the horizon; but this method requires another set of dark glasses, to darken the direct rays of the sun; such a set is applied to the best instruments, and this method of adjustment is explained in the following description of the sextant.

TO ADJUST THE INDEX GLASS.

This adjustment consists in setting the plane of the index glass perpendicular to that of the instrument.

Method 1. By means of the two adjusting tools, represented at fig. 2 and 3, which are two wooden frames, with two lines on each, exactly at the same distance from the bottom.

Place the quadrant in an horizontal position on a table, put the index about the middle of the arch, turn back the dark glasses, place one of the above-mentioned tools near one end of the arch, the other at the opposite end, the side with the lines towards the index glass; then look down the index glass, directing the sight parallel to the plane of the instrument, you will see one of the tools by direct vision, the other by reflection in the mirror; by moving the index a little, they may be brought exactly together. If the lines coincide, the mirror is rightly fixed; if not, it must be restored to it's proper situation by loosening the screw c, and tightening the screw d, or vice versa, by tightening the screw c, and releasing the screw d.

Method 2. Hold the quadrant in an horizontal position, with the index glass close to the eye; look nearly in a right line down the glass, and in such manner, that you may see the arch of the quadrant by direct view, and by reflection at the same time. If they join in one direct line, and the arch seen by reflection forms an exact plane with the arch seen by direct view, the glass is perpendicular to the plane of the quadrant; if not, the error must be rectified, by altering the position

position of the screws behind the frame, as directed above.

TO ASCERTAIN WHETHER THE FORE HORIZON GLASS BE PERPENDICULAR TO THE PLANE OF THE INSTRUMENT.

Having adjusted the index and horizon glasses agreeable to the foregoing directions, set the index division of the nonius exactly against 0 on the limb; hold the plane of the quadrant parallel to the horizon, and observe the image of any distant object at land, or at sea the horizon itself; if the image of the horizon at the edge of the silvered part coincide with the object seen directly, the glass is perpendicular to the plane of the instrument. If it fall above or below, it must be adjusted. If the image seen by reflection be higher than the object itself seen directly, release the fore screw, and tighten the back screw, and vice versa, if the image seen by reflection be lowest, release the back screw and screw up the fore one: and thus proceed till both are of an equal height, and that by moving the index you can make the image and the object appear as one.

Or: Adjust the fore horizon glass as directed in page 252; then incline the quadrant on one side as much as possible, provided the horizon continues to be seen in both parts of the glass. If, when the instrument is thus inclined, the edge of the sea continues to form one unbroken line, the quadrant is perfectly adjusted; but if the reflected horizon be separated from that seen by direct vision, the speculum is not perpendicular to the plane of the quadrant. And if the observer is inclined to the right, with the face of the quadrant upwards, and the reflected sea appears higher than
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the real sea, you must slacken the screw before the horizon glass, and tighten that which is behind it; but if the reflected sea appears lower, the contrary must be performed.

Care must be always taken in these adjustments to loosen one screw before the other is screwed up, and to leave the adjusting screws tight, or so as to draw with a moderate force against each other.

This adjustment may be also made by the sun, moon, or star; in this case the quadrant may be held in a vertical position; if the image seen by reflection appears to the right or left of the object seen directly, then the glass must be adjusted as before by the two screws.

OF THE ADJUSTMENTS FOR THE BACK OBSERVATION.

The back observation is so called, because the back is turned upon one of the two objects whose angular distance is to be measured.

The adjustment consists in making the reflected image of the object behind the observer coincide with another seen directly before him, at the same time that the index division of the nonius is directly against the 0 division of the arch.

The method, therefore, of adjusting it, consists in measuring the distance of two objects nearly 180 degrees apart from each other; the arc passing through each object must be measured in both it's parts, and if the sum of the parts be 360 degrees, the speculums are adjusted; but if not, the axis of the horizon glass must be moved till this sum is obtained.

Set the index as far behind 0 as twice the dip of the horizon amounts to; then look at the horizon through the slit near G, and at the same time the
opposite

opposite edge of the sea will appear by reflection inverted, or upside down. By moving the lever of the axis (if necessary), the edges may be made to coincide, and the quadrant is adjusted.

There is but one position in which the quadrant can be held with the limb downwards, without causing the reflected horizon to cross the part seen by direct vision.

If, on trial, this position be found to be that in which the plane of the quadrant is perpendicular to the horizon, no farther adjustment is necessary, than the fore-mentioned one; but if the horizons cross each other when the quadrant is held upright, observe which part of the reflected horizon is lowest.

If the right-hand part be lowest, the sunk screw which is before the horizon-glass must be tightened after slackening that which is behind the glass; but if the right hand is highest, the contrary must be performed: this adjustment is, however, of much less importance than the preceding, as it does not so much affect the angle measured.

INCONVENIENCIES AND INACCURACIES OF THE BACK OBSERVATION.

The occasions, in which the back observation is to be used, are when the altitude of the sun or a star is to be taken, and the fore horizon is broken by adjacent shores; or when the angular distance between the moon and sun, or a star, exceeds 90 degrees, and is required to be measured for obtaining the longitude at sea: but there are objections to it's use in both cases; for if a known land lie a few miles to the north or southward of a ship, the latitude may be known from it's bearing and distance, without having recourse to observation:

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and

and again, if the distance of the land in miles exceed the number of minutes in the dip, as is almost always the case in coasting along an open shore, the horizon will not be broken, and the fore observation may be used: and lastly, if the land be too near to use the fore observation, it's extreme points will in general be so far asunder, as to prevent the adjustment, by taking away the back horizon. In the case of measuring the angular distances of the heavenly bodies, the very great accuracy required in these observations, makes it a matter of importance that the adjustments should be well made, and frequently examined into. But the quantity of the dip is varied by the pitching and rolling of the ship; and this variation, which is perceptible in the measuring altitude by the fore observation, is doubled in the adjustment for the back observation, and amounts to several minutes. It is likewise exceeding difficult, in a ship under way, to hold the quadrant for any length of time, so that the two horizons do not cross each other, and in the night the edge of the sea cannot be accurately distinguished. All these circumstances concur to render the adjustment uncertain; the fore observation is subject to none of these inconveniencies.*

* Nicholson's Navigator's Assistant.

DESCRIP-

DESCRIPTION, USE, AND METHOD OF ADJUSTING HADLEY'S SEXTANT.

As the taking the angular distances of the moon and the sun, or a star, is one of the best and most accurate methods of discovering the longitude, it was necessary to enlarge the arch of the octant to the sixth part of a circle; but as the observations for determining the longitude must be made with the utmost accuracy, the framing of the instrument was also altered, that it might be rendered more adequate to the solution of this important problem. Hence arose the present construction of the sextant, in the description of which, it is presumed that the foregoing pages have been read, as otherwise we should be obliged to repeat the same observations.

Sextants are mostly executed (some trifling variations excepted) on two plans; in the one, all the adjustments are left to the observer: he has it in his power to examine and rectify every part of his instrument. This mode is founded on this general principle, that the parts of no instrument can be so fixed as to remain accurately in the same position they had when first put out of the maker's hand; and that therefore the principal parts should be made moveable, that their positions may be examined and rectified from time to time by the observer.

In the second construction, the principal adjustment, or that of the horizon glass for the index error, is rejected; and this rejection is grounded upon two reasons:

S 2

1. That

1. That from the nature of the adjustment it frequently happens that a sextant will alter even during the time of an observation, without any apparent cause whatever, or without being sensible at what period of the observation such alteration took place, and consequently the observer is unable to allow for the error of adjustment.

2. That this adjustment is not in itself sufficiently exact, it being impossible to adjust a sextant with the same accuracy by the coincidence of two images of an object, as by the contact of the limbs thereof; and hence experienced and accurate observers have always directed the index error to be found and allowed for, which renders the adjustment of the horizon glass, in this direction, useless; for it is easy to place it nearly parallel to the index glass, when the instrument is made, and then to fix it *firmly* in that position by screws. The utility of this method is confirmed by experience: many sextants, whose index had been determined previous to their being carried out to India, have been found to remain the same at their return.

Notwithstanding the probable certainty of the horizon glass remaining permanently in its situation, the observer ought from time to time to examine the index error of his instrument, to see if it remains the same; or make the proper allowances for it, if any alteration should have taken place.

One material point in the formation of a sextant is so to construct it, that it may support its own weight, and not be liable to bend by any inclination of the plane of the instrument, as every flexure would alter the relative position of the mirrors, on which the determination of an angle depends.

DESCRIP-

DESCRIPTION OF THE SEXTANT.

Fig. 4, plate 19, represents the sextant, so framed, as not to be liable to bend. The arch AA is divided into 120° , each degree is divided into three parts, of course equal to 20 minutes, which are again subdivided by the nonius into every half minute, or 30 seconds: see the nature of the nonius, and the general rule for estimating the value thereof, in the preceding part of these essays. Every second division, or minute, on the nonius is cut longer than the intermediate ones. The nonius is numbered at every fifth of these longer divisions, from the right towards the left, with 5, 10, 15, and 20, the first division towards the right hand being to be considered as the index division.

In order to observe with accuracy, and make the images come precisely in contact, an adjusting screw B is added to the index, which may be moved with greater accuracy than it can by hand; but this screw does not act until the index is fixed by the finger screw C. Care should be taken not to force the adjusting screw when it arrives at either extremity of it's adjustment. When the index is to be moved any considerable quantity, the screw C at the back of the sextant must be loosened; but when the index is brought nearly to the division required, this back screw should be tightened, and then the index may be moved gradually by the adjusting screw. A small shade is sometimes fixed to that part of the index where the nonius is divided, this being covered with white paper, reflects the light strongly upon the divisions.

There are four tinged glasses at D, each of which is set in a separate frame turning on a center: they

are used to screen and save the eye from the brightness of the solar image and the glare of the moon, and may be used separately, or together, as occasion requires.

There are three more such glasses placed behind the horizon glass at E, to weaken the rays of the sun or moon, when they are viewed directly through the horizon glass. The paler glass is sometimes used in observing altitudes at sea, to take off the strong glare of the horizon.

The frame of the index glass I is firmly fixed by a strong cock to the center plate of the index. The horizon glass F is fixed in a frame that turns on the axes or pivots, which move in an exterior frame: the holes in which the pivots move may be tightened by four screws in the exterior frame; G is a screw by which the horizon glass may be set perpendicular to the plane of the instrument; should this screw become loose, or move too easy, it may be easily tightened by turning the capstan-headed screw H, which is on one side the socket, through which the stem of the finger-screw passes.

The sextant is furnished with a plain tube, *fig. 7*, without any glasses; and to render the objects still more distinct, it has also two telescopes, one, *fig. 5*, representing the objects erect, or in their natural position. The longer one, *fig. 6*, shews them inverted; it has a large field of view, and other advantages; and a little use will soon accustom the observer to the inverted position, and the instrument will be as readily managed by it as by the plane tube alone. By a telescope, the contact of the images is more perfectly distinguished; and by the place of the images in the field of the telescope it is easy to perceive whether the sextant is held in the proper plane for observation. By

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sliding the tube that contains the eye-glasses in the inside of the other tube, the object is suited to different eyes, and made to appear perfectly distinct and well defined.

The telescopes are to be screwed into a circular ring, at K; this ring rests on two points against an exterior ring, and is held thereto by two screws: by turning one, and tightening the other, the axis of the telescope may be set parallel to the plane of the sextant. The exterior ring is fixed on a triangular brass stem that slides in a socket, and by means of a screw at the back of the quadrant, may be raised or lowered so as to move the center of the telescope to point to that part of the horizon-glass which shall be judged the most fit for observation. *Fig. 8*, is a circular head, with tinged glasses to screw on the eye-end of either of the telescopes, or the plane tube. The glasses are contained in a circular plate, which has four holes; three of these are fitted with tinged glasses, the fourth is open. By pressing the finger against the projecting edge of this circular plate, and turning it round, the open hole, or any of the tinged glasses, may be brought between the eye-glass of the telescope and the eye.

Fig. 9, a small screw-driver. *Fig. 10*, a magnifying glass.

TO FIND THE INDEX ERROR OF THE SEXTANT.

To find the index error, is, in other words, to shew what number of degrees and minutes is indicated by the nonius when the direct and reflected images of an object coincide with each other.

The most general and most certain method of ascertaining this error, is to measure the diameter of the sun, by bringing the limb of it's image to coincide with the limb of the sun itself seen directly

both on the quadrantal arch, and on the arch of excess.

If the diameter taken by moving the index forward on the quadrantal arch be greater than that taken on the arch of excess, then half the difference is to be *subtracted*; but if the diameter taken on the arch of excess be greater than that by the quadrantal arch, half the difference is to be added. If the numbers shewn on the arch be the same in both cases, the glasses are truly parallel, and there is no index error; but if the numbers be different, then half the difference is the index error.*

It is, however, to be observed, that when the index is on the arch of excess, or to the right of 0, the compliment of the numbers shewn on the nonius to 20 ought to be set down.

Several observations of the sun's diameter should be made, and a mean taken as the result, which will give the index error to very great exactness.

Example. Let the numbers of minutes shewn by the index to the right of zero, when the limbs of the two images are in contact, be 20 minutes, and the odd number shewn by the nonius be 5, the compliment of this to 20 is 15, which, added to 20, gives 35 minutes; and, secondly, that the number shewn by the index, when on the left of zero, and the opposite limbs are in contact, be 20 minutes, and by the nonius 9' 30", which makes together 29' 30"; this subtracted from 35' gives 5' 30", which divided by 2, gives 2' 45" for the index error; and because the greatest of the two numbers thus found was when the index was to

* In other words, the difference between the degree and minute shewn by the index: 1st, when the lower reflected limb of the sun is exactly in contact with the upper limb of the sun; and secondly, when the upper edge of the image is in contact with the lower edge of the object, divided by 2 will be the index error.

to the right of the 0, this index error must be added to the number of degrees shewn on the arch at the time of an observation.

TO SET THE HORIZON GLASS PERPENDICULAR TO THE PLANE OF THE SEXTANT.

Direct the telescope to the sun, a star, or any other well-defined object, and bring the direct object and reflected image to coincide nearly with each other, by moving the index; then set the two images parallel to the plane of the sextant, by turning the screw, and the images will pass exactly over each other, and the mirror will then be adjusted in this direction.

TO SET THE AXIS OF THE TELESCOPE PARALLEL TO THE PLANE OF THE SEXTANT.

We have already observed, that in measuring angular distances the line of sight, or plane of observation, should be parallel to the plane of the instrument, as a deviation in that respect will occasion great errors in the observation, and this is most sensible in large angles: to avoid these, a telescope is made use of, in whose field there are placed two wires parallel to each other, and equidistant from the center. These wires may be placed parallel to the plane of the sextant, by turning the eye-glass tube, and consequently, by bringing the object to the middle between them, the observer may be certain of having the axis of vision parallel to the plane of the quadrant.

TO ADJUST THE TELESCOPE.

Screw the telescope in it's place, and turn the eye-tube round, that the wires in the focus of the eye-glass may be parallel to the plane of the instrument:

ment: then seek two objects, as the sun and moon, or the moon and a star, whose distance should, for this purpose, exceed 90 degrees; the distance of the sun and moon is to be taken great, because the same deviation of the axis will cause a greater error, and will consequently be more easily discovered. Move the index, so as to bring the limbs of the sun and moon, if they are made use of, exactly in contact with that wire which is nearest to the plane of the sextant; fix the index there; then, by altering a little the position of your instrument, make the images appear on the wire furthest from the sextant. If the nearest limbs be now precisely in contact as they were before, then the axis of the telescope is in it's right situation. But if the limbs of the two objects appear to separate at the wire that is furthest from the plane of the instrument, it shews that the object end of the telescope inclines towards the plane of the instrument, and must be rectified by tightening the screw nearest the sextant, having previously unturned the screw furthest from it. If the images overlap each other at the wire furthest from the sextant, the object end of the telescope is inclined from the plane of the sextant, and the highest screw must be turned towards the right, and the lowest screw towards the left: by repeating this operation a few times, the contact will be precisely the same at both wires, and consequently the axis of the telescope will be parallel to the plane of the instrument.

TO EXAMINE THE GLASSES OF A SEXTANT, OR QUADRANT.

1. To find whether the two surfaces of any one of the reflecting glasses be parallel, apply your eye at one end of it, and observe the image of some object

object reflected very obliquely from it; if that image appears single, and well-defined about the edges, it is a proof that the surfaces are parallel: on the contrary, if the edge of the reflected images appear misted, as if it threw a shadow from it, or separated like two edges, it is a proof that the two surfaces of the glass are inclined to each other: if the image in the speculum, particularly if that image be the sun, be viewed through a small telescope, the examination will be more perfect.

2. To find whether the surface of a reflecting glass be plane. Chuse two distant objects, nearly on a level with each other; hold the instrument in an horizontal position, view the left hand object directly through the transparent part of the horizon glass, and move the index till the reflected image of the other is seen below it in the silvered part; make the two images unite just at the line of separation, then turn the instrument round slowly on it's own plane, so as to make the united images move along the line of separation of the horizon glass. If the images continue united without receding from each other, or varying their respective position, the reflecting surface is a good plane. The observer must be careful that he does not give the instrument a motion about the axis of vision, as that will cause a separation if the planes be perfect.

3. To find if the two surfaces of a red or darkening glass are parallel and perfectly plane. It is difficult, nay almost impossible to procure the shades perfectly parallel and good; they will therefore, according to their different combinations, give different altitudes or measures of the sun and moon.

The best way to discover the error of the shades,

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is to take the sun's diameter with a piece of smoaked glass before your telescope, all the vanes being removed; then take away the smoaked glass, and view the sun through each shade and the several combinations thereof. If the two images still remain in contact, the glasses are good; but if they separate, the error is to be attributed to the dark glasses, which must either be changed, or the error found in each combination must be allowed for in the observations. If you use the *same* dark glasses in the observation as in the adjustment, there will be no error in the observed angle.

AN IMPORTANT REMARK.

The following very important observation of Dr. Blair's did not occur soon enough to be inserted in it's proper place, and yet was of too much consequence to be altogether neglected. Besides the errors already mentioned, to which the Hadley's quadrant, &c. is liable, there is another, which seems inseparable from the construction and materials of the instrument. It arises from the bending and elasticity of the index, and the resistance it meets with in turning round it's center.

To obviate this error, let the observer be careful always to finish his observations, by moving the index in the same direction which was used in setting it to 0 for adjusting, or in finding the index error.

The direction of the motion is indeed indifferent; but as the common practice in observing is to finish the observations by a motion of the index in that direction which increases the angle, that is, in the fore observation from 0 towards 90, it would

would be well if the observer would adopt it as a general rule, to finish the motion of the index, by pushing it from him, or turning the screw in that direction which carries it farther from him. By finishing the motion of the index, we mean that the last motion of the index should be for some minutes of a degree at least in the required direction.

OF HADLEY'S SEXTANT, AS USED IN SURVEYING.

No instrument can be so conveniently used for taking angles in maritime surveying, as Hadley's sextant. It is used with equal facility at the mast head, as upon deck, by which it's sphere of observation is much extended: for supposing many islands to be visible from the mast head, and only one from deck, no useful observation can be made by any other instrument. But by this, angles may be taken at the mast head from the one visible object with great exactness; and further, taking angles from heights, as hills, or a ship's mast head, is almost the only way of exactly describing the figure and extent of the shoals.

It has been objected to the use of Hadley's sextant for surveying, that it does not measure the horizontal angles, by which alone a plan can be laid down. This observation, however true in theory, may be obviated in practice by a little caution.

If an angle be measured between an object on an elevation, and another near to it in a hollow, the difference between the base, which is the horizontal angle, and the hypotenuse, which is the angle observed, may be very great; but if these objects are measured, not from each other, but from some very distant object, the difference between

tween the angles of each from the very distant object, will be very near the same as the horizontal angle. This may be still further corrected, by measuring the angle not between an object on a plane and an object on an elevation, but between the object on a plane and some object in the same direction, as the elevated object of which the eye is sufficiently able to judge.

HOW TO OBSERVE THE HORIZONTAL ANGLE, OR
ANGULAR DISTANCE, BETWEEN TWO OBJECTS.

First adjust the sextant, and if the objects are not small, pitch on a sharp top, or corner, or some small distinct part in each to observe; then having set the index to 0 deg. hold the sextant horizontally, as above directed, and as nearly in a plane passing through the two objects as you can; direct the sight through the tube to the left-hand object, till it is seen directly through the transparent part of the horizon glass: keeping that object still in sight there, move the index till the other object is seen by reflection in the silvered part of the horizon glass; then bring both objects together by the index, and by the inclination of the plane of the sextant when necessary, till they unite as one, or appear to join in one vertical line in the middle of the line which divides the transparent and reflecting parts of the horizon glass: the two objects thus coinciding, or one appearing directly below the other, the index then shews on the limb the angle which the two objects subtend at the naked eye. This angle is always double the inclination of the planes of the two reflecting glasses to one another; and therefore every degree and minute the index is actually moved from 0, to bring the two objects together, the angle subtended by

by them at the eye will be twice that number of degrees and minutes; and is accordingly numbered so on the arch of the sextant; which is really an arc of 60 degrees only, but graduated into 120 degrees.

The angle found in this manner between two objects that are near the observer, is not precise; and may be reckoned exact only when the objects are above half a mile off. For, to get the angle truly exact, the objects should be viewed from the center of the index glass, and not where the sight vane is placed; therefore, except the objects are so remote that the distance between the index glass and sight vane vanishes, or is as nothing compared to it, the angle will not be quite exact. This inaccuracy in the angle between near objects is called the parallax of the instrument, and is the angle which the distance between the index glass and sight vane subtends at any near object. It is so small, that a surveyor will seldom have occasion to regard it; but if it shall happen that great accuracy is required, let him chuse a distant object exactly in a line with each of the near ones, and take the angles between them, and that will be the true angle between the near objects. Or, observe the angle between near objects, when the sextant has been first properly adjusted by a distant object; then adjust it by the left-hand object, which will bring the index on the arch of excess beyond 0 degrees: add that excess to the angle found between the objects, and the sum will be the true angle between them. If one of the objects is near, and the other distant, and no remote object to be found in a line with the near one, adjust the sextant to the near object, and then take the angle between them, and the parallax will be found.

Example. To measure the horizontal angle ABC , *fig. 19, plate 9*, with the sextant.

1. Set up such marks at A and C , as may be seen when you are standing at B . 2. Set 0 on the index to coincide with 0 on the quadrantal arch. 3. Hold the sextant in an horizontal position, look through the sight and horizon glass at the mark A , and observe whether the image is directly under the object; if not, move the index till they coincide. 4. If the index division of the nonius is on the arch of excess, the indicated quantity is to be added to the observed angle; but if it be on the quadrantal arch, the quantity indicated is to be subtracted: let us suppose 5 to be added. 5. Now direct the sight through the transparent part of the horizon glass to A , keep that object in view, move the index till the object A is seen by reflexion in the silvered part of the horizon glass. 6. The objects being now both in view, move the index till they unite as one, or appear in one vertical line, and the index will shew the angle subtended at the eye by the two objects; suppose $75. 20$. to which add 5 for the index error, and you obtain $75. 25$. the angle required; if the angle be greater than 120° , which seldom happens in practice, it may be subdivided by marks, and then measured.

No instrument can be more convenient or expeditious than the sextant, for setting of offsets. Adjust the instrument, and set the index to 90 degrees; walk along the station line with the octant in your hand, always directing the sight to the farther station staff; let the assistant walk along the boundary line; then if you wish to make an offset from a given point in the station line, stop at that place, and wait till you see your assistant by reflexion,

flexion, he is then at the point in the boundary through which that offset passes; on the other hand, if you wish an offset from a given point or bend in the boundary, let the assistant stop at that place, and do you walk on in the station line till you see the assistant by reflexion in the octant, and that will be the point where an offset from the proposed point or bend will fall.

The manner of using this instrument for the solution of those astronomical problems that are necessary in surveying, will be shewn in it's proper place.

TO SURVEY WITH THE CHAIN ONLY.

The difficulties that occur in measuring with accuracy a strait line, render this method of surveying altogether insufficient for measuring a piece of ground of any extent; it would be not only extremely tedious, but liable to many errors that could not be detected; indeed there are very few situations where it could be used without *King's surveying quadrant*, or some substitute for it. The method is indeed in itself so essentially defective, that those who have praised it most, have been forced to call in some instrument, as the surveying cross and optical square, to their aid. Little more need be said, as it is evident, as well from the nature of the subject, as from the practice of the most eminent surveyors, that the measuring of fields by the chain, can only be proper for *level ground* and *small inclosures*; and that even then, it is better to go round the field and measure the angles thereof, taking offsets from the station lines to the fences. That this work may not be deemed imperfect, we shall introduce an example or two selected from some of the best writers on the subject;

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ject; observing, however, that fields that are plotted from measured lines, are always plotted nearest to the truth, when those lines form at their junction, angles that approach nearly to a right angle.

Example 1. To survey the triangular field ABC, fig. 22, plate 9, by the chain and cross. Set up marks at the corners, then begin at one of them and measure from A to B, till you imagine that you are near the point D, where a perpendicular would fall from the angle C, then letting the chain lie in the line AB, fix the cross at D, so as to see through one pair of the sights the marks at A and B; then look through the other pair towards C, and if you see the mark there, the cross is at it's right place; if not, you must move it backwards and forwards on the line AB, till you see the mark at C, and thus find the point D; place a mark at D, set down in your field book the distance AD, and complete the measure of AB, by measuring from D to B, 11. 41. Set down this measure, then return to D, and measure the perpendicular DC, 6.43. Having obtained the base and perpendicular, the area is readily found: it is on this principle that irregular fields may be surveyed by the chain and cross; the theodolite, or Hadley's sextant, may even here be used to advantage for ascertaining perpendicular lines. Some authors have given the method of raising perpendiculars by the chain only; the principle is good, but the practice is too operose, tedious, and even inaccurate to be used in surveying; for the method, see geometry on the ground.

Example 2. To measure the four sided figures ABCD, fig. 34, plate 9.

AE

A E 214

D E 210

A F 362

B F 306

A C 592.

Measure either of the diagonals as A C, and the two perpendiculars D E, B F, as in the last problem, which gives you data for completing the figure.

Example 3. To survey the irregular field, *fig. 22, plate 13.*

Having set up marks or station staves wherever it may be necessary, walk over the ground, and consider how it can be most conveniently divided into triangles and trapeziums, and then measure them by the last two problems.

It is best to subdivide the field into as few separate triangles as possible, but rather into trapeziums, by drawing diagonals from corner to corner, so that the perpendicular may fall within the figure; thus the figure is divided into two trapeziums A B C G, G D E F, and the triangle G C D. Measure the diagonal A C, and the two perpendiculars G M, B N, then the base G C, and the perpendicular D q; lastly, the diagonal D F, and the two perpendiculars, p E, O G, and you have obtained sufficient for your purpose.

OF SURVEYING BY THE PLAIN TABLE.

We have already given our opinion of this instrument, and shewn how far only it can be depended upon where accuracy is required; that there are many cases where it may be used to advantage, there is no doubt; that it is an expeditious mode of surveying, is allowed by all. I shall, therefore, here lay down the general modes of surveying with it, leaving it to the practitioner to select those best adapted to his peculiar circumstances, recommending him to use the modes laid

down in example 3, in preference to others where they may be readily applied. He will also be a better judge than I can be, of the advantages of Mr. Break's method of using the plain table.

To take by the plain table the plot of a piece of land *ABCDE*, *fig. 36, plate 9*, at one station near the middle, from whence all the corners may be seen.

Let *RTSV*, *fig. 37, plate 9*, represent the plain table covered with a sheet of paper, on which the plan of the field, *fig. 36*, is to be drawn; go round the field and set up objects at all the corners thereof, then put up and level your plain table, turning it about till the south point of the needle points to the fleur de lys, or 360° in the compass box; screw the table fast in that position, and then draw a line *Pp* parallel to one of the sides for a meridian line. Now chuse some point on the paper for your station line, and make there a fine hole with a small circle of black lead round it; this is to represent the station point on the land, and to this the edge of the index is to be applied when directed to an object.

Thus, apply the edge of the index to the point $\odot$, and direct the sight to the object at *A*, when this is cut by the hair, draw a blank line along the chamfered edge of the index from $\odot$ towards *A*, after this move the index round the point $\odot$ as a center, till you have successively observed through the sights, the several marks at *A, B, C, D, E*; and when these marks coincide with the sights, draw blank or obscure lines by the edge of the index to $\odot$. Now measure the distance from the station point on the ground to each of the objects, and set off by your scale (which should be as large as your paper will admit of) these measures on their respective

respective lines; join the points A B, B C, C D, D E, E A, by lines for the boundaries of the field, which if the work be properly executed, will be truly represented on the paper.

N. B. It is necessary, before the lines are measured, to find by a plumb line the place on the ground under the mark $\odot$ on the paper, and to place an arrow at that point.

Example 2. Let *fig. 33, plate 9*, represent the piece of ground to be surveyed from one station point, whence all the angles may be seen, but not so near the middle as in the foregoing instance; go round the field and set up your objects at all the corners, then plant the table where they may all be conveniently seen; and if in any place a near object and one more remote are in the same line, that situation is to be preferred. Thus in the present case, as at $\odot$, *g* coincides with *b*, and *c* with *b*; the table is planted thereon, making the lengthway of the table correspond to that of the field. Make your point-hole and circle to represent the place of the table on the land, and apply the edge of the index thereto, so as to see through the slit the mark at *a* cut by the hair; then with your pointrel draw a blank line from $\odot$ towards *a*, do the same by viewing through the sights the several marks *c, d, e, f, g*, keeping the edge of the index always close to $\odot$, and drawing blank lines from $\odot$ towards each of these marks.

Find by a plumb line the place on the ground under $\odot$ on the paper, and from this point measure the distances first to *g*, and proceed on in the same line to *b*, writing down their lengths as you come to each, then go to *a*, and measure from it to $\odot$, then set off from your scale the respective distance of each on it's proper line; after this measure

sure to c , and continue on the line to b , and set off their distances; then measure from $\odot$ to d , and from e to $\odot$; and lastly, from $\odot$ to f , and set off their distances. Then draw lines in ink from each point thus found to the next for boundaries, and a line to cross the whole for a meridian line.

Fig. 33, may be supposed to be two fields, and the table to be planted in the N. E. angle of the lower field, where the other angles of both fields may be observed and measured to.

Example 3. To survey a field represented at *fig. 32*, *plate 9*, by going round the same, either within or without, taking at the same time offsets to the boundaries.

Set up marks at a, b, c, d , at a small distance from the hedges; but at those places which you intend to make your station points.

Then beginning at $\odot$, plant your instrument there, and having adjusted it, make a fine point $\odot 1$ on that part of the paper, where it will be most probable to get the whole plan (if not too large) in one sheet; place the index to $\odot 1$, and direct the sight to the mark at $\odot 6$, draw a blank line from $\odot 1$ to $\odot 6$, then direct the index to the tree near the middle of the field, and afterwards to the mark at $\odot 2$, then dig a hole in the ground under $\odot 1$ in the plan, and taking up the table, set up an object in it exactly upright, and measure from it towards $\odot 2$, and find that perpendicular against 218, the offset to the angle at the boundary is 157 links, which set off in the plan; then measuring on at 375 the offset is but 6, and continues the same to 698, at both which set off 6 in the plan; then measure on to $\odot 2$, and find the whole 1041 links, which set off in the blank line drawn for it, and mark it $\odot 2$; then

then taking out the object, plant the table to have $\odot 2$ over the hole, when placed parallel to what it was at $\odot 1$; that is, the edge of the index-ruler touching both stations, the hair must cut the object at $\odot 1$, and then screw it fast.

Now setting up objects in the by-angles a and b , first turn the index to view that at a , and draw a blank line from $\odot 2$ towards it; then do the same towards b , $\odot 3$, and the tree, which last crossing that drawn towards it from $\odot 1$, the intersection determines the place of the tree, which being remarkable, as seen from all the stations, mark it in the plan; then measure to a 412 links, and from b 353, and set them off in the blank lines drawn towards them; then set off the distance to the boundaries in the two station-lines produced, *viz.* 151 in the next produced backwards, and 15 in the first produced forwards; after this draw the boundaries from the angle where the first offset was made to the next, and so on round by a , b , through the 151 to the next angle; then taking up the table, fix again the object as before, and measure on to $\odot 3$, which set off 564 links, and the offset to the angle in the boundary 27, and then draw the boundary from it through the 15 to the angle at meeting that last drawn.

Now taking out the object at $\odot 3$, plant the table so as to have $\odot 3$ over the hole, when placed parallel to what it was at the former stations, and screwed fast; then turn the index to make the edge touch the place of the tree and $\odot 3$ in the plan, and finding the hair cuts the tree, turn the index to view $\odot 4$, and draw a blank line towards it; then taking up the table, fix the object as before, and measure on to $\odot 4$, which set

off 471 links, and the offset 23, and draw the boundary from the last angle through it to the next; then measure on in the station-line produced to the next boundary 207 links, and the distance of $\odot 4$, from the nearest place in the same boundary 173, both which set off, and draw the boundary from this last through the 207 to the angle.

Now taking out the mark, plant the table to have $\odot 4$ over the hole, when screwed fast in the same parallelism as at the other stations; then, after viewing again the tree, turn the index to view $\odot 5$, and draw a blank line towards it; then taking up the table, fix the object as before, and measuring on towards $\odot 5$, at 125 the nearest place of the boundary is distant 121, which set off bearing forwards, as the *figure* shews; at 388 the perpendicular offset is 9, and at 712 it is 12, both which set off in your plan; then measure on to $\odot 5$, and set it off at 912 links.

Take out the mark, plant the table to have $\odot 5$ over the hole, when screwed fast in the same parallelism as before; then set up objects in the by-angles *c* and *d*, and after viewing the tree, turn the index to view the objects at *c*, *d*, and $\odot 6$, and draw a blank line towards each; then measure to *c* 159, and from *d* 245, both which set off in your plan, and also the distance to the boundary in the next station-line produced backward 95; and now make up the boundary round by the several offsets to the angles *c* and *d*; then taking up your table, fix the object as before, and measuring towards $\odot 6$, find at 162 the offset is 32, which set off; measure on to $\odot 6$, and set it off at 708, and the offset from it to the boundary is 36 links.

Finding the blank line drawn from $\odot 1$ to intersect the point-hole here made for $\odot 6$, do not plant the table at $\odot 6$, but begin measuring from it towards $\odot 1$, and finding at right angles to the line at $\odot 6$, the offset to the angle is 42, set that off in your plan; then measuring to $\odot 1$, 582, which, measuring the same by the scale in the plan, proves the truth of the work; the offset is here also 42, which set off, and draw the boundary from d round by the several offsets through this last to the angle; then measure on in the station-line produced to the next boundary 88 links, and set that off also, and draw the boundary from the angle at the first offset, taken through it to the angle at meeting the last boundary; and then if a meridian-line be drawn, as in the former, the rough plan is completed.

But if $\odot 6$ had not met in the intersection, or it's distance from $\odot 1$ been too much, or too little, you would very likely have all your work, except the offsets, to measure and plot over again.

The plain-table *surveyors*, says Mr. Gardner, when they find their work not to close right, do often close it wrong, not only to save time and labour, but the acknowledging an error to their assistants, which they are not sure they can amend, because in many cases it is not in their power, and may be more often the fault of the instrument than the *surveyor*; for in uneven land, where the table cannot at all the stations be set horizontal, or in any other one plane, it is impossible the work should be true in all parts: but to prevent great errors, at every $\odot$ after the second view, wherever it is possible, the object at some former $\odot$, besides that the table was last planted

ed at ; because if the edge of the index-ruler do not quite touch, or but very little covers that $\odot$ in the plan, whilst it touches the $\odot$ you are at, the error may be amended before it is more increased, and if it varies much, it may be examined by planting again the table at the former station or stations.

If a field is so hilly, that you cannot, without increasing the number of stations, see more than one object backward, and another forward, and there is nothing fit within the field, as the supposed tree in *fig. 32*, then set up an object on purpose to be viewed from all the stations, if possible, for such a rectifier.

The lengthening and shortening of the paper, as the weather is moister or drier, often causes no small error in plotting on the plain-table ; for between a dewy morning, and the sun shining hot at noon-day, there is great difference, and care should be taken to allow for it ; but that cannot be done in large surveys, and so ought not to be expected ; indeed those working by the degrees, without having their plan on it, are not liable to this error, though they are to the former ; but both ways are liable to another error, which is, that the station-lines drawn, or the degrees taken, are not in the line between the objects, nor parallel thereto ; neither will this error be small in short distances, and may be great, if each $\odot$ on the plan, or the center used with the degrees, is not exactly over the station-holes ; but to be most exact, it is the line of their sights, that should be directly over the hole.

Mr. Beighton made such improvements to his plain-tables, by a conical ferril fixed on the same slaves as his theodolet, that the above errors,

errors, except that of the paper, are thereby remedied; for the line of the sights in viewing is always over the center of the table, which is as readily set perpendicular over the hole, as the center of the theodolite, and the station-lines drawn parallel to those measured on the land; and the table is set horizontal with a spirit-level by the same four screws that adjust the theodolite; therefore some chuse to have both instruments, that they may use either, as they shall think most convenient.

Let *fig. 32*, now be a wood, to be measured and plotted on the outside; if on coming round to the first $\odot$, the lines meet as they ought, the plan will be as truly made, as if done on the inside; but here having no rectifier of the work as you go on, you must trust to the closing of the last measured line; and if that does not truly close with the first, you must go over the work again, and without a better instrument than the common plain-tables, you cannot be sure of not making an error in this case.

Suppose the table planted at $\odot 1$ on the outside, with paper fixt on it, and objects set up at all the other stations on the outside, and dry blank lines drawn from $\odot 1$ on the paper towards $\odot 6$ and $\odot 2$; these done, take up the table, and set up an object at $\odot 1$; then measuring from it towards $\odot 2$, you find at 20 the offset to the first angle is 38, then at 280 the offset to the next angle is 26 links, both of which set off in the plan; then at 394 the perpendicular offset to the next angle is 206; then at 698 the distance of the same angle is 366 bearing backward, as may be seen in the *figure*, that by the intersection of these two offset-lines the angle may be more truly plotted;

ted; then measure the distance from this angle to the next 323, and from that to 698 place in the station-line 280, which is the perpendicular offset; then by the intersection of these, that angle will be well plotted; then at 776 the offset to angle *a* is 48, and at 1012 the offset to *b* is 22, both which set off in the plan, and at 1306 you make $\odot 2$; now draw the boundaries from the first offset to the next, &c. to the angle *b*. As there is no difficulty in taking the offsets from the other station lines, we shall not proceed farther in plotting it on the outside; for a sight of the figure is sufficient.

Some surveyors would plant their table at a place between 394 and 698 in the first station-line, and take the two angles (which are here plotted by the intersection of lines) as the by-angles *a* and *b* were taken at $\odot 2$ within the field; but if the boundary should not be a strait line from one angle to the other, then their distance should be measured, and offsets taken to the several bends in it.

You plot an inaccessible distance in the same manner as the tree in *fig. 32*; for if you could come no nearer to it than the station-line, yet you might with a scale measure it's distance from $\odot 1$, or $\odot 2$, or any part of the line between them in the plan, the same as if you measured it with the chain on the land; observing to make the stations at such a distance from one another, that the lines drawn towards the tree may intersect each other as near as possible to right angles, drawing a line from each to $\odot 1$;* writing down the degree the

* In general the mark $\odot$ always denotes a station or place where the instrument is planted. The dotted lines leading from

the north end of the needle points to, as it should point to the same degree at each station; remove the table, and set up a mark at $\odot$ 1.

The imperfection in all the common methods of using the plain table are so various, so tedious, and liable to such inaccuracies, that this instrument, so much esteemed at one time, is now disregarded by all those who aim at correctness in their work. *Mr. Break* has endeavoured to remedy the evils to which this instrument is liable, by adopting another method of using it; a method which I think does him considerable honour, and which I shall therefore extract from his complete system of land-surveying, for the information of the practitioner.

Example 1. To take the plot of a field ABC DEF, *fig. 11, plate 13*, from one station therein.

Chuse a station from whence you can see every corner of the field, and place a mark at each, numbering these with the figures, 1, 2, 3, 4, &c. at this station erect your plain table covered with paper, and bring the south point of the needle to the fleur-de-lys in the box, then draw a circle OPQR upon the paper, and as large as the paper will hold.

Through the center of this circle draw the line NS parallel to that side of the table which is parallel to the meridian line in the box, and this will be the meridian of the plan.

Move the chamfered edge of the index on $\odot$ till you observe through the sights the several marks A, B, C, and the edge thereof will cut the circle in the points, 1, 2, 3, &c. Then having taken

from one station to another, are the station lines; the black lines the boundaries; the dotted lines from the boundary to the station line are offsets.

taken and protracted the several bearings, the distances must be measured as shewn before.

To draw the plan through the center $\odot$ and the points 1, 2, 3, &c. draw lines $\odot A$, $\odot B$, $\odot C$, $\odot D$, $\odot E$, $\odot F$, and make each of them equal to it's respective measure in the field; join the points A, B, C, &c. and the plan is finished.

Example 2.—To take the plot of a field ABCD, &c. from several stations. Fig. 14, plate 13.

Having chosen the necessary stations in the field, and drawn the circle $OPQR$ (which you must ever observe to do in every case), set up your instrument at the first station, and bring the needle to the meridian (which is called adjusting the instrument); move the index on the center $\odot$, and take an observation at $A, B, C, \odot 2, H$, and the fiducial edge thereof will intersect the circle in 1, 2, 3, &c. Then remove your instrument to the second station in the field, and applying the edge of the index to the center $\odot$ and the mark $\odot 2$ in the circle, take a back sight to the first station, and fasten the table in this position; then move the index on the center $\odot$, and direct the sights to the remaining angular marks, so will the fiducial edge thereof cut the circle in the points 4, 5, 6, &c. The several distances being measured with a chain, the work in the field is finished.

The FIELD-BOOK Explained.

N ^o .	DIST.	N ^o .	DIST.
	⊙ ₁		⊙ ₂
1	520	3	
2	344	4	370
3	360	5	470
⊙ ₂	730	6	550
8	386	7	550
		8	

To draw the Plan.

Having chosen ⊙₁ upon paper to represent the first station in the field, lay the edge of a parallel ruler to ⊙ and the mark 1, and extend the other edge till it touch (or lay upon) ⊙₁, and close by it's edge draw a line 11=520. Then lay the ruler as before to ⊙ and the mark 2, and extending the other edge to ⊙₁, draw thereby the line 12=344, which gives the corner B, as the line 11 does the corner A. After the same manner project ⊙₂, together with the corners C H. Again, apply the edge of the ruler to ⊙ and the point 4, and extend the other edge till it touch ⊙₂, and draw the line 24=370, which will give the point or corner D. Thus project the remaining corners E, F, G, and the plan is ready for closing.

Example 3.—To take the plot of several fields ABC D, BECI, DIHK, and IFGH, from stations chosen at or near the middle of each. Fig. 15, plate 13.

Adjust your plain-table at the first station in
ABCD,

ABCD, and draw the protracting circle and meridian *NS*; then by prop. 1, project the angles or corners of *ABCD*, *BEFC*, and also the 2d station, into the points 1, 2, $\odot$ 2, 3, 4, 5, 6. Again, erect your instrument at the 1st station, and lay the index on the center $\odot$ and the mark $\odot$ 2, and direct the sights (by turning the table) to the 2d station, then move the index till you observe the 3d station, and the edge thereof will cut the circle in $\odot$ 3. Then remove the instrument to the 3d station, lay the index on the center $\odot$ and the mark $\odot$ 3, and take a back observation to the first station; after which by the last prop. find the points 7, 8, $\odot$ 4, 9 in the circle *OPQR*. As to the measuring of distances both in this and the two succeeding prop. that shall be passed over in silence, having sufficiently displayed the same heretofore; what I intend to treat of hereafter, is the method of taking and protracting the bearings in the field, with the manner of deducing a plan therefrom.

To draw the Plan.

Chuse any point ($\odot$ 1) for your first station; apply the edge of a parallel ruler to the center $\odot$ and the point 1, and having extended the other edge to $\odot$ 1, draw the line $11=370$, which will give the corner *A*: In like manner find the other corners *B*, *C*, *D*, together with $\odot$ 2; which being joined, finishes the field *ABCD*. After the same method construct the other fields *BEFC*, *DIHK*, *IFGH*, and you have done.

The

The FIELD-BOOK.

REMARKS.	Nº.	DIST.	REMARKS.
The open	1	⊙ ₁ 370	In <i>ABCD</i>
	2	380	Field.
	⊙ ₂	580	<i>BEFI</i> .
	3	403	<i>DIHK</i> .
Sir Will. Jones's	4	440	Ground.
		⊙ ₂	In <i>BEFI</i> .
John Simpson's	2		
	5	400	Ground.
	6	460	<i>IFGH</i> .
<i>BEFI</i> closes at	7		
	3		
Return to		⊙ ₁	In <i>ABCD</i> .
	⊙ ₃	600	
		⊙ ₃	In <i>DIHK</i> .
South	7		<i>IFGH</i> .
	⊙ ₄	630	
	8	432	Field.
	9	400	Ground.
John Spencer's	4		
<i>DIHK</i> closes at		⊙ ₄	In <i>IFGH</i> .
Ed. Johnstone's	6		Ground.
	10	380	Field.
	8		

Example 4.—To take the plot of a field *ABCDEF*, by going round the same. Fig. 12, plate 13.

Set up your plain table at the first station in the field; move the fiducial edge of the index on the
 U center

center $\odot$, and take an observation at the mark placed at the 2d station, then will the same fiducial edge cut the circle $OPQR$ in the point 1. Then remove your instrument to the 2d station, and placing the edge of the index on $\odot$ and the point 1, take a back-sight to the 1st (or last) station; then directing the index on the center $\odot$ to the 3d (or next) station, the edge thereof will cross the circle in the point 2. In like manner the instrument being planted at every station, a back-sight taken to the last preceding one, and the index directed forward to the next succeeding station, will give the protracted points, 3, 4, 5, 6.

The FIELD-BOOK.

REMARKS.	O.	$\odot$ L.	O.	REMARKS.
		$\odot 1$		In $ABCDEF$.
	70	$\circ$		
	50	250		
	85	550		
		$\odot 2$		In Ditto.
	84	$\circ$		
Corner	65	440		
		$\odot 3$		In Ditto.
		$\circ$		
	60	465		
		$\odot 4$		In Ditto.
	72	$\circ$		
	58	365		
	80	750		

The

The FIELD-BOOK.

REMARKS.	O.	⊙ L.	O.	REMARKS.
		⊙ 5		In Ditto.
	40	0		
	68	302		
	60	680		
		—		
		⊙ 6		In Ditto.
	58	0		
	50	355		
	67	663	to ⊙ 1	Close Ditto.

To draw the Plan.

Chuse any point $\odot 1$, to denote the first station. lay the edge of a parallel ruler on the center $\odot$ and the point 1, and extend the other edge till it touch $\odot 1$, and draw by the side thereof the line $1\ 2 = 550$; then apply the ruler to $\odot$ and the mark 2, and extend the other edge to $\odot 2$, and draw thereby the line $2\ 3 = 440$; again, lay the edge of the ruler to $\odot$ and the point 3, and the other edge being extended to $\odot 3$, draw the line $3\ 4 = 465$: after the same method lay down the remaining stations, and the traverse is delineated. As for drawing the hedges, that shall be left for the learner's exercise.

Example 5.—To take the plot of several fields A, B, C, D, by circulation. Fig. 16, plate 13.

From the projecting point $\odot$ (by last example) project the stations in *A*, into the points 1, 2, 3, 4; then the instrument being planted at the 2d station, from the same projecting point $\odot$ project that station (the 2d in *A*) into the point 2^2 (2^2 denoting

U 2

noting

noting the instrument being planted a second time at that station) which is done thus : lay the index to $\odot$ and the point 2, and take a back-sight to the first station (that being the station immediately preceding that you are at in the field-book) ; then on the center $\odot$ take a fore-observation at the next succeeding station, and the index will cut the circle in the point 2². Thus project every other remaining station.

The FIELD-BOOK.

REMARKS.	O.	$\odot$ L.	O.	REMARKS.
		$\odot$ 1		In Field <i>A</i> .
	53	0		
	80	290		
Corner	85	610		
	60	695		
		$\odot$ 2		In Ditto.
	70	0		
	65	560		
		$\odot$ 3		In Ditto.
	60	0		
	75	368		
	55	680		
		$\odot$ 4		In Ditto.
	50	0		
	48	440	to $\odot$ 1	Close here <i>A</i> .
Return to		$\odot$ 2		In <i>A</i> .
		0		
Corner	60	80	into	<i>B</i> .
	50	402		

The

The FIELD-BOOK.

REMARKS.	O.	OL.	O.	REMARKS.
		⊙5		In B.
	50	0		
	60	340		
	50	650		
		⊙6		In Ditto.
	50	0		
Ag. Hedge	51	500		
		560	by ⊙	for closing C.
Closes at Cor. of A	50	620	+	the Hedge,
		682	to ⊙3	in A.
		⊙4		In A.
		0		
		60	70	
		into C.		
		380	60	
		682	68	
		⊙7		In C.
		0	40	
		300	60	
		720	50	
		⊙8		In Ditto.
		0	70	
		400	24	
		780	63	Close C.
		Cross hedge		
		820	bye	⊙at 56el. 6,4.

The FIELD-BOOK.

REMARKS.	O.	⊙ L.	O.	REMARKS.
Return to		⊙ 8		In Ditto.
		o		
		60	50	Corner.
		into	D.	
		229	50	
		554	40	
		⊙ 9		In D.
		o	40	
		250	38	
		⊙ 10		In Ditto.
		o		
		64	50	
		530	64	Cloſe D at Cor. B,
		into	B.	
		570	to ⊙ 6	in B.

The bearing being protracted, the plan may be readily drawn from what has been already deſcribed.

GENERAL METHODS OF SURVEYING DETACHED
PIECES OF GROUND.

To ſurvey the triangular field, fig. 22, plate 9, with any inſtrument uſed for meaſuring angles.

1. Set up marks at the three corners A, B, C.
2. Meaſure the angle A C B. 3. Meaſure the two ſides A C, C B.

This method of meaſuring two ſides and the included angle, is far more accurate than the old method of going round the field and meaſuring all the

the angles. It was first introduced into practice by Mr. Talbot.

If the field contain four sides, *fig. 23, plate 9*, begin at one of the corners A. 1. Measure the angle B A D. 2. Measure the side A B. 3. Take the angle A B C. 4. Measure the side B C, and angle B C D. 5. Measure the side C D, and the angle C D A, and side D A, the dimensions are finished; add the four angles together, and if the sum makes 360° , you may conclude that your operations are correct; the above figure may be measured by any other method as taught before, by measuring the diagonal, &c.

If the field contain more than four sides, *fig. 24, 25, 26, plate 9*, having set up your marks, endeavour to get an idea of the largest four-sided figure, that can be formed in the field you are going to measure; this figure is represented in the figures by the dotted lines.

Then beginning at A, take the angle B A D, measure in a right line towards B, till you come against the angle *f*, there with your sextant or cross let fall the perpendicular *f e*, (as taught in the method of the triangle (observing at how many chains and links this offset or perpendicular falls from the beginning or point A, which note in your field-book, and measure *e f*, noting it also in your field-book; then continue the measure of the line A B to B; take the angle A B C, and measure the line B C; take the angle B C D, and measure the line C D; take the angle C D A, and measure the line D A; observing as you go round to let fall perpendiculars where necessary, and measure them as specified in the line A B.

Example to *figure 26*, containing seven sides.

FIELD-BOOK.

$\angle A =$	86°, 35'	
$Ae =$	2,08	80 = ef .
	5,32	= 1st side A B.
$\angle B =$	88°, 25'	
	4,80	= 2d side B C.
$\angle C =$	93°, 00'	
$Cg =$	1,20	88 = gb .
	4,92	= 3d side C D.
$\angle D =$	92°, 00'	
$Di =$	2,00	1,08 = ik .
	4,84	= 4th side D A.

N. B. Set the offsets to the right or left of your column of angles and chains, according as they fall to the right and left of your chain line in the field.

TO USE THE COMMON CIRCUMFERENTER.

We have already observed that this instrument should never be used where much accuracy is required, for it is scarce possible to obtain with any certainty the measure of an angle nearer than to two degrees, and often not so near; it has therefore long been rejected by accurate surveyors.*

This instrument takes the bearing of objects from station

* See Gardner's Practical Surveying, p. 54.

station to station, by moving the index till the line of the sights coincides with the next station mark; then counting the degrees between the point of the compass box marked N, and the point of the needle in the circle of quadrants.

Thus let it be required to survey a large wood, *fig. 31, plate 9*, by going round it, and observing the bearing of the several station lines which encompass that wood.*

The station marks being set up, plant the circumferenter at some convenient station as at *a*, the flower de lys in the compass box being from you; direct the sights to the next station rod *b*, and set down the division indicated by the north end of the needle, namely $260^{\circ} 30'$, for the bearing of the needle.

Remove the station rod *b* to *c*, and place the circumferenter exactly over the hole where the rod *b* was placed, measuring the station lines, and the offsets from them to the boundaries; now move the instrument, and place the center thereof exactly over the hole from whence the rod *b* was taken. The flower de lys being from you, turn the instrument till the hair in the sights coincides with the object at the station *c*, then will the north end of the needle point to $292^{\circ} 12'$, the bearing of *bc*; the instrument being planted at *c*, and the sights directed to *d*, the bearing of *cd*, will be $331^{\circ} 45'$. In the same manner proceed to take the bearing of other lines round the wood, observing carefully the following general rule:

Keep the flower de lys from you, and take the bearing of each line from the north end of the needle.

lines.

* Wild's Practical Surveyor, p. 77.

lines.	bearings.	links.
a b	260. 30	1242
b c	292. 12	1015
c d	331. 45	1050
d c	59. 00	1428
c f	112. 15	645
f a	151. 30.	1806

Instead of planting the circumferenter at every station in the field, the bearings of the several lines may be taken if it be planted only at every other station.

So if the instrument had been planted at *b*, and the flower de lys in the box kept towards you when you look back to the station *a*, and from you when you look forwards to the station *c*, the bearings of the lines *ab*, and *bc*, would be the same as before observed; also the bearings of the lines *cd*, and *de*, might be observed at *d*, and *ef*, and *fa*, at *f*; so that instead of planting the instrument 6 times, you need in this case plant it but 3 times, which saves some labour.

But since you must go along every station line, to measure it or see it measured, the trouble of setting down the instrument is not very great, and then also you may examine the bearing of each line as you go along; and if you suspect an error in the work by the needles being acted on by some hidden magnetic power, or from your own mistake, in observing the degrees that the needle points to, you may correct such error at the next station before you proceed.

As when the instrument was planted at *a*, and the sights directed to *b*, the flower de lys from you, the north end of the needle pointed to 260° 30'; now being come to *b*, direct the sights back to a mark

a mark at *a*, keeping the flower de lys towards you: so shall the north end of the needle point to $260^{\circ} 30'$, as before at *a*, and then you may be sure the bearing of the line *ab*, is truly observed.

But if the needle doth not point to the same number of degrees, &c. there hath been some error in that observation which must be corrected,

OF THE IMPROVED CIRCUMFERENTER, WITH MR. GALE'S METHOD OF USING IT; A METHOD THAT IS APPLICABLE WITH EQUAL ADVANTAGE TO THE THEODOLET, &c.

For the sake of perspicuity, it will be necessary to give again the example before used in page 219, and that not only because it will exhibit more clearly the advantages of Mr. Gale's method,* but because we shall have occasion to refer it to when we come to his improved method of plotting; and further, because I have thought this mode so advantageous, and the tables so conducive to accuracy and expedition, that I have caused occasionally the traversing quadrant to be engraved in smaller figures under the usual one of the limb of the theodolet.

Set the circumferenter up at B, *fig. 1, plate 18*, take the course and bearing of BC, and measure the length thereof, and so proceed with the sides CD, DE, EF, FF, GA, all the way round to the place of beginning, noting the several courses or bearings, and the lengths of the several sides in a field-book, which let us suppose to be as following:

			°	'	chains.	links.
1.	AB	North	7		West	21. 00.
2.	BC	North	55	15	East	18. 20.
3.	CD	South	62	30	East	14. 40.
4.	DE	South	40		West	11.
						5. EF

* See page 217 of this work, for a fuller account of this method.

				°	'	chains.	links.
5.	E F	South	4	15	East	14.	
6.	F G	North	73	45	West	12.	40.
7.	G A	South	52		West	9.	17.

N. B. By north 7° west, is meant seven degrees to the westward (or left hand) of the north, as shewn by the needle; by north $55^{\circ} 15'$ east, fifty-five degrees fifteen minutes to the eastward, or right hand of the north, as shewn by the needle.

In like manner by south $62^{\circ} 30'$ east, is meant sixty-two degrees and thirty minutes to the eastward, or left hand of the south; and by south 40° west, forty degrees to the westward, or right hand of the south.

The 21 chains, 18 chains 20 links, &c. are the lengths or distances of the respective sides, as measured by the chain.

To survey a field or tract of land having irregular boundaries.

When the boundaries of a survey have crooks and bends in them, it is by no means necessary to take a new course for every small bend; the best and most usual way, is to proceed in a strait line from one principal corner to another, and when you are opposite to any bend in the boundary, to measure the rectangular distance, termed the offset from the strait line to the bend, noting the same in the field-book, together with the distance on the strait line, from whence such offsets were made. The offsets, as already observed, are generally measured with an offset staff.

For the purpose of noting these offsets, it is necessary that the field book should be ruled into FIVE COLUMNS. The middle column to contain the courses and distances; the adjoining columns

on the right and left hand to contain the measure of the offsets, made to the right or left hand respectively; and the outside columns on the right and left hand, to contain remarks made on the right and left hand, respectively, such as the names of the adjoining fields, or the bearing of any remarkable object, &c.

Example. Let *fig. 3, plate 18*, represent a field to be surveyed, whose boundaries are crooked.

Set up the instrument at or near any convenient corner as at *1*, and take the course and bearing as before directed, *north 7° west*, note this down in the middle column of the field book, and measure with the chain as before directed, till you come opposite to the first bend, so that the bend be at right angles to the station line; note the distance thus measured, 3 chains, 60 links, in the middle column, and measure the offset from thence to the bend 40 links, noting the same in the adjoining left hand column, (because the boundary is on the left hand of the station line) and note in the outside column, the name or owner of the adjoining field, proceeding in the same manner all round the field, noting the courses, distances, offsets, and remarks, as in the following.

FIELD-

FIELD-BOOK.

Remarks.	Off-sets.	Station lines.	Off-sets.	Remarks.
1st. At the corner, against Wm. Humphreys, and H. Derman's land.				
N. 7° W.				
H. Derman's land	0 0	0 0		
	0 40	3 60		
	0 10	8 45		
	0 65	15 60		
A corner	0 0	21 00		
2d.				
N. 55° 15 E.				
W. Huggin's land	0 0	0 0	0 0	
		6 10	0 60	
A corner	0 0	18 20	0 0	To the boundary of the field.
3d.				
S. 62° 30 E.				
D. Horne's land	0 0	0 0	0 0	
		7 40	0 90	
A corner	0 0	14 40	0 0	To the boundary of the field.
4th.				
S. 40° W.				
C. Ward's land	0 0	0 0		
	0 40	8 00		
To a corner	0 95	11 00		
5th.				
S. 4° 15 E.				
To the above corner	0 68	0 66		
C. Ward's land	1 25	7 50		
A corner	0 0	14 00		

Remarks.

FIELD-BOOK.

Remarks.	Off-sets.	Station lines.	Off-sets.	Remarks.
W. Humphrey's land	○ ○	6th. N. 73° 45' N.	○ ○	
		3 20	○ 30	To the bound-
		7 50	○ 25	ary of the field.
		12 40	○ ○	A corner.
W. Humphrey's land	○ ○	7th. S. 52° W.	○ ○	
		4 10	○ 60	To the bound-
		9 17	○ ○	ary of the field.

To the place of beginning.

To survey a tract of land, consisting of any number of fields lying together.

1. Take the outside boundaries of the whole tract as before directed, noting in your field-book where the particular fields intersect the outside boundaries; and then take the internal boundaries of the several fields from the place where they so cut on the outside bounds.

Let fig. 4, plate 18, represent a tract of land to be surveyed, consisting of three fields.

First begin at any convenient corner as at A, and proceed taking the courses, distances, offsets, and remarks, as in the following field-book.

Remarks.

FIELD-BOOK.

Remarks.	Off-sets.	Station lines.	Off-sets.	Remarks.
		1st. At a corner against Winterton farm, and H. Smith's land.		
Lands of Henry Smith.	0 00	S. 87° W. 0 00 5 50 11 30 15 00	0 00 1 10 0 00 0 60	East field. To the corner of east and west fields. Corner of west field.
Hunterdon farm.		2d. N. 18° 30. W. 0 58 9 60	0 16 0 00	The above corner.
		3d. N. 12° 15. E. 0 00 4 10 8 90	0 00 0 60 0 00	West field.
		4th. N. 37° E. 5 40 17 00	0 40 0 00	To the corner of west field and north field.
Lands of Jacob Williams.	0 70 0 00	5th. S. 76° 45. E. 6 50 16 00	0 00 0 00	North field.
Winterton farm.		6th. S. 5 15. W. 9 60 14 30 19 00 25 00	0 56 0 10 0 70 0 00	to the corner of north and east fields. N. B. A gate into each field 20 links from the corner.
		7th. S. 74° 30. W. 2 30 7 65	0 86 0 00	East field.
To the place of beginning.				

FIELD-BOOK.

Remarks.	Off-sets.	Station lines.	Off-sets.	Remarks.
		8th. At the corner of east and west field. viz. The first offset on the first station line.		
West field.		North. 8 30 16 61	0 75 0 00	East field.
		9th. East.		East field.
North field.	0 00	0 5 40 13 77	0 00 0 60 0 00	to the corner of north and east fields against Winterton farm.
West field.		10th. Back to the last station.		
Corner of west and north fields against Hunten-don farm.	0 00 0 70 0 00	N. 62° 03. W. 0 8 40 7 92	0 00 0 00 0 00	North field.

To take a survey of an estate, manor, and lordship.

An estate, manor, or lordship, is in reality a tract of land, consisting of a number of fields; it differs in no respect from the last article, (excepting in the number of fields it may contain, and the roads, lanes, or waters that may run through it,) and is of course surveyed in the same manner.

It is best in the first place to take the whole of the outside boundaries, noting as above directed the several offsets, the several places where the boundaries are intersected by roads, lanes, or waters, the places where the boundaries of the respective fields butt on the outside bounds, and where the gates lead into the respective fields, and whatever other objects, as windmills, houses, &c. that may happen to be worthy of being taken notice of. If, however, there should be a large stream

X

of

of water running through the estate, thereby dividing it into two parts, and no bridge near the boundary, then it will be best to survey that part which lies on one side of the stream first, and afterwards that part which lies on the other side thereof: if the stream be of an irregular breadth, both it's banks (forming boundaries to several fields) should be surveyed, and it's breadth, where it enters and where it leaves the estate, be determined by trigonometry.

In the next place, take the lanes or roads (if any such there be) that go through the estate, noting in the same manner as before, where the divisions between the several fields butt on those lanes or roads, and where the gates enter into those fields, and what other objects there may be worth noticing. Where a lane runs through an estate, it is best to survey in the lane, because in so doing, you can take the offsets and remarks both on the right hand and the left, and thereby carry on the boundaries on each side at once. If a large stream run through and separate the estate, it should be surveyed as above mentioned; but small brooks running through a meadow, require only a few offsets to be taken from the nearest station line, to the principal bends or turning in the brook.

In the last place, take the internal divisions or boundaries between the several fields, beginning at any convenient place, before noted in the field-book, where the internal divisions butt on the outside of the grounds, or on the lanes, &c. noting always every remarkable object in the field-book.

Example. Let *fig. 5, plate 18*, represent an estate to be surveyed; begin at any convenient place as at *A*, where the two lanes meet, proceed (noting the

the courses, distances, &c. as before directed) from A to B, from B to C, and so on to D, E, F, G, H, I, K, L, M, and A, quite round the estate.

Then proceed along the lane from A to N, O and I, setting the courses, distances, offsets, and remarks, as before.

This done, proceed to the internal divisions, beginning at any convenient place as O, and proceed (always taking your notes as before directed) from O to P, Q and R, so will you have (with the notes previously taken) the dimensions of the north field; go back to Q, and proceed from Q to H, and you obtain the dimensions of the copse: Take E S and S P, and you will have the dimensions of the home field; go back to S, and take S T, and you will have the dimensions of the land, applied to domestic purposes of buildings, yards, gardens, and orchards, the particulars and separate divisions of which being small, had better betaken last of all; go down to N, and take N D, (noting the offsets as well to the brook, as to the fences) which divide the meadow from the south and west fields, so will you have the dimensions of the long meadow, together with the minutes for laying down the brook therein; go back to U, and take U B, and you obtain the dimensions of the west field, and also of the south field; go back again to N, and take N W, (noting the offsets as well to the brook, as to the fence which divides the meadow from the east field,) thus will you have the dimensions of the east field, and the minutes for laying down the brooks in the meadow; then go to L, and take L X, which gives you the dimensions of the east meadow, and of the great field; and lastly, take the internal divisions of the land, appropriated to the domestic purposes of buildings, yards, orchards, gardens, &c.

The method of taking the field notes, is so entirely similar to the examples already given, that they would be altogether unnecessary.

To the surveyor there can need no apology for introducing, in this place, the method used by Mr. Milne, one of the most able and expert surveyors of the present day; and I think he will consider himself obliged to Mr. Milne, for communicating with so much liberality, his deviations from the common practice, as those who have hitherto made any improvements in the practical part of surveying, have kept them as profound secrets, to the detriment of science, and the young practitioner. As every man can describe his own methods, in the clearest and most intelligible manner, I have left Mr. Milne's in his own language.

MR. MILNE'S METHOD OF SURVEYING.

The method I take to keep my field notes in surveying land, differing materially from those yet published; if upon examination you shall think it may be useful or worthy of adoption, you may, if you please, give it a place in your treatise on surveying.

What I, as well as all surveyors aim at, in going about a survey, is accuracy and dispatch; the first is only to be acquired by care and good instruments, the latter by diligence and long practice.

From twenty years experience, in the course of which I have tried various methods, the following is what I at last adopted, as the most eligible for carrying on an extensive survey, either in England, Scotland, or any other cleared country.

Having taken a cursory view of the ground that I am first to proceed upon, and observed the plainest and clearest tract, upon which I can measure

sure

sure a circuit of three or four miles, I begin at a point convenient for placing the theodolite upon, and making a small hole in the ground as at A, *plate 20*, one assistant leading the chain, the other having a spade for making marks in the ground; I measure in a direct line from the hole at A to B, noting down as I go along on the field-sketch the several distances from place to place, and sketching in the figure of the road, as 75 links to the east side of the avenue, 400 links, touching the south side of the road, with an offset of 40 links to the other side, 500 links to the corner of a wood, and 817 to the corner of another wood; which last, not allowing me to carry the line farther, I make a mark at B, where I mean to plant the instrument, and beginning a new line, measure along the side of the road 885 links to C, where I make a mark in the ground, writing the same in my field-sketch, taking care always to stop at a place from which the last station can be seen, when a pole is placed at it. Beginning a new line, and measuring in a direct line towards D, I have 345 links to a line of trees, 390 links opposite a corner of paled inclosures where I make a mark to have recourse to, and 893 links to the end of the line, where I make a mark in the ground at D, from thence measuring in a direct line to E, I here have 600 links of steep ground, therefore make a mark in the ground at the bottom of the steepness, so that when I come to take the angles with the theodolite, I may take the depression thereof; continuing out the line, I have 1050 links to a bridge, with an offset of 20 links to the bridge, and 80 links to the paling; at 1247 there is a line of trees on the left, and here also the line comes to the north side of the road, and 1388 links to the end of the line at E, and so

X 3.

proceed

proceed in like manner round the circuit to F, G, H, I, K, L, M, N, and from thence to A where I began. The holes or marks made in the ground, are represented by round dots on the field-sketch; steepness of ground is expressed on the sketch by faint strokes of the pen, as at D, F, &c. Figures ending a station line, are written larger than the intermediate ones; offsets are written down opposite the places they were taken at, and are marked either to the right, or left of the line on the field-sketch, just as they happen to be on the ground. If the field-sketch here given was enlarged, so as to fill a sheet of paper, there would then be room for inserting the figures of all the offsets, which the smallness of this does not admit of.

In measuring these circuits or station lines, too much care cannot be taken by the surveyor to measure exact; I therefore, in doing them, always chuse to hold the hindermost end of the chain myself.

The next thing to be done is to take the angles or bearings of the above described circuit, and also the altitude or depression of the different declivities that have been measured up or down.

Having previously prepared a sheet of dutch paper with meridian lines drawn upon it, as in *plate 21*, also a horn protractor, with a scale of chains upon the edge thereof, and a small ruler about a foot in length; and having two assistants provided with a pole each, to which are attached plumb lines for keeping them perpendicular, and a third assistant for carrying the theodolite; I proceed to plant the instrument where I began to measure, or at any other angular point in the circuit; if the wind blows high I chuse a point to begin at, that is sheltered from it, so that the needle

dle may settle steady at the magnetic north, which is indispensably necessary at first setting off, at the same time taking care that no iron is near enough the place to attract the needle.

The best theodolet for this purpose is the large one, (see *fig. 2, plate 16,*) the manner of using of which I shall here describe.

The spirit level C, having been previously adjusted to the telescope A, and the two telescopes pointing to the same object, I begin by levelling the instrument, (by means of the four screws, M playing between the two parallel plates N,) first in a line with the magnetic north, and then at right angles thereto; this accomplished, I turn the moveable index by means of the screw G, till it coincides with 180° and 360° , and with the magnetic north nearly, screwing it fast by means of the nut H, and also the head of the instrument by means of the nut L; I make the north point in the compass box coincide with the needle very exactly, by turning the screw K; both telescopes being then in the magnetic meridian, I look through the lower one I, and notice what distant object it points to.

Then unscrewing the moveable index by the aforesaid nut H, (the first assistant having been previously sent to the station point marked N,) I turn about the telescope A or moveable index by means of the nut G, till it takes up the pole now placed at N, raising or depressing the telescope by means of the nut E, till the cross hairs or wires cut the pole near the ground. This done, I look through the lower telescope to see that it points to the same object it did at first; if so, the bearing or angle is truly taken, and reading it upon the limb of the instrument F, find it to be $40^{\circ} 55'$ S. W. I then take my sheet of paper, and placing the

horn protractor upon the point A, *plate 21*, along the meridian line, passing through it I prick off the same angle, and with the first ruler draw a faint line with the pen, and by the scale set off the length of the line, which I find to be (by my first sketch, *plate 20*,) 7690 links; and writing both bearing and distance down, as in *plate 21*, and again reading off the angle to compare it with what I have wrote down, I then make a signal to the first assistant to come forward with his pole; in the mean time I turn the moveable index about, till the hair or wire cut the pole which the second assistant holds up at B, and looking through the lower telescope to see that it points on the same object as at first, I read the angle upon the limb of the theodolet, $48^{\circ} 55'$ S. E. and plotting it off upon the sketch with the horn protractor, draw a strait line, and prick off by the scale the length to B 817 links; writing the same down on the field-sketch, and again reading the angle to see that I have wrote it down right, I screw fast the moveable index to the limb, by means of the screw H, and making a signal to second assistant to proceed with his pole and plant it at 3d station C, while I with the theodolet proceed to B, leaving first assistant with his pole where the instrument stood at A.

Planting the instrument by means of a plumb line over the hole, which the pole made in the ground at the 2d station, and holding the moveable index at $48^{\circ} 55'$ as before, and the limb levelled, I turn the limb till the vertical wire in the telescope cuts the pole at A nearly, and there screw it fast. I then make the vertical wire cut the pole very nicely by means of the screw K; then looking through the lower telescope to see and remark what distant object it points on, I loosen the index-screw, and turn the moveable index till the
vertical

vertical hair in the telescope cuts the pole at C nearly, and making the limb fast, I make it do so very nicely by means of the screw K, and then fix the index; reading the degrees and minutes which the index now points to on the limb, I plot it in my field-sketch as before, the bearing or angle being $48^{\circ} 30'$, and the length 885 links. Sending the theodolite to next station C, the same operation is repeated; coming to D, besides taking the bearing of the line D E, I take the depression from D, to the foot of the declivity 600 links below it, which I perform thus: Having the instrument and the double quadrant level, I try what part of the body of the assistant I have with me for carrying the instrument, the telescope thereof touches, and then send him to stand at the mark at the bottom of the declivity, and making the cross wires of the telescope cut the part of his body, equal the height of the instrument, I find the depression pointed out upon the quadrant to be $\frac{3}{4}$ of a link upon each chain, which upon six chains is four links to be subtracted from 1388 links, the measured length of the line, leaving 1384 links for the horizontal length, which I mark down in my field-sketch, plate 2, in the manner there written. Also from the high ground at D, seeing the temple O, I take a bearing to it, and plotting off the same in my field-sketch, draw a straight line; the same temple being seen from station L, and station M, I take bearings to it, from each; and from the extension of these three bearings intersecting each other in the point O, is a proof that the lines have been truly measured, and the angles right taken.

Again from station E to station F, the ground rises considerably; therefore, besides the bearing, I take the altitude in the same manner I did the depression;

depression; and do the same with every considerable rise, or fall, round the circuit.

Coming to station N, and having the moveable index at $61^{\circ} 15'$ N. W, taking up the pole at M, and making fast the limb, I loosen the index screw, and turn it till I take up the pole at first station A. This bearing ought to be $40^{\circ} 55'$ N. E. being the same number of degrees and minutes it bore from A south-west, and if it turn out so, or within a minute or two, we call it a good closure of the circuit, and is a proof that the angles have been accurately taken.

But if a greater error appears, I rectify the moveable index to $40^{\circ} 55'$ and taking up the pole at A, make fast the limb, and take the bearing to M, and so return back upon the circuit again, till I find out the error. But an error will seldom or never happen with such an instrument as here described, if attention is paid to the lower telescope; and besides the needle will settle at the same degree (minutes by it cannot be counted) in the box, as the moveable index will point out on the limb, provided it is a calm day, and no extraneous matter to attract it. If it does not, I then suspect some error has been committed, and return to the last station to prove it, before I go farther.

With regard to plotting off the angles with the horn protractor in the field, much accuracy is not necessary; the use of it being only to keep the field-sketch regular, and to preserve the figure of the ground nearly in it's just proportion.

Coming home, I transfer from the first field-sketch, *plate 20*, all the intermediate distances, offsets, and objects, into the second, *plate 21*, nearly to their just proportions; and then I am
ready

ready to proceed upon surveying the interior parts of the circuit. For doing this, the little light theodolite represented at *fig. 4, plate 15,** will be sufficiently accurate for common surveying. It is unnecessary for me to describe farther the taking the angles, and mensurations of the interior subdivisions of the circuit, but in doing of which, the young surveyor had best make use of the horn protractor and scale, because if he mistakes a chain, or takes an angle wrong, he will be soon sensible thereof by the lines not closing as he goes along; the old, or experienced, will be able to fill in the work sufficiently clear and distinct without them. The field sketch will then be such as is given in *plate 22*, which indeed appears rather confused, owing to the smallness of the scale made use of, to bring it within compass; but if this figure was enlarged four times, so as to fill a sheet of paper, there would then be room for entering all the figures and lines very distinctly.

Having finished this first circuit, before I begin to measure another, I examine the chain I have been using, by another that has not been in use, and find that it has lengthened more or less, as the ground it has gone through was rugged, or smooth, or as the wire of which it is made, is thick, or small; the thick or great wire drawing or lengthening more than the small.

The careful surveyor, if he has more than two or three days chain work to do, will take care to have a spare chain, so that he may every now and then correct the one, by the other. The offset staff is inadequate for this purpose, being too short.

* Since the writing of this work, a small theodolite has been contrived useful for the foregoing, and many other purposes; it is very portable, and is furnished with a telescope.

short. For field surveying, when no uncommon accuracy is required, cutting a bit off any link on one side, 50 links, and as much on the other side, answers the purpose better than taking away the rings.

The above method is what I reckon the best for taking surveys to the extent of 100,000 acres; beyond this, an error from small beginnings in the mensuration becomes very sensible, notwithstanding the utmost care; therefore surveying larger tracts of country, as counties or kingdoms, requires a different process. Surveys of this kind are made from a judicious series of triangles, proceeding from a base line, in length not less than three miles, measured upon a horizontal plane with the greatest possible accuracy.

THOMAS MILNE,

OF P L O T T I N G, OR MAKING A DRAUGHT OF THE LAND FROM THE FIELD NOTES.

OF PLOTTING, AND OF THE INSTRUMENTS USED IN PLOTTING.

BY plotting, we mean the making a draught of the land from the field notes. As the instruments necessary to be used by the surveyor in taking the dimensions of land, are such wherewith he may measure the length of a side, and the quantity of an angle in the field; so the instruments commonly used in making a plot or draught thereof, are such wherewith he may lay down the length of a side, and the quantity of an angle on paper. They therefore consist in *scales of equal parts* for laying down the lengths or distances, and *protractors* for laying down the angles.

Scales of equal parts are of different lengths and differently divided; the scales commonly used by surveyors, are called feather-edge scales; these are made of brass, ivory, or box; in length about 10 or 12 inches, but may be made longer or shorter at pleasure. Each scale is decimally divided, (the whole length,) close by the edges, which are made sloping in order to lay close to the paper, and numbered, 0, 1, 2, 3, 4, &c. which are called chains, and every one of the intermediate divisions is 10 links, the numbers are so placed as to reckon backwards and forwards; the commencement of the scale is about 2 or 3 of the larger divisions from the fore end of the scale; these are numbered
backwards

backwards from 0 towards the left hand with the numbers or figures, 1, 2, 3, &c. these scales are often sold in sets.

The application and use of this scale is easy and expeditious, for to lay down any number of chains, from a given point in a given line; place the edge of the scale in such a manner that the 0 of the scale may coincide with the given point; and the edge of the scale with the given line; then with the protracting pin, point off from the scale the given distance in chains, or chains and links.

Fig. F G H, plate 2, represents a new scale or rather scales of equal parts, as several may be laid down on the same instrument; each scale is divided to every 10 links or tenths of a chain, which are again subdivided by their respective nonius divisions into single links; the protracting pin is moved with the nonius by means of the screw *d*, so that the distances may be set off with such great accuracy, as not to err a single link in setting off any extent, which in the scale of four chains in an inch, does not amount to more than the 400th part of an inch.

Fig. I K L, plate 2, answers the same purpose as the foregoing instrument, and may be used as a protractor also.

For Mr. Gale's method of plotting, a method which will recommend itself to every attentive surveyor, two scales should be used, one of about 15 inches long, the other about 10 inches, each divided on the edge from one end to the other; a clasp should be fixed on the shorter scale whereby it may at pleasure be so fixed to the other scale, as to move along the edge thereof at right angles, for the purpose of laying off perpendicular lines.

Fig. F G H, plate 2, represents an instrument
for

for this purpose, with different scales on the part FG; both this and that represented at *fig. I K L*, form excellent parallel rules. They are also made only with the clasp, if desired.

The protractor is a circle or semicircle of thin brass, divided into degrees, and parts of a degree, on the outer edge.

The common semicircular protractor, *fig. 1, plate 3*, is of 6 or 8 inches diameter, the limb divided into 180 degrees, and numbered both ways, 10, 20, 30, &c. to 180; each degree is subdivided into two parts. In the middle of the diameter is a small mark, to indicate the center of the protractor; this mark must be always placed on the given angular point.

The common circular protractor is more useful than the semicircular one; the outside edge is divided into 360 degrees, and numbered, 10, 20, 30, to 360; each degree is subdivided into halves. In the middle of the diameter is a small mark, which is to be placed on the angular point, when an angle is to be protracted; the diameter (representing a meridian) must be placed on the meridian of any plan, where the bearing of any object is to be laid down; the application of this instrument is so easy and simple, that it is unnecessary to fill up paper with examples of it's use.

Fig. 7, plate 11, and *4 and 5, plate 17*, represent the three best circular protractors; the principle is the same in all; the difference consists in superiority of execution, and the conveniencies that arise from the construction. *Fig. 4, plate 17*, is a round protractor, the limb accurately divided to 360°, each degrees is divided into 2 parts, which are again subdivided to every minute, by the nonius, which moves round the limb of the protractor,

tor, on a conical center; that part of the index beyond the limb has a steel point fixed at the end, whose use is to prick off the angles; it is in a direct line with the center of the protractor, and the index division of the nonius, all which is evident from the figure.

OF PLOTTING OR MAKING A DRAUGHT OF THE
LAND FROM THE FIELD NOTES.

The common method of plotting is this: take a sheet of paper of convenient size, draw a line thereon, to represent the magnetic meridian, and assign any convenient point therein to represent the place where the survey was begun; lay the edge of the protractor on this meridian line, and (bringing the center thereof to the point so assigned to represent the place of beginning) mark off the degrees and minutes of the first course or bearing by the limb of the protractor, and draw a line from the place of beginning through the point so marked, laying off it's proper length or distance by the scale of equal parts; this line will represent the first line of the survey. Through the point or termination of the said first line of the survey, draw another line representing the magnetic meridian parallel to the former; and lay off the course or bearing of the second line of the survey by the protractor, and it's length by the scale of equal parts as before, and so proceed till the whole be laid down; and you will have a plot or draught of the survey.

For example. Let it be required to make a plot, or draught of the field notes.

1st. Draw any line, as N A S, *fig. 1, plate 18*, to represent the magnetic meridian, and assign any convenient point therein, as A, to represent the place

place of beginning the survey; lay the edge of the protractor on the line N A S, with the center thereof at the point A, and mark off *seven degrees on the limb* to the westward or left hand of the north, and draw the line A B, through the point so marked off, making the length thereof 21 chains by the scale of equal parts. 2dly. Draw another meridian line N B S, through the point B, parallel to the former.* Lay the edge of the protractor on this second meridian line N B S, with the center thereof at the point B, and mark off $55^{\circ} 15'$ on the limb to the right hand, or eastward of the north; and draw the line B C through the point so marked off, making the length thereof 18 chains 20 links by the scale of equal parts. 3dly. Draw another meridian line N C S, through the point C, parallel to the former: lay the edge of the protractor to this third meridian line N C S, with the center thereof at the point C, and lay down the third course and distance C D, in the same manner as before, and so proceed with all the other lines D E, E F, F G, G A; and if the last line shall terminate in the place of beginning, the work *closes* as it is called, and all is well. But if the last line do not terminate in the place of beginning, there must have been a mistake either in taking the notes, or in the protraction of them; in such case, therefore, it will be necessary to go over the protraction again, and if it be not found then, it must of course be in the field notes, to correct which, if the error is material, they must be taken again.

Mr.

* These parallel lines may be drawn with the least trouble by a parallel rule.

MR. GALE'S METHOD OF PLOTTING.

The foregoing method of plotting is liable to some inaccuracies of practice, on account of having a new meridian for every particular line of the survey, and on account of laying off every new line *from the point of termination of the preceding one*, whereby any little inaccuracy that may happen in laying down of one line is communicated to the rest. But there is a second method of plotting, by which these inconveniences are avoided, and by which also the accuracy or inaccuracy of the field work is decided with precision and certainty; I would therefore recommend this second method to the practitioner as far preferable to any other I have seen.

2d method. Take out from the first table in the appendix, the *northings, southings, eastings, and westings*, made on each of the several lines of the survey, placing them in a kind of table in their respective columns; and if the sum of the northings be equal to the sum of the southings, and the sum of the eastings equal to the sum of the westings, the work is right, otherwise not.*

Then in an additional column put the whole quantity of northing or of southing made at the termination of each of the several lines of the survey; which will be determined by adding or subtracting

* The truth of this observation cannot but appear self-evident to the reader. For the meridians within the limits of an ordinary survey, having no sensible difference from parallelism, it must necessarily follow, that if a person travel any way soever with such small limits, and at length come round to the place where he set out; he must have travelled as far to the northward, as to the southward, and to the eastward, as to the westward; though the practical surveyor will always find it difficult to make his work close with this perfect degree of exactness.

tracting the northing or southing made on each particular line, *to* or *from* the northing or southing made on the preceding line or lines. And in another additional column, put the whole quantity of easting or westing made at the termination of each of the several lines of the survey, which will be determined in like manner, by adding or subtracting the easting or westing made on each particular line, *to* or *from* the easting or westing made on the preceding line or lines.

The whole quantity of the northings or southings, and of the eastings or westings, made at the terminations of each of the several lines, being thus contained in these two additional columns, the plot may be easily laid down from thence, by a scale of equal parts, without the help of a protractor.

It is best, however, to use a pair of scales with a clasp,* whereby the one may at pleasure be so fastened to the other as to move along the edge thereof at right angles, so that the one scale may represent the meridian, or north and south line, the other an east and west line. It may also be observed, that in order to avoid taking the scales apart during the work, it will be necessary that the whole of the plot lay on one side of that scale which represents the meridian; or in other words, the stationary scale should represent *an assumed meridian*, laying wholly on one side of the survey. On this account it will be necessary to note in a third additional column of the preparatory table, the distances of each of the corners or terminations of the lines of the survey, east or west from such assumed meridian. In practice, it is rather more convenient that this assumed meridian should lay

Y 2

on

* Or one of those *fig.* at F G H, or I K L, *plate 2.*

on the west side, than on the east side of the survey.

The above-mentioned additional column, as has been already observed, contains the distance of each of the corners or terminations of the lines of the survey, eastward or westward from that magnetic meridian which passes through the place of beginning the survey. In order, therefore, that the assumed meridian should lay entirely without the survey, and on the west side thereof, it will only be necessary, that it's distance from that meridian which passes through the place of beginning, should be somewhat greater than the greatest quantity of westing, contained in the said second additional column. Let then an assumed number, somewhat greater than the greatest quantity of westing contained in the second additional column, be placed at the top of the third additional column, to represent the distance of the place of beginning of the survey, from the assumed meridian. Let the several eastings contained in the second additional column, be added to this assumed number, and the several westings subtracted from it; and these sums and remainders being respectively placed in the third column, will shew the distance of the several corners or terminations of the lines of the survey, from the assumed meridian.

The preparatory table being thus made, take a sheet of paper of a convenient size, or two or more sheets pasted together with a little paste or mouth glue, in case a single sheet should not be large enough; and on the left hand of the intended plot, draw a pencil line to represent the assumed meridian, on which lay the stationary scale. Place the moveable scale to any convenient point on the edge of the former, and point off by the edge of the

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the latter, (according to any desired number of chains to an inch) the assumed distance or number of chains, contained at the top of the above-mentioned *third* column of the preparatory table. Move the moveable scale along the edge of the stationary one, to the several north or south distances contained in the first of the above-mentioned additional columns; so will the points thus marked off, represent the several corners or terminations of the lines of the survey; and lines being drawn from one point to the other, will of course represent the several lines of the survey.

Example. Let it be required to make a plot or draught of the field notes, page 219, according to Mr. Gale's method.

The preparatory table will be as follows.

Courses and distances as taken in the field.	Northings, southings, eastings, and westings, made on those respective courses and distances.				N. and S. distances from the place of beginning to the end of each line.	E. and W. distances from the place of beginning to the end of each line.	Distances of the ends of each line from the assumed meridian.
	N.	S.	E.	W.			
1. N. 7° W. 21.00 ch.	20 84			2 56	N. 20 86	W. 2 56	E. 3 00
2. N. 55 15 E. 18 20	10 37		14 95		N. 31 21	E. 12 39	E. 0 44
3. S. 62 30 E. 14 40		6 65	12 77		N. 24 56	E. 25 16	15 39
4. S. 40° W. 11		8 43		7 97	N. 16 13	E. 18 09	28 10
5. S. 4 15 E. 14		13 96	1 04		N. 2 17	E. 19 13	21 09
6. N. 73 45 W. 12 40	3 47			11 90	N. 5 64	E. 7 23	22 13
7. S. 52° W. 9 17		5 64		7 23	0 00	00 0	10 23
	34 68	34 68	28 76	28 76			3 00

The northings and southings, eastings and westings, in the above table, are taken from the first table in the appendix; thus, first find the course 7 degrees in the table, and over against 21 chains in the column marked dist. you have 20.843 in the column marked N. S. which (rejecting the

right hand figure 3 for it's insignificancy) is 20 chains, 84 links, for the quantity of *northing*; and in the column marked E. W. you have 2.559, very near 2 chains, and 56 links for the westing made on that course and distance. 2. Find the next course 55.15 in the table, and over against 18 chains in the column dist. you have 10 chains, 26 links in the column marked N. S. and 14 chains, 79 links in the column marked E. W. and over against 20 links in the column dist. you have 11 links in the column N. S. and 16 links in the column marked E. W. which put together, make 10 chains, 37 links, for the *northing*, and 14 chains, 95 links, for the *easting* made on the second course; and so of the rest.

The north and south distances made from the place of beginning to the end of each line, contained in the next column, are determined thus. On the first course, 20 chains, and 84 links of *northing* was made; on the second course, 10 chains, 37 links of *northing*, which added to the preceding, makes 31 chains, 21 links of *northing*; on the third course was made, 6 chains, 65 links of *southing*, which subtracted from the preceding, 31 chains, 21 links, of *northing*, makes 24 chains, 56 links, of *northing*; and so of the rest.

The east and west distances made from the place of beginning to the end of each line, contained in the next right hand column, are determined in the same manner; thus, on the first course was made, 2 chains, 56 links of *westing*; on the second course, 14 chains, 25 links, of *easting*, from which subtracting the preceding, 2 chains, 56 links, of *westing*, there remains 12 chains, 39 links, of *easting*; on the third course was made, 12 chains, 77 links, of *easting*, which added to the preceding

ding 12 chains, 39 links, makes 25 chains, 16 links, of easting; and so of the rest.

The distances of the end of each line from the assumed meridian, contained in the next right hand column, are thus determined. The first assumed number may be taken at pleasure, provided only, that it exceeds the greatest quantity of westing contained in the preceding column, whereby the assumed meridian shall be entirely out of the survey. In the foregoing example, the greatest quantity of westing contained in the preceding column, is 2 chains, 56 links; the nearest whole number greater than this, is 3 chains, which is accordingly taken and placed at the top, to represent the distance between the assumed meridian and the place of beginning of the survey; from this 3 chains, subtract 2 chains 56 links of westing, there remains 44 links for the distance between the terminations of the first line and the assumed meridian. The 12 chains, 39 links, of easting, in the next step, is added to the assumed 3 chains, which make 15 chains, 39 links, for the distance of the termination, by the second line from the assumed meridian. The 25 chains, 16 links of easting, in the next step, being added in like manner to the assumed 3 chains, makes 28 chains, 16 links, for the distance of the termination of the third line from the assumed meridian, and so on, always adding the eastings, and subtracting the westings from the first assumed number.

The preparatory table being completed, take a sheet of paper or more, joined together if necessary, and near the left hand edge thereof, rule a line as N. S. *fig. 2, plate 18*, to represent the assumed meridian; on this line lay the stationary scale, and assuming *a* as a convenient point therein, to re-

present the point directly west from the place of beginning; bring the moveable scale to the point *a*, and lay off the first number contained in the last-mentioned column of the preparatory table, (viz. 3 chains) from *a* to *A*, and *A* will represent the place of beginning of the survey. Move the moveable scale along the stationary one to the first north and south distance from the place of beginning, (viz. 20.84. from *a* to *b*) and lay off the corresponding distance from the assumed meridian (0.44) from *b* to *B*, and draw *AB*, so will *AB* represent the first line of the survey; again move the moveable scale along the stationary one, to the third north and south distance from the place of beginning, (viz. N. 31. 21. from *a* to *c*) and lay off the corresponding distance from the assumed meridian, viz. 15. 39. from *c* to *C*, and draw *BC*, so will *BC* represent the second line of the survey. Again move the moveable scale along the stationary one, to the third north and south distance from the place of beginning, (viz. 24. 56. from *a* to *d*) and lay off the corresponding distance from the assumed meridian, (viz. 28. 16.) and draw *CD*, which will be the third line of the survey. Proceed in the same manner till the whole be laid down, and *ABCDEFGH* will be the required plot.

This method of plotting is by far the most perfect, and the least liable to error of any that has been contrived. It may appear to some to require more labour than the common method, on account of the computations required to be made for the preparatory table. These computations are however made with so much ease and expedition, (by the help of the table in the appendix,) that this objection would vanish, even if the computation were of no other use but merely for plotting; but it must be observed,

observed, that these computations are of much further use in determining the area or quantity of land contained in the survey, which cannot be ascertained with equal accuracy in any other way. When this is considered, it will be found that this method is not only preferable on account of it's superior accuracy, but is attended with less labour on the whole than the common method.

If a pair of scales, such as are above recommended, be not at hand, the work may be laid down from a single scale, by first marking of the N. and S. distance on the line N. S. and afterwards laying off the corresponding east distances at right angles thereto.

To plot the field notes, p. 302. 1. Lay down all the station lines (*viz.* 7° W. 21 chains, N. 55, 15. E. 18. 20. &c.) contained in the middle column of the field book as before directed, without paying any regard to the offsets, until all the station lines (represented in *fig. 3*, by dotted lines) be laid down; then lay off the respective offsets at right angles from the station line, at the respective distances at which they were taken, as is done at *fig. 3*, a bare inspection of which will make the work perfectly plain.

To plot the field notes, p. 304. 1. Lay down the whole of the outside boundaries, the station lines first, the offsets afterwards, as directed in the preceding articles, and then lay down the internal divisions or boundaries of the respective fields, in the same manner as is done in *fig. 4*.

OBSERVATIONS ON PLOTTING, BY MR. MILNE.

The protractor, whether a whole or a semi-circle, ought not to be less in diameter than seven or eight

eight inches, to insure the necessary degree of accuracy in plotting the angles of a survey. The degrees on the limb are numbered in various ways, but most commonly from 10° , $20'$, &c. to 360° . It would be right to repeat the numbers the contrary way, when the breadth of the limb will admit of it. Others are numbered 10° , $20'$, &c. to 180° , and the contrary. Surveyors will suit themselves with that kind best adapted to their mode of taking angles in the field; so that in which ever way the limb of the instrument they survey with is graduated, the protractor had best be the same.

It is true, a surveyor of much practice will read off an angle mentally very readily, in which ever way the instrument happens to be graduated.

For instance, I have in the field taken the angles made with the magnetic meridian, in which case I count no angle above $89^{\circ} 59'$; if the angle comes to $90^{\circ} 0'$, I call it due E. or due W; if it is $89^{\circ} 59'$, I write to it N. E. or S. E. or else N. W. or S. W. as it happens to turn out.

Now $89^{\circ} 59'$ N. E. upon a protractor numbered to $360^{\circ} 0'$, and placing 360 to the north, reads the same; but $89^{\circ} 59'$ S. E. reads $90^{\circ} 1'$, and $89^{\circ} 59'$ S. W. reads $269^{\circ} 59'$; and lastly, $89^{\circ} 59'$ N. W. reads $270^{\circ} 1'$.

Again, if the angle or bearing is due south, it reads on this protractor 180° , if $0^{\circ} 1'$ S. E. it reads $179^{\circ} 59'$, and if $0^{\circ} 1'$ S. W. it is $180^{\circ} 1'$; and so of any other intermediate angle or bearing.

The great inducement to surveyors, for taking the angles by the bearings the lines make with the magnetic meridian, is the having the needle as
a check

a check in the course of the survey; and when the circuit comes to be plotted, having it in his power to prick off all the angles, or bearings of the circuit, at once planting of the protractor.

This mode of surveying, or taking angles is also more expeditious, and more accurate than any other, provided the index of the instrument is furnished with a clamp for making it fast to the limb, while it is carrying from one station to another.

The protractor for plotting this way of surveying, had best be graduated 10, 20, &c. to 180, on the right hand, and the same repeated on the left; and again repeated contrary to the former; but as protractors for general use are graduated 10, 20, &c. to 360, I shall here describe a very useful and convenient one of this latter kind, and then proceed to give an example of plotting all the angles of a circuit by it, from one station.

This protractor is represented at *fig. 5, plate 17*. It's diameter from pointer to pointer, is $9\frac{1}{2}$ inches; the center point is formed by two lines crossing each other at right angles, which are cut on a piece of glass. The limb is divided into degrees, and half degrees, having an index with a nonius graduated to count to a single minute, and is furnished with tooth and pinion, by means of which the index is moved round, by turning a small nut. It has two pointers, one at each end of the index, furnished with springs for keeping them suspended while they are bringing to any angle; and being brought, applying a finger to the top of the pointer, and pressing it down, pricks off the angle. There is this advantage in having two pointers, that all the bearings round a circuit may be laid,
or

or pricked off, although the index traverses but one half of the protractor.*

Let *fig. 5, plate 13*, be the circuit to be plotted. I draw the magnetic meridian N. S. *fig. 6, plate 13*, and assigning a point therein for station 1, place the center of the above described protractor upon it, with 360° exactly to the north, and 180° to the south of the magnetic meridian line N. S. For the conveniency of more easily reading off degrees and minutes, I bring the nonius to the south side of the protractor, or side next to me; and seeing by the field sketch, or eye draught, *fig. 5*, that the bearing from station 1, to station 2, is $87^\circ 30'$ S. E. I readily conceive it wanting $2^\circ 30'$ of being due east, or $90^\circ 0'$; therefore adding mentally $2^\circ 30'$ to $90^\circ 0'$, I make $92^\circ 30'$ on this protractor, to which I bring the nonius, and then make a prick, by pressing down the pointer of the protractor, and with my lead pencil mark it 1. The bearing from station 2, to station 3, being $10^\circ 15'$ N. E. (see *fig. 5*.) I bring the nonius for the sake of expedition and ease of reading off, to $10^\circ 15'$ S. W. which is upon this protractor $190^\circ 15'$,

* The protractor, represented *fig. 7, plate 11*, is circular, and of the same diameter as the foregoing; the center is also formed by the intersection of two lines at right angles to each other, which are cut on glass, that all parallax may thereby be avoided; the index is moved round by a tooth and pinion. The limb is divided into degrees, and half degrees, and subdivided to every minute by the nonius; the pointer may be set at any convenient distance from the center, as the socket which carries it moves upon the bar B C, and is fixed thereto by the nut D. At right angles to the bar B C, (and moveable with it) there is another bar E F; on this bar different scales of equal parts are placed, so that by moving a square against the inner edge thereof, angles may be transferred to any distance from the center, containing the same number of degrees marked out by the index, the use of which will be evident from Mr. Milne's observations.

15', and pressing down the north pointer, it will prick off $10^{\circ} 15'$ N. E. which I mark with my pencil 2.

The bearing from station 3, to station 4, being $85^{\circ} 13'$ N. E. I readily see this is wanting $4^{\circ} 47'$ of being due east; therefore I bring the nonius to within that of due west, which upon this protractor is $265^{\circ} 13'$, and pressing down the N. E. pointer, it will prick off $85^{\circ} 13'$ N. E. which I mark 3.

The bearing from station 4, to station 5, being $11^{\circ} 5'$ N. W. I bring the nonius to $168^{\circ} 55'$, and with the opposite, or north-west pointer prick off $11^{\circ} 5'$ N. W. which I mark 4.

In like manner I prick off the bearings from station 5 to 6, from 6 to 7, and from 7 to 1, which closes the circuit; marking them severally with my pencil 5, 6, 7.

Then laying aside the protractor, and casting my eye about the tract traced by the pointer of the protractor for the bearing from station 1, marked 1, I apply the ruler to it and station 1, and drawing a line, mark off it's length by compasses and scale, eight chains, which fixes station 2.

Again, casting my eye about the tract traced by the pointer of the protractor, for the 2 bearing, marked 2, I apply the parallel ruler to it, and station 1, and moving the ruler parallel eastward, till it's edge touches station 2, I from thence draw a line northward, and by scale and compasses mark it's length five chains, to station 3. Thus the bearing $10^{\circ} 15'$ N. E. at station 1, is transferred to station 2, as will easily be conceived by supposing the points a, b, moved eastward, preserving their parallelism, till they fall into the points of station 2 and 3, forming in *fig. 6*, the parallelogram, a, b, 3, 2. It

It is evident this bearing of $10^{\circ} 15'$ N. E. is by this means as truly plotted, as if a second meridian line had been drawn parallel to the first, from station 2, and the bearing $10^{\circ} 15'$ N. E. laid off from it, as was the common method of plotting, previous to the ingenious Mr. William Gardiner's communicating this more expeditious, and more accurate mode, to the public. However, it's accuracy depends entirely upon using a parallel ruler that moves truly parallel, which the artist will do well to look to, before he proceeds to this way of plotting.

Station 3 being thus fixed, to find station 4 I cast my eye about the tract traced by the pointer of the protractor, for the bearing mark 3, and applying the parallel ruler to it, and station 1, move the ruler parallel thereto northward, till it touch station 3, and from thence draw a line north-eastward, and by scale and compasses marking it's length, three chains, station 4 is found.

Now conceive the points c, d, moved northward, preserving their parallelism, till they fall into the points of stations 3 and 4; and thus station 4 has the same bearing from station 3, as c, or d, has from station 1.

In the same manner I proceed to the next bearing 4; and placing the ruler to the point made by the pointer of the protractor at 4, and station 1, and moving the ruler parallel, till it's edge touch station 4; from thence draw a line, and mark off it's length, five chains, to station 5. Here e, f, is transferred to 4th and 5th stations, and the 5th station makes the same angle with the meridian from the 4th station, as e, or f, does with the meridian from the first station.

In like manner applying the parallel ruler to
station

station 1, and the several other bearings of the circuit 5, 6, and 7; the points g, h, will be transferred to 5th and 6th; and i, k, to 6th and 7th; and lastly, the bearing from the 7th station will fall exactly into the first, which closes the plot of the circuit.

It is almost unnecessary to observe, that the dotted lines marked a, b, c, &c. *fig. 2*, are drawn there only for illustrating the operation: all that is necessary to mark in practice are the figures 1, 2, 3, &c. round the tract traced by the pointer of the protractor.

T. MILNE.



OF

OF DETERMINING THE AREA OR CONTENT OF LAND.

1. The *area* or content of land is denominated by acres, roods, and perches.

40 square perches make a *rood*.

4 roods (or 160 square perches) make an *English acre*.

And hence 10 square chains (of 4 perches each) make an English acre.

2. The most simple surface for admeasurement, is the rectangular parallelogram, which is a plane four-sided figure, having it's opposite sides equal and parallel, and the adjacent sides rectangular to one another; if the length and breadth be equal, it is called a *square*; but if it be longer one way than the other, it is commonly called an *oblong*.

TO DETERMINE THE AREA OF A RECTANGULAR PARALLELOGRAM.

Rule. Multiply the length by the breadth, and that product will be the area.*

It has been already observed, that the best method of taking the lengths is in *chains* and *links*; hence, the above-mentioned product will give the area in chains and decimal parts of a chain. And as 10 square chains make an acre, the area in acres will be shewn by pointing off one place of decimals more in the *product*, than there are both in the *multiplicand* and *multiplier*; which answers the same purpose as dividing the area in chains by 10; and the

* Euclid. Book 1. Prop. 35. Scholium.

the roods and perches will be determined by multiplying the decimal parts by 4, and by 40.

Example. Suppose the side of the foregoing square be 6 chains and 75 links, what is the area?

Multiply 6.75
by 6.75

3375
4725
4050

Acres 4.55625

4 Roods in an acre.

Roods 2.22500

40 Perches in a rood.

Perches 9.00000

Acres. Roods. Perches.

Anf. 4 2 9.

Suppose the length of the foregoing oblong be 9 chains and 45 links, and the breadth 5 chains and 15 links, how many acres doth it contain?

Multiply 9.45
by 5.15

4725
945
4725

Acres 4.86675

(carried over)

Z

Acres

Acres 4.86675

4

Roods 3.46700

40

Perches 18.68000

A. R. P.

Anf. 4 3 18. (The decimal parts .68 are not worth regarding.)

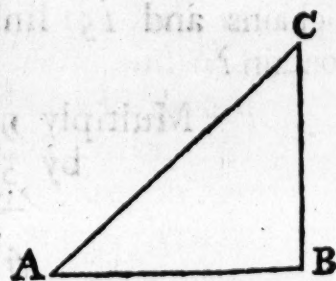
If the parallelogram be not rectangular, but oblique angled, the length must be multiplied by the perpendicular breadth, in order to give the area.*

3. After the parallelogram, the next most simple surface for admenfuration is the triangle.

TO DETERMINE THE AREA OF A TRIANGLE.

Rule. Multiply the base (or length) by half the perpendicular, let fall thereon from the opposite angle, and that product will be the area of the triangle.†

Examples. In the triangle ABC, right angled at B, let the base AB, be 8 chains and 25 links, and the perpendicular BC, 6 chains and 40 links; how many acres doth it contain?



Multiply

* See Euclid, Book 1, Prop. 35, and 36.

† Euclid, Book 1, Prop. 41, Scholium. Every triangle is equal to the one half of a parallelogram, having the same base and altitude.

Multiply 8.25
by (the half of 6.40 viz.) 3.20

$$\begin{array}{r} 16500 \\ 2475 \\ \hline \end{array}$$

Acres 2.64000

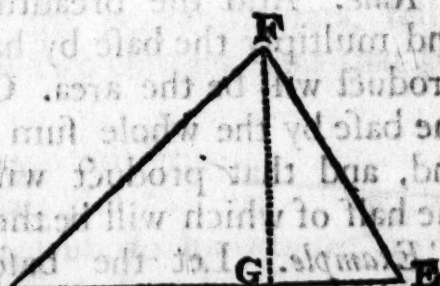
$$\begin{array}{r} 4 \\ \hline \text{Roods } 2.56000 \\ 40 \\ \hline \end{array}$$

Perches 22.40000

A. R. P.

Ans. 2 2 22.

In the oblique angled triangle DEF, let the base DE be 12 chains and 25 links, and the perpendicular FG, (let fall on the base DE from the D opposite angle F) be 7 chains and 30 links, how many acres doth it contain?



Multiply 12.25
by (the half of 7.30 viz.) 3.65

$$\begin{array}{r} 6125 \\ 7350 \\ 3675 \\ \hline \end{array}$$

Acres 4.47125

4

Roods 1.88500 (carried over)
Z 2 Roods

Roods 1.88500

40

Perches 35.40000

A. R. P.

Ans. 4 1 35.

It would be the same thing, if the perpendicular were to be multiplied by half the base; or, if the base and perpendicular were to be multiplied together, and half the product taken for the area.

4. The next most simple surface for admeasurement is a four-sided figure, broader at one end than at the other; but having it's ends parallel to one another, and rectangular to the base. It's area is determined by the following

Rule. Add the breadths at each end together, and multiply the base by half their sum, and that product will be the area. Or otherwise; multiply the base by the whole sum of the breadths at each end, and that product will be double the area, the half of which will be the area required.

Example. Let the base be 10 chains and 90 links; the breadth at the one end 4 chains and 75 links, and the breadth at the other end 7 chains and 35 links; what is the area?

$$\begin{array}{r}
 4.75 \} \\
 7.35 \} \\
 \hline
 2)12.10 \text{ sum} \\
 \hline
 6.05 \text{ half sum} \\
 10.90 \\
 \hline
 54450 \text{ (carried over)} \\
 54450
 \end{array}$$

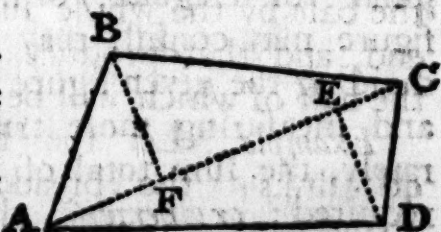
$$\begin{array}{r}
 54450 \\
 6050 \\
 \hline
 \text{Acres } 6.59450 \\
 4 \\
 \hline
 \text{Roods } 2.37800 \\
 40 \\
 \hline
 \text{Perches } 15.12000 \\
 \hline
 \hline
 \end{array}$$

A. R. P.
 Ans. 6 2 15.

5. Irregular four-sided figures are called *trapezia*. The area of a trapezium may be determined by the following

Rule. Take the diagonal length from one extreme corner to the other, as a base, and multiply it by half the sum of the perpendiculars, falling thereon from the other two corners, and that product will be the area.

Example. In the trapezium $ABCD$, let the diagonal AC , be 11 chains and 90 links; the perpendicular BF , 4 chains and 40 links, and the perpendicular DE , 3 chains and 90 links; what is the area?



BF 4.40
 DE 3.90

$$\begin{array}{r}
 2) 8.30 \quad \text{sum} \\
 \hline
 4.15 \quad \text{half sum} \\
 \text{Z } 3 \quad (\text{carried over}) \quad 4.15
 \end{array}$$

$$\begin{array}{r}
 4.15 \text{ half sum} \\
 \text{AC } 11.90 \\
 \hline
 37850 \\
 415 \\
 \hline
 415 \\
 \hline
 \text{Acres } 4.93850 \\
 4 \\
 \hline
 \text{Roods } 3.75400 \\
 40 \\
 \hline
 \text{Perches } 30.16000
 \end{array}$$

A. R. P.

Ans. 4 3 30.

If the diagonal A C were multiplied by the whole sum of the perpendiculars, that product would be *double* the area; the half of which would be the area required.

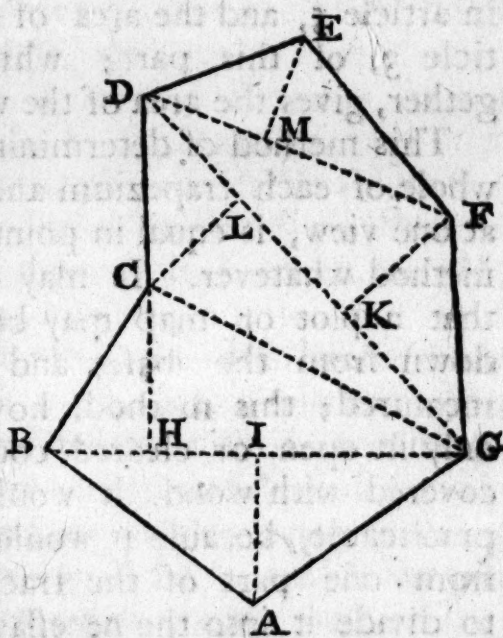
6. The area of all other figures, (whether regular * or irregular,) of how many sides soever the figure may consist, may be determined either by dividing the given figure into triangles or trapezia, and measuring those triangles or trapezia separately, the sum total of which will be the area required: *or otherwise*, the area of any figure may be determined by a computation made from the courses and distances of the boundary lines, according to an universal theorem, which will be mentioned presently.

7. The

* *The regular figures* (polygons and circles) do not occur in practical surveying. On this head, therefore, I shall only observe, that the method of determining the area of a regular polygon, (which is a figure containing any number of

7. The method of determining the area by dividing the given figure into trapezia and triangles, and measuring those trapezia and triangles separately (as in article 5, and article 3 of this part,) is generally practised by those land measurers, who are employed to ascertain the number of acres in any piece of land, when a regular land surveyor is not at hand; and for this purpose, they measure with the chain, the bases and perpendiculars of the several trapezia and triangles in the field.

Thus, for example, suppose the annexed figure be a field to be measured according to this method, the land measurer would measure with the chain, the base BG , and the perpendiculars HC , and IA , of the trapezium $ABCG$; the base GD , and the perpendiculars KF , and LC , of the trapezium $GCDF$: and the base DF , and perpendicular ME , of the triangle DEF .



In (of equal sides and equal angles) is, to multiply the length of one of the sides by the number of sides which the polygon contains, and then to multiply that product, by half the perpendicular let fall from the center to any one of the sides, and this last product will be the area.

The length of the perpendicular, if not given, may be determined by trigonometry, thus:

Divide 360 degrees, by double the number of sides contained

Z 4

In order to determine where on the base lines the perpendiculars shall fall, the land measurer commonly makes use of a *square*, consisting of a piece of wood about 3 inches square, with apertures therein, made with a fine saw, from corner to corner, thereby crossing each other at right angles in the center; which, when in use, is fixed on a staff.

The bases and perpendiculars being thus measured, the area of each trapezium is determined as in article 5, and the area of the triangle as in article 3, of this part; which being added together, gives the area of the whole figure required.

This method of determining the area, where the whole of each trapezium and triangle can be seen at one view, is equal in point of accuracy to any method whatever. It may likewise be observed, that a plot or map may be very accurately laid down from the bases and perpendiculars thus measured; this method, however, is practicable only in open or cleared countries; in countries covered with wood, it would be altogether impracticable, because it would be impossible to see from one part of the tract to another, so as to divide it into the necessary trapezia and triangles.

tained in the polygon, and that quotient will be half the angle at the center: then say,

As the *tangent* or half the angle at the center,

Is to half the length of one of the sides:

So is *radius*,

To the perpendicular sought.

The area of a **CIRCLE** is thus determined; if the diameter be given, say as 1: is to 3.141592: so is the diameter: to the circumference; or, if the circumference be given, say, as 3.141592 is to 1:: so is the circumference: to the diameter. Then multiply the circumference by $\frac{1}{4}$ of the diameter, and that product will be the area,

gles. But in open countries, the only objection to this method, is the great labour and tediousness of the field work; in a large survey, the number of the trapezia and triangles would be so very great, that the measuring of all the several bases and perpendiculars would be a very tedious business. The same ends are as effectually answered, with infinitely less labour, by taking the courses and distances of the boundary lines, in manner mentioned page 299.

8. Many surveyors who take their field notes in courses and distances, and make a plot or map thereof, and then determine the area from the plot or map so made, by dividing such plot or map into trapezia and triangles, as in the above figure, and measuring the several bases and perpendiculars by the scale of equal parts, from which the plot or map was laid down.

By this method, the area will be determined something near the truth, but it falls short of that accuracy which might be wished, because there is no determining the lengths of the bases and perpendiculars by the scale, within many links; and the smaller the scale shall be by which the plot is laid down, the greater of course must be the inaccuracy of this method of determining the area.

This method, however, notwithstanding it's inaccuracy, will always be made use of, where tables of the northing, southing, easting, and westing, are not at hand.

9. Proper tables of the northing, southing, easting, and westing, fitted purposely for the surveyors use, (such as are contained in the appendix to this tract) ought to be in the hands of every surveyor. By these tables the area of any survey is determined with great accuracy and expedition,

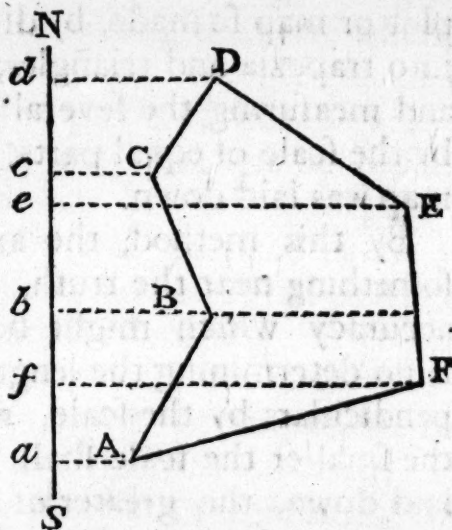
pedition, by an easy computation from the following

UNIVERSAL THEOREM.

*If the sum of the DISTANCES, (on an east and west line,) of the two ends of each line of the survey, from any assumed meridian laying entirely out of the survey, be multiplied by the respective NORTHING, or SOUTHING, made on each respective line; the DIFFERENCE between the sum of the NORTH PRODUCTS, and the sum of the SOUTH PRODUCTS, will be double the area of the survey.**

For

* *Demonstration.* Let $ABCDEF$ be any survey, and let the east and west lines Aa, Bb, Cc, Dd, Ee , and Ff , be drawn from the ends of each of the lines of the survey to any assumed meridian, as $N. S.$ laying entirely out of the survey.



Then, I say, the difference between the sum of the products $Aa + Bb \times ab$, $Bb + Cc \times bc$, $Cc + Dd \times cd$; and the sum of the products $Dd + Ee \times de$, $Ee + Ff \times ef$, $Ff + Aa \times fa$; will be double the area of $ABCDEF$.

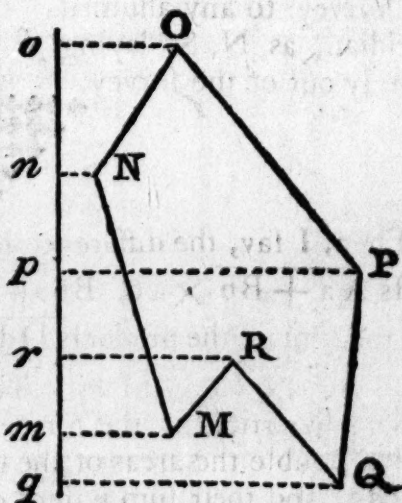
For, by article 4, the former products will be, respectively, double the areas of the spaces $ABba$, $BCcb$, and $CDdc$, and their sum will of course be double the area of the space $AadDCBA$. In like manner, the latter products will be, respectively, double the areas of the spaces $DEed$, $EFfe$, and $FAaf$; and their sum will of course be

For example. Let it be required to determine the area of the survey mentioned in page 299, from the field notes there given.

The northings, southings, eastings, and westings; as also the *north and south distances*, and the *east and west distances* made from the place of beginning to the end of each line; and likewise the distances of the ends of each line from the assumed meridian, have been already placed in their respective columns, and explained in the preparatory to the explanation of the method of plotting there recommended; but it will not be improper to give them a place here again, together with the further columns for determining the area; in order that the

be double the area of the space A a d D E F A. But it is evident, that the difference between the areas of the spaces A a d D E F A, and A a d D C B A, must be the area of the space A B C D E F A; consequently, the difference between their doubles, must be double the area of A B C D E F A.

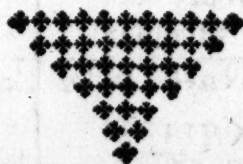
Again, the difference between the sum of the products $\overline{Mm} + \overline{Nn} + \overline{mn}$, $\overline{Nn} + \overline{Oo}$ $\times n o$, $\overline{Qq} + \overline{Rr} \times q r$; and the sum of the products $\overline{Oo} + \overline{Pp} \times o p$, $\overline{Pp} + \overline{Qq} \times p q$, $\overline{Rr} + \overline{Mm} \times r m$; will be double the area of M N O P Q R M.



For reasoning as before, the sum of the first mentioned products

the whole, together with the respective uses of the several parts, may appear under one view, as follows.

products will be double the area of $\overline{Qq o O N M R Q + M R r m M}$; and the sum of the last mentioned products will be double the area of $\overline{Qq o O P Q + M R r m M}$. But it is evident, that $\overline{Qq o O P Q - Qq o O N M R Q} = \overline{M N O P Q R M}$: consequently, $\overline{Qq o O P Q + M R r m M - Qq o O N M R Q + M R r m M} = \overline{M N O P Q R M}$; and consequently, the difference of their doubles, must be equal to double $\overline{M N O P Q R M}$. And the like in every other possible figure.



Field

For the Area.

For the Plotting.

East and west Distances of the ends of

North and

Field Notes.

Field Notes.		For a Proof of the Field Work.			For the Plotting.		For the Area.	
Courses and distances as taken in the field.		Northings, southings, eastings, and westings, made on those respective courses and distances.			North and south distances made on the whole from the place of beginning to the end of each line.		East and west distances made on the whole, from an assumed meridian, laying out of the survey.	
		N.	S.	E.	W.	Distances of the ends of each line		Sums of the distances of the two ends of each line, North products, in acres.
1. N. 7	ch. 1.							
2. N. 55 15	W. 21 00	20 84			2 56			
3. S. 62 30	E. 18 20	10 37		14 95				
4. S. 40	E. 14 40		6 65	12 77				
5. S. 4 15	W. 11 00		8 43		7 07			
6. N. 73 45	E. 14 00	3 47	13 96	1 04				
7. S. 52	W. 12 40		5 64		11 90			
	W. 9 17				7 23			
Totals.		34 68	34 68	28 76	28 76			
						North prod. Subtract		34 81359
								2) 103 46175
						Area.		Acres 51 73087 4
								Roods 2 92348 40
								Perches 36 93920

The three last columns, relating to the *area*, are the only ones that remain to be explained.

The first of these three columns contains the sums of the distances of the two ends of each line of the survey from the assumed meridian; which (according to the above universal theorem) are *multipliers* for determining the area. These are formed by adding each two of the successive numbers in the preceding left hand column together. Thus 3.00 is the distance of the place of beginning from the assumed meridian; and 0.44 is the distance of the end of the first line from the assumed meridian; and their sum is 3.44 for the first multiplier. Again 0.44 added to 15.39 makes 15.83 for the second multiplier. And 15.39 added to 28.16 makes 43.55 for the third multiplier; and so of the rest.

The north products contained in the next column, are the products of the multiplication of the several *northings* contained in the columns marked N, by their corresponding *multipliers* in the last mentioned column. Thus 7.16896 is the product of the multiplication of 20.84 by 3.44; an additional place of decimals being cut off, to give the product in acres.

The south products contained in the next right hand column are, in like manner, the products of the multiplication of the several *southings* in the column marked S, by their corresponding *multipliers*. Thus 28.96075, is the product of the multiplication of 6.65 by 43.55; an additional place of decimals being cut off, as before, to give the product in acres: and so of the rest.

The *north products* being then added together into one sum, and the *south products* into another sum, and the lesser of these sums being subtracted

from the greater; the remainder (by the above universal theorem) is double the area of the survey: which, being divided by Q , gives the area required: viz. 51.73087, equal (when reduced) to 51 acres, 2 roods, and 36 perches 9-10.*

Nothing can be more simple or easy in the operation, or more accurate in answering the desired ends, both with respect to the proof of the work, the plotting, and the computation of the area, than this process. The northings, southings, eastings, and westings, which give the most decisive proof of the accuracy or inaccuracy of the field work, are shewn by inspection in table 1. From these the numbers or distances for the plotting, are formed by a simple addition or subtraction. And the multipliers, for determining the area, are formed from thence, by a simple addition, with much less labour, than by the common method of dividing the plot into trapezia, and taking the multipliers by the scale of equal parts.

The superior accuracy and ease with which every part of the process is performed, cannot, it is imagined, fail to recommend it to every practitioner, into whose hands this tract shall fall.

10. If the boundary lines of the survey shall have crooks and bends in them, which is generally the case in old settled countries, those crooks and bends are taken by making offsets from the station lines;

as

* It must be observed, that, in order to determine the area by this method of computation, it is indispensibly necessary that the several lines of the survey be arranged regularly, one after another, all the way round the survey, in case they should not have been so taken in the field. Because, without such arrangement, the northings or southings made on each line, and the distances from the assumed meridian, would not correspond with one another.

as explained in page 300. In these cases the area comprehended within the station lines is determined as in the last example; and the areas comprehended between the station lines and the boundaries, are determined separately, (as in the *third* and *fourth* articles of this part,) from the *offsets* and *base lines*, noted in the field book; and are *added* or *subtracted* respectively, according as the station lines shall happen to be *within* or *without* the field surveyed; which, of course, gives the area required.

For example. Let it be required to determine the area of the survey mentioned in the example, page 301, from the field book there given.*

The *bearings* and *lengths* of the station lines are the same as in the last example; and consequently the area comprehended within those station lines, must be the same; viz. 51.73087.

Then, for the area comprehended between the first station line, and the boundary of the field, we have (see the field book, page 302,) first, a base line 3.60, and a perpendicular offset 0.40; which (according to article 3 of this part) contains an area of 0.07200. *Secondly*, a base line 4.85 (being the difference between 3.60 and 8.45, on the station line, at which points the offsets were taken), and an offset or breadth at the one end 0.40, and at the other end 0.10, which (according to article 4 of this part) contains an area of 0.12125. *Thirdly*, a base line 7.15 (being the difference between 8.45 and 15.60 on the station line), and an offset or breadth at the one end 0.10, and at the other end 0.65; which

* A plot of this survey is given in *plate 18, fig. 3.*

which (according to the 4th article of this part) contains an area of 0.26812: *Fourthly*, a base line 3.40 (being the difference between 15.60 and 21.00 on the station line), and a perpendicular offset 0.65; which (according to the third article of this part,) contains an area of 0.17550. These added together, make 0.63687 for the area of the offsets on the first station line; and, as this station line is *within* the survey, this area must of course be *added* to the above-mentioned area comprehended within the station lines.

Next, for the area comprehended between the second station line and the boundary of the field, we have a base line 18.20, and a perpendicular or offset 0.60; which (according to article 3 of this part,) contains an area of 0.54600. And, as this station line is *without* the survey, this area must of course be *subtracted* from the above-mentioned area.

The areas of all the other offsets are determined in the same manner, and are respectively as follows; viz.

Areas of the Offsets.	
To be added.	To be subtracted.
On the <i>first</i> station line A. 0.63687	
On the <i>second</i> - - -	0.54600
On the <i>third</i> - - -	0.64800
On the <i>fourth</i> - - -	
On the <i>fifth</i> - - -	0.36250
On the <i>sixth</i> - - -	1.08875
On the <i>seventh</i> - - -	0.68750
	0.27510
Totals - - -	2.08812
	2.15660

The area comprehended within the station lines, as above	-	-	A	51.73087
Offsets to be added	-	-	-	2.08812
	Sum	-		53.81899
Offsets to be subtracted	-			2.15660
Remains the area of the survey required	-	-	A.	51.66239
				4
	Roods	-		2.64956
				40
	Perches	-		25.98240

page 304, of the several fields therein contained; from the field book there given.*

The bearings and lengths of the station lines on the outside boundaries are as follows; viz.

			Ch. L.
1st.	S. 87°	W. -	15.00
2d.	N. 18° 30'	W. -	9.60
3d.	N. 12° 15'	E. -	8.90
4th.	N. 37°	E. -	17.00
5th.	S. 76° 45'	E. -	16.00
6th.	S. 5° 15'	W. -	25.00
7th.	S. 74° 30'	W. -	7.65

The area comprehended within these station lines (computed according to the above-mentioned universal theorem) is

A. 66.90493

The area of the offsets to be added,

0.56715

(computed as in the last example)

Sum - 67.47208

The area of the offsets to be subtracted,

2.56715

The area of the whole survey

Acres - 64.90493

4

Roods - 3.61972

40

Perches - 24.78880

Then

* See the plot of this survey in plate 18, fig. 4.

Then for determining the area of the east field, we have the following lines, viz.

1st. Part of the first station	Ch. L.
line in the field book, viz.	S. 87° W. 5.50
2d. The offset to the corner	
of the field, - - -	N. 3° W. 1.10
3d. The eighth station line	
in the field book, - -	North - 16.61
4th. The ninth station line	
in the field book, - -	East - 13.77
5th. The offset from the	
corner of the field to	
the sixth station line in	
the field book, - -	S. 84° 45' E. 0.56
6th. Part of the sixth	
station line in the field	
book, - - -	S. 5° 15' W. 15.40
7th. The seventh station	
line in the field book, -	S. 74° 30' W. 7.65

The area comprehended within these lines, (computed according to the foregoing universal theorem,) is - - - A. 22.91936

There are no offsets to be added in this field, the areas of the offsets to be subtracted (computed as in the last example) are - - 2.22052

Remains the area of the east field, - - - Acres - 20.69884

Roods - 2.79536

Perches - 31.81440

For

For ascertaining the area of the west field, we have the following lines, viz.

1st. The offset from the south-easterly corner to the first station line in the field book,	-	S. 3°	E.	Ch. L. 1.10
2d. Part of the first station line in the field book,	-	S. 87°	W.	9.50
3d. The second station line in the field book,	N. 18° 30'	W.	9.60	
4th. The third station line in the field book,	N. 12° 15'	E.	8.90	
5th. Part of the fourth station line in the field book,	-	N. 37°	E.	5.40
6th. The offset to the corner,	-	S. 53°	E.	0.40
7th. The tenth station line in the field book, reversed,	-	S. 62° 30'	E.	7.92
8th. The eighth station line in the field book, reversed,	-	South	-	16.61

The area comprehended within these lines, (computed according to the above universal theorem,) is A. 22.18048

The areas of the offsets to be added (computed as in the last example) - - - 0.62287

Sum - 22.80335

The areas of the offsets to be subtracted - - - 1.15900

A a 3 Remains

Remains the area of the west field,

Acres - 21.64435
4

Roods - 2.57740
40

Perches - 23.09600

For ascertaining the area of the north field, we have the following lines, viz,

- | | | | |
|--|---|------------|----------------|
| 1st. The offset from the south-westerly corner to the fourth station line in the field book, | - | N. 53° | Ch. L. W. 0.40 |
| 2d. Part of the fourth station line in the field book, | - | N. 37° | E. 11.60 |
| 3d. The fifth station line in the field book, | - | S. 76° 45' | E. 16.00 |
| 4th. Part of the sixth station line in the field book, | - | S. 5° 15' | W. 9.60 |
| 5th. The offset to the corner, | - | N. 84° 45' | W. 0.56 |
| 6th. The ninth station line in the field book, reversed, | - | West. | 13.77 |
| 7th. The tenth station line in the field book, | | N. 62° 30' | W. 7.92 |

The area comprehended within these lines (computed according to the above universal theorem,) is A. 21.81422
The

(brought up) 21.81422

The areas of the offsets to be added (computed as in the last example,) - - - 1.25030

Sum = 23.06452

The areas of the offsets to be subtracted, - - - 0.50080

Remains the area of north field,

Acres - 22.56372
4

Roods - 2.25488
40

Perches - 10.19520

		A.	R.	P.
The area of the East field,	-	20	2	31
West field,	-	21	2	23
North field,	-	22	2	10

Total - A. 64 3 24

The agreement of this *total* with the area of the *whole* as first ascertained, shews the work to be right. We must not however, in actual practice, expect to find so perfect an agreement as the above. A difference of a few perches, in actual practice, is no impeachment either of the field work, or of the computations.

A 24

Of

OF MARITIME SURVEYING.

IT was not my intention at first to say any thing concerning maritime surveying, as that subject had been already very well handled by Mr. Murdoch Mackenzie, whose treatise on *maritime surveying* ought to be in every person's hands who is engaged in this branch of surveying, a branch which has been hitherto too much neglected. A few general principles only can be laid down in this work; these, however, it is presumed, will be found sufficient for most purposes; when the practice is seen to be easy, and the knowledge thereof readily attained, it is to be hoped, that it will constitute a part of every seaman's education, and the more so, as it is a subject in which the safety of shipping and sailors is very much concerned.

1. Make a rough sketch of the coast or harbour, and mark every point of land, or particular variation of the coast, with some letter of the alphabet; either walk or sail round the coast, and fix a staff with a white rag at the top, at each of the places, marked with the letters of the alphabet. If there be a tree, house, white cliff, or other remarkable object at any of these places, it may serve instead of a station staff.*

2. Chuse some level spot of ground upon which a right line, called a *fundamental base*, may be measured either by a chain, a measuring pole, or a piece of log-line marked into feet; generally speaking, the longer this line is, the better: it's situation must be such, that the whole or most part of the station-staves may be seen from both ends thereof; and it's length and direction must, if possible,

* See Nicholson's Navigator's Assistant.

possible, be such, that the bearing of any station-staff, taken from one of it's ends, may differ at least ten degrees from the bearing of the same staff taken from the other end; station-staves must be set at each end of the fundamental base.

If a convenient right line cannot be had, two lines and the interjacent angle may be measured, and the distance of their extremes found by construction, may be taken as the fundamental base.

If the land measured has a sensible and gradual declivity, as from high-water mark to low-water, then the length measured may be reduced to the horizontal distance (which is the proper distance) by making the perpendicular rise of the tide, one side of a right-angled triangle, the distance measured along the land the hypotenuse, and from thence finding the other side trigonometrically, or by protraction on paper; which will be the true length of the base-line. If the plane measured is on the dry land, and there is a sensible declivity there, the height of the descent must be taken by a spirit-level, or by a quadrant, and that made the perpendicular side of the triangle.

If, in a bay, one strait line of a sufficient length cannot be measured, let two or three lines, forming angles with each other, like the sides of a polygon, be measured on the land along the circuit of the bay: these angles carefully taken with a theodolite, and exactly protracted, or calculated, will give the strait distance between the two farthest extremities of the first and last line.

3. Find the bearing of the fundamental base by the compass, as accurately as possible, with Hadley's quadrant, or any other instrument equally exact; take the angles formed at one end of the base, between the base line and lines drawn to each

each of the station-staves; take likewise the angles formed between the base line and lines drawn to every remarkable object near the shore, as houses, trees, windmills, churches, &c. which may be supposed useful as pilot-marks; from the other end of the base, take the angles formed between the base and lines drawn to every one of the station staves and objects; if any angle be greater than the arc of the quadrant, measure it at twice, by taking the angular distance of some intermediate object from each extreme object; enter all these angles in a book as they are taken.

4. Draw the fundamental base upon paper, from a scale of equal parts, and from it's ends respectively draw unlimited lines, forming with it the angles taken in the survey, and mark the extreme of each line with the letter of the station to which it's angle corresponds. The intersection of every two lines whose extremes are marked with the same letter, will denote the situation of the station or object, to which, in the rough draught that letter belongs; through or near all the points of intersection which represent station-staves, draw a waving line with a pencil, to represent the coast.

5. At low water sail about the harbour, and take the soundings, observing whether the ground be rocky, sandy, shelly, &c. These soundings may be entered by small numeral figures in the chart, by taking at the same time the bearings of two remarkable objects; in this excursion, be particular in examining the ground off points of land which project out into the sea, or where the water is remarkably smooth, without a visible cause, or in the vicinity of small islands, &c, observe the set and velocity of the tide of flood,
by

by heaving the log while at anchor, and denote the same in the chart by small darts. The time of high water is denoted by Roman numeral letters; rocks are denoted by small crosses; sands by dotted shading, the figures upon which usually shew the depth at low water in feet; good anchoring places are marked by a small anchor; upon coming near the shore, care must be taken to examine and correct the outline of the chart, by observing the inflections, creeks, &c. more minutely.

6. In a small sailing vessel go out to sea, and take drawings of the appearance of the land, with it's bearings; sail into the harbour, observe the appearance of it's entrance, and particularly whether there be any false resemblance of an entrance by which ships may be deceived into danger; remark the signs or objects, by attending to which, the harbour may be entered in safety; more especially, where it can be done, let the ship steer to the anchoring place, keeping two remarkable objects in one, or on a line,

7. Coasts are shaded off on the land side; houses, churches, trees, &c. on shore, are drawn small in their proper figures; in a proper place of the chart, draw a mariner's compass, to denote the situation of the rhumbs, &c. and on one side of the fleur-de-lis, draw a faint half fleur-de-lis at the point of north by compass; draw also a divided scale of miles, or leagues, which must be taken from the same original scale as the fundamental base.

Charts or plans may be neatly drawn with indian ink, or the pen and common ink, and are as useful as any others, but they are frequently done in water colours; for which purpose the beginner will derive more advantage from viewing a proper drawing,

drawing, or from overlooking a proficient at work, than from a multitude of written instructions.

An example to illustrate the foregoing precepts.

Let A B C D E F G H, *fig. 11, plate 23*, represent a coast to be surveyed, and I K L, an adjacent island; it is required to make an accurate chart of the same.

To make the actual observations on shore.

By sailing or walking along the shore, a rough sketch is made, and station staves set up at the points of land, A, B, C, D, E, F, G, H, and also at I, K, L, on the island, (precept 1.) During this operation, it is observed, that no proper place offers itself at which a fundamental base can be drawn, so as to command at once a view of all the stations; it will therefore be necessary to survey the coast in two separate parts, by making use of two base lines; if the coast had been more extended, or irregular, a greater number of base lines might have been necessary. Between the points B and C, the ground is level, and a considerable number of the southermost points may be seen from both points, conformable to precept 2; make B C, therefore, the first fundamental base, it's length by admeasurement is found to be 812 fathoms, and it's bearing by compass from B to C, is N. $11^{\circ} 14'$ E. from each end of this base, measure with Hadley's quadrant the angles formed between it, and lines drawn to each station in sight, (precept 3.) enter or tabulate them as follows.

First fundamental base B C = 812 fathom,
bearing of C from B, N. $11^{\circ} 14'$ E.

Angle

Angle A B C = 135 07	Angle A C B = 22 54
L B C = 85 00	L C B = 71 02
K B C = 56 28	K C B = 91 40
E B C = 43 32	E C B = 113 50
D B C = 13 30	D C B = 145 20

After having made these observations, it will be requisite to proceed to the northern part of the coast; in all cases where a coast is surveyed in separate parts, it is best to measure a new fundamental base for each part, when it can be conveniently done; a line from the station E drawn towards F, is well adapted to our purpose; let E P, therefore, be the second base line, it's length by admeasurement is found to be 778 fathoms, and it's bearing by compass from E to P, N. 38° 20' E. measure the angles formed by lines drawn from each end of this base, as before directed, and tabulate them as follows.

Second fundamental base E P = 778 fathom, bearing of P, from E. N. 38° 20' E.

Angle H E P = 88 30	Angle H P E = 73 00
I E P = 71 32	I P E = 28 54
G E P = 43 59	G P E = 111 00
F E P = 27 51	F P E = 127 18

It is sufficiently apparent, that the connection between the two parts of this survey, is preserved by the second fundamental base being drawn from the point E, whose situation was before determined by observations from the first base line; if this particular position of the second base had not been convenient, and it had been taken at a distance from every point determined in situation from the first

first base, the connection would have required an observation of the bearing of one of the said points from each end of the second base: thus, suppose the line IP to be the second base line, instead of EP ; the position of IP with respect to the given point E , may be known by taking the bearings of E , from I and P .

All the observations which are required to be made on shore, being now completed, it will be adviseable to construct the chart, before we proceed to the other observations on the water, for, by this method, an opportunity is offered of drawing the waving line of the coast with more correctness, than could otherwise be obtained.

GEOMETRICAL CONTRUCTION. *Fig. 12, plate 23.*

On the point B as a center, describe the circle $n e s w$, the diameters ns and we being at right angles to each other, and representing the meridian and parallel; from n set off the arc nN , equal to the quantity of the variation of the compass; draw the diameter NS , and the diameter WE at right angles to the same; these lines will represent the magnetic meridian, and it's parallel of latitude.

From the center B , draw the first base line $BC = 812$ fathoms, $N. 11^{\circ} 14' E.$ by compass; from one extremity B , draw the unlimited lines Ba, Bc, Bk, Be, Bd , forming respectively with the base, the angles observed from thence; from the other extremity C , draw the unlimited lines Ca, Cc, Ck, Ce, Cd , forming respectively with the base, the angles observed from thence; the intersections of lines terminated by the same letter, will give the stations $A, L, K, E, D.$ (precept 4.)

Draw the line B Q, N. $38^{\circ} 20'$ E. by compass, and from E, draw EP = 778 fathoms in the same direction or parallel to it, EP will be the second base line; from one extremity E, draw the unlimited lines Eh, Ei, Eg, Ef, forming respectively with the base, the angles observed from thence; from the other extremity P, draw the unlimited lines Ph, Pi, Pg, Pf; the intersections of lines terminated by the same letter, will give the stations H, I, G, F. (precept 4.)

Near the points A, B, C, D, E, F, G, H, draw a waving or irregular line to represent the coast, draw also by the help of the points I, K, L, a line to represent the coasts of the islands.

The chart being thus far delineated, it will be proper to make the observations which are required on the water.

TO MAKE THE ACTUAL OBSERVATIONS ON THE WATER.

Precept 5, sufficiently explains the business, to which it is necessary to attend on the water; nothing more, therefore, need be added here, except an elucidation of the method, by the place at which any observation is made, may be found on the chart; the place of an observer may be determined by the bearings of two known objects; the intelligent practitioner will find reasons for occasionally preferring each of these methods; but, as the distance in the second method is known only by estimation, we have given the preference to the first, in the present example; all the soundings, &c. are supposed to be laid down by these methods, but, to prevent confusion and prolixity, the lines of bearing are drawn only from the points M, N, and O.

From

From the southern extremity M, of the reef of rocks, which runs off the point L, the bearing of the stations C and D are taken by the compass, as follows, viz. C, N. 62° E. and B, N. 85° E.

On the chart, therefore, from C, draw the opposite rhumb B m, S. 62° W. and from C, parallel to the rhumb B R, draw the opposite rhumb C m, S. 85° W. their intersection M, will be the extremity of the reef of rocks.

Again, from the sand head N, the bearings of the stations L and K are taken by compass, as follows, viz. L, N. 17° W. and K, N. 5° E.

On the chart draw the rhumb B n, N. 17° W. and from L, parallel to the same, draw the opposite rhumb L n; draw likewise the rhumb B T, N. 5° E. and from K, parallel to the same, draw the opposite rhumb K n; the intersection of these two last lines will be the sand head, and the situation of the point of land O, which lies so that it cannot be seen from either of these base-lines, is determined in like manner from the bearings of L and K, though in practice it may have been more convenient to settle it's place from a single bearing and distance of the station A.

The position of the compass is given from that of the circle N E S W, made use of in the construction, and the scale of geographical miles is found from the consideration, that 1021 fathoms make one geographical mile; take therefore 1021 from the same scale of equal parts, as was used in laying down the fundamental bases, and it will give one primary division for the intended scale of miles.

How to measure a strait line on the surface of the sea.

First, prepare a measuring-line of strong cord, two or three hundred yards in length, with small pieces of cork of equal thickness made fast to it at
small

small distances, all along, like a fishing-net, so that it may float strait on the surface of the water: if the line has been well stretched, or much used before, it is the better: also prepare two ropes somewhat longer than the greatest depth of the water to be measured, with a pig of lead or iron ballast (which we call an anchor) 50 or 60 pounds weight, tied by the middle to one end of each rope, that when it is at the bottom it may be able to anchor a boat, and bear to be stretched strait without shifting the place of the anchor. Let the measuring-line be thoroughly wet immediately before you begin to use it, and then stretched on the water close by the shore, and it's length measured there with a pole. Then, in the direction intended to be measured, take two remarkable sharp objects on the land in a line, one near the shore, the other as far up in the country as you can: if such are not to be had, place buoys on the water at proper distances in that direction.

2. Take the objects, or buoys, in a line, and holding one end of the measuring-line fast on the shore, carry out the other in a boat, in that direction, till it is stretched strait at it's full length by one man in the boat, and exactly at the end of the line let another man drop the anchor, which will mark one length of it. There keep the boat, and the end of the measuring-line, close to the anchor-rope, drawn tight up and down, till another boat takes in the other end which was on the shore, and rows farther on, and lays it strait in the direction of the land-marks, or buoys, and there drops another anchor, which will mark the second length of the measuring-line. Go on thus till the whole proposed distance is measured: and immediately after let the measuring-line be again measured with a pole on the

water near the shore, as at first, and if the lengths differ, take the mean between them for the true length. It is obvious, that to measure with any exactness this way, the sea must not only be smooth, but void of a swell, and of all stream of tide; either of which will hinder the line to lie strait. This method of measuring a strait line may be convenient on some occasions; and if care is taken to keep the anchor-rope right up and down when the measuring-line is applied to it, will be found sufficiently exact for many purposes, but not for a fundamental base-line from which other distances are to be deduced.

There is another way of measuring a strait line, mechanically, on the sea, which is so well known to seamen, that it is needless to describe it particularly here: and that is, by heaving the log over a ship's stern while she is under sail, and observing how many knots of the log-line run out in half a minute; for the line is so divided that the ship will run (or is supposed to run) so many miles in an hour, in a strait course; and twice as much in two hours, and so on. But this conclusion is founded on three suppositions, neither of which is certain, viz. That the log remains in the same place during the whole half minute that the line is running out from the ship's stern; that the ship continues to sail with the same velocity, and also in the same direction, during an hour, or two, that she did during the half minute: the contrary of which is more likely in most cases. For the log-line may shrink, or stretch, while it is running out; or may drag after the vessel by the weight of the line, or by not running easily and readily off the reel; the swell of the sea may alter the place
of

of the log; and currents or streams of tide, stronger or weaker below the surface than at it, an unsteady helm, lee-way, and varying winds may change the direction, or celerity of the ship's motion; for neither of which can any certain allowance be made. This way, therefore, of measuring a strait line, or distance, is not to be depended on as exact; but is mentioned here, because rocks, shoals, or islands sometimes lie so far from the coast, that there is no other way of forming any notion of their distance. If any such distance is to be measured after this manner, let the log-line be thoroughly wet when it is measured; let the length between each knot be 51 feet, which is the hundred-and-twentieth part of a geometrical mile, as half a minute is $\frac{1}{120}$ th part of an hour; chuse neap-tide, as much slack water as can be got, and a moderate breeze of following wind; let the line be run off the reel so as never to be stretched quite strait; and if the half minute is measured by a watch that shews seconds, rather than by a glass, it will generally be more exact. Perhaps one second should be allowed for the loss of time in calling out at the beginning, and stopping it at the end of the time; except the person who holds the watch can contrive to observe the going out of the red rag at the beginning, and also to stop the line himself at the end of the time; which does not seem a difficult matter.

To find the distance of two places, by the flash and report of a gun.

Sound moves 1142 English feet in one second of time, or 6120 feet, which make a geographical mile in $5\frac{1}{3}$ " nearly; therefore, let a gun be fired at one place at an appointed time, and observe the

Bb 2

time

time that elapses between the flash and report, and so many seconds as you observe, so many times 1142 feet, are you distant from the place; the operation should be repeated two or three times for greater certainty; the distance to be measured in this way, should never be less than two miles, on account of errors that may arise in taking the time.

As a second method of measuring distance, by the sound of a gun, or other loud noise, may be used, it is necessary to know the time that elapses between the flash and report, and so many seconds as you observe, so many times 1142 feet, are you distant from the place; the operation should be repeated two or three times for greater certainty; the distance to be measured in this way, should never be less than two miles, on account of errors that may arise in taking the time.

To find the distance by the flash and report of a gun, or other loud noise, it is necessary to know the time that elapses between the flash and report, and so many seconds as you observe, so many times 1142 feet, are you distant from the place; the operation should be repeated two or three times for greater certainty; the distance to be measured in this way, should never be less than two miles, on account of errors that may arise in taking the time.

TO TRANSFER A PLAN FROM ONE PAPER TO ANOTHER.

Method 1. By points. Lay the rough plan upon the clean paper, on which you intend to draw the fair copy, press them close together by weights, and keep them as flat as possible; then with a pointrel or needle point, prick through all the corners of the plan to be copied, separate the papers, and join by lines the points on the clean paper. This method can only be used in plans, whose figures are small, regular, and bounded by strait lines.

Method 2. By tracing paper. Rub the back of the rough plan with black-lead powder, and having wiped off the superfluous lead, lay the blacked part upon the clean paper, or place a sheet of black tracing paper between the rough plan and the clean paper; weights are to be placed as in the former method, to maintain the papers in the same position.

Then with a blunt point of brass, steel, or ivory, trace exactly the lines of the plan, pressing the paper so much, that the black lead under the lines may be transferred to the clean paper; when the whole of the plan has been thus delineated, go over the black-lead marks with common or Indian ink.

Method 3. By squares. See prob. 65, page 91.

Method 4. By a copying glass. This is a large square or rectangular piece of looking glass, fixed in a frame of wood, which can be raised to any angle like a desk, the lower end resting upon the table; a screen of blue paper fits to the upper edge, and stands at right angles to it.

Place this frame at a convenient angle against a strong light: fix the old plan and clean paper firmly together by pins, the clean paper uppermost, and on the face of the plan to be copied; lay them with the back of the old plan next the glass, namely, that part which you intend to copy first. The light through the glass will enable you to perceive distinctly every line of the plan upon the clean paper, and you can easily trace over them with a pencil; and having finished that part which covers the glass, slide another part over it, and copy this, and thus continue till the whole be copied.

Method 5. By the assistance of proportionable and triangular compasses. These will in many instances assist the draftsman very much, and lessen the labour of copying.

Method 6. By the pantographer. There is no method so easy, so expeditious, nor even so accurate as the pantographer. It is an instrument as useful to the experienced draftsman, as to those who have made but little progress in the art. It saves a great deal of time, either in reducing, enlarging, or copying of the same size, giving the outlines of any drawing, however crooked or complex, with the utmost exactness; nor is it confined to any particular kind, but may with equal facility be used for copying figures, plans, sea-charts, maps, profiles, landscapes, &c.

DESCRIPTION AND USE OF THE PANTOGRAPHER.

I have not been able to ascertain who was the inventor of this useful instrument. The earliest account I find is that of the Jesuit Scheiner, about the year 1631, in a small tract entitled *Pantographice, sive ars nova delineandi*. The principles are self-evident to every geometrician; the mechanical construction

construction was first improved and brought to it's present state of perfection by MY FATHER, about the year 1750. It is one, among many other scientific improvements and inventions completed by him, that others have ingloriously, and many years after, assumed to themselves.

The pantographer is usually made of wood, or brass, and consists of four flat rules, *fig. 19, plate 28*, two of them long, and two short. The two longest are joined at the end A, by a double pivot, which is fixed to one of the rules, and works in two small holes placed at the end of the other. Under the joint is an ivory castor, to support this end of the instrument. The two smaller rules are fixed by pivots at E and H, near the middle of the larger rules, and are also joined together at their other end, G.

By the construction of this instrument, the four rules always form a parallelogram. There is a sliding box on the longer arm, and another on the shorter arm. These boxes may be fixed at any part of the rules by means of their milled nuts; each of these boxes are furnished with a cylindric tube, to carry either the tracing point or crayon or fulcrum.

The fulcrum or support K, is a leaden weight inclosed in a mahogany box, on this the instrument moves when in use; there are two moveable rollers, to support and facilitate the motions of the pantographer, their situation may be varied as occasion requires.

The graduations are placed on two of the rules, on each of them are two scales, the fiducial edges of the boxes are to be set to these, according to the work to be performed by the instrument.

The crayon, the tracer, and fulcrum, must in all cases be in a right line, so that when they are

set, if a string be stretched over them, and they do not coincide with it, there is an error either in the setting or graduations.

The long tube which carries the pencil or crayon, moves easily up or down another tube; there is a string affixed to the long or inner tube, passing afterwards through the holes in the three small knobs to the tracing point, where it may, if necessary, be fastened. By pulling this string, the pencil is lifted up occasionally, and thus prevented from making false or improper marks upon the copy.

TO USE THIS INSTRUMENT WHEN THE COPY IS TO BE OF THE SAME SIZE AS THE ORIGINAL.

Place the instrument upon a large table, and set the sliding boxes B and D, to the divisions marked 12. Put the crayon into the box B, place the box D upon the fulcrum or leaden foot; the tracing point at C. Then lay a piece of paper under the crayon, and the original drawing under the tracer, and move the tracing point over the principal strokes of the original, and the crayon will form the required copy.

To reduce a drawing, &c. to half the size of the original. Set the boxes B and D, to the divisions marked $\frac{1}{2}$, place the fulcrum at B, the crayon at D, and the tracer at C.

To reduce a drawing, &c. to less than $\frac{1}{2}$ the original. Suppose $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, &c. Place the fulcrum at B, crayon at D, and tracer at C, and slide the boxes B and D, to the divisions marked $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, &c. on the longer scales.*

For

* It may be proper to observe here, that if the copy is to be less than $\frac{1}{2}$ the original, or when it is required greater than the original, the longer scales are to be used.

For greater than $\frac{1}{2}$ the original drawing. Suppose it be required to make a drawing, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{2}$, &c. Set the boxes B and D, to corresponding divisions, as $\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{2}$, &c. on the shorter scales, place the fulcrum at D, the crayon at B, and tracer at C.

When the original drawing is to be enlarged. Suppose $\frac{3}{2}$, $\frac{4}{3}$, &c. set the boxes B and D, to $\frac{3}{4}$, $\frac{2}{3}$, &c. on the longer scales, place the fulcrum at D, the crayon at C, and tracer at B.

Where the copy is required of a size differing from the fractional parts laid down on the instrument. For this purpose there are two scales laid down, containing 100 unequal parts, one scale numbered from 10 to 80, the other from 50 to 100.

If the copy is to be under $\frac{1}{2}$ the original size, place the boxes B and D, to any two corresponding divisions under 50, the fulcrum at B, and crayon at D.

If the copy is to be larger than $\frac{1}{2}$ the original, place the boxes B and D, to corresponding divisions between 50 and 100; the fulcrum at B, and crayon at D.

To change the situation of the pantographer.

Copy first as much as the pantographer will take in; then make three points on the original, and as many corresponding points on the copy. Then remove the fulcrum to another situation, but so, that when the tracing point is applied to the three points marked on the original, the crayon may exactly coincide with the other three points on the copy, and proceed as before; and so on for every change in the situation of your instrument, and by this means a pantographer of $2\frac{1}{2}$ feet in length, will copy a drawing of any size whatsoever.

Or

OF LEVELLING.

The necessity of finding a proper channel for conveying water, occurs so often to the surveyor, that any work on that subject which neglected to treat on the art of levelling, would be manifestly imperfect; I shall therefore endeavour to give the reader a satisfactory account of the instruments used, and the mode of using them.

A DESCRIPTION OF THE BEST SPIRIT LEVEL.

Fig 3, plate 17, represents one of the best spirit levels, mounted on a best staff, similar to those affixed to a theodolite.

The telescope is moveable, either in the plane of the horizon, or with a small inclination thereto; so as to cut any object whose elevation, or depression, from that plane, does not exceed 12 degrees; the telescope is about 2 feet long, is furnished with cross wires for determining the axis of the tube, and forming a just line of sight.

The tube *cc* with the spirit bubble, is fixed to the telescope by a joint at one end, and a capstan head at screw at the other, to raise or depress it, for the adjustment. The supporters on which the telescope is placed, are nearly in the shape of the letter Y, the inner sides of the Y's are tangents to the cylindric ring of the telescope. The lower end of these supporters are let perpendicularly into a strong bar; to the lower end of the support A, is a milled nut, to bring the instrument accurately to a level; between the two supports is a compass box, divided into 4 ninety's, with a magnetical needle, and a contrivance to throw the needle off the center; when it is not used,

used, the staff is the same in every respect as that belonging to the best theodolite.

Two staves often accompany this instrument, they are called station staves, they consist of two parts, each part is about five feet 10 inches long, so that when they are pulled out to their greatest extent, they form a ten feet rod; and every foot is divided into 100 equal parts. To each of these staves there is a sliding vane with a brass wire across a little hole made in the vane; when this wire coincides with the horizontal wire of the telescope, it shews the height of the apparent level above the ground at that place.

It is evident from the nature of this instrument, that two adjustments are necessary. 1. To place the intersection of the wires in the telescope, so that it shall coincide with the axis of the cylindrical rings on which the telescope turns. 2. To render the level parallel to this axis. These adjustments are performed in a manner exactly similar to those of the telescopic theodolites.

OF LEVELLING.

Levelling is the art which instructs us in finding how much higher or lower any given point on the surface of the earth is, than another given point on the same surface; or in other words, the difference in their distance from the center of the earth.

Those points are said to be *level*, which are equi-distant from the center of the earth. The art of levelling consists therefore: 1st, In finding and marking two, or more level points that shall be in the circumference of a circle, whose center is that of the earth. 2dly, In comparing the points thus found, with other points, in order to ascertain

tain the difference in their distances from the earth's center.

Let *fig. 1, plate 23*, represent the earth, A it's center; the points B, C, D, E, F, upon the circumference thereof are level, because they are equally distant from the center; such are the waters of the sea, lakes, &c.

To know how much higher the point B, *fig. 2, plate 23*, is than C, and C lower than D. We must find and mark the level points, E, F, C, upon the radii A B, A C, A D, thereby comparing B with E, C with F, and D with G; we shall discover how much B is nearer the circumference of the circle than C, and consequently how much further from the center of the earth, and so of the other points.

OF THE DIFFERENT METHODS OF MARKING OUT THE LEVEL POINTS.

The first, which is the most simple and independent, is by the tangent of a circle, for the two extremities of the tangent give the true level points, when the point of contact is precisely in the middle of the line. But if the point of contact with the circumference be at one of the extremities of the line, or in any other part except the middle, it will then only shew the apparent level, as one of it's extremities is further from the circumference than the other. Thus the tangent B C, *fig. 3, plate 23*, marks out two true level points at B and C, because the point of contact D, is exactly in the middle of the line, B C, and it's two extremities are equally distant from the circumference and the center A.

But the tangent B C, *fig. 7, plate 23*, marks two points of apparent level, because the point D, where

where it touches the circumference, is not the middle of the line; and therefore one of it's extremities, B, is nearer to the center than the other D; D is further from the center in proportion as it is more distant from the point of contact B; which constitutes the difference between the true and apparent level, of which we shall speak presently; every point of the apparent level, except the point of contact, is higher than the true level.

As the tangent to a circle is perpendicular to the radius, we may make use of the radius to determine the tangent, and thus mark the level points. Thus let A, *fig. 5, plate 23.* represent the center of the earth, A B the radius, and C, B, D, the tangent; the two extremities C, D, are equally distant from the point of contact B, consequently the angles B C A, B D A, will be equal; the angles at the tangent point are right angles, and the radius common to both triangles, and the sides C A, D A, are equal, and the points C D, are two level points, because equally distant from the center.

It is evident from this, that if from any point of the radius two lines be drawn, one on each side, making equal angles with it, and being of an equal length, the extreme points of these lines will be level points. Thus, if from B, of the radius B A, two equal lines B D, C D, *fig. 6, plate 23.* be drawn, making equal angles, C B A, D B A, then will C, and D, be equally distant from the center; though the level may be obtained by these oblique lines, yet it is far easier to obtain it by a line perpendicular to the radius.

When the level line is perpendicular to the radius, and touches it at one of it's extremes, the other

other extremity will mark the apparent level, and the true level is found by knowing how much the apparent one exceeds it in height.

To find the height of the apparent above the true level for a certain distance, square that distance, and then divide the product by the diameter of the earth, and the quotient will be the required difference; it follows clearly, that the heights of the apparent level at different distances, are as the squares of those distances, and consequently that the difference is greater or smaller, in proportion to the extent of the line, for the extremity of this line separates more from the circumference of the circle, in proportion as it recedes from the point of contact. Thus let *A*, *fig. 7, plate 23*, be the center of the earth, *BC* the arc which marks the true level, and *B, E, D*, the tangent that marks the apparent level; it is evident, that the secant *AD*, exceeds the radius *AB*, by *CD*, which is the difference between the apparent and true level; and it is equally evident, that if the line extended no further than *E*. this difference would not be so great as when it is extended to *D*, and that it increases as the line is lengthened.

When the distance does not exceed 25 yards, the difference between the two levels may be neglected; but if it be 50, 100, &c. yards, then the error resulting from the difference will become sensible, and require to be noticed.

T A B L E,

T A B L E,

Which shews the quantity of curvature below the apparent level, in inches for every chain up to 100.

Chain	Inches	Chain	Inches	Chain	Inches	Chain	Inches
1	0,00125	14	0,24	27	0,91	40	2,00
2	0,005	15	0,28	28	0,98	45	2,28
3	0,01125	16	0,32	29	1,05	50	3,12
4	0,02	17	0,36	30	1,12	55	3,78
5	0,03	18	0,40	31	1,19	60	4,50
6	0,04	19	0,45	32	1,27	65	5,31
7	0,06	20	0,50	33	1,35	70	6,12
8	0,08	21	0,55	34	1,44	75	7,03
9	0,10	22	0,60	35	1,53	80	8,00
10	0,12	23	0,67	36	1,62	85	9,03
11	0,15	24	0,72	37	1,71	90	10,12
12	0,18	25	0,78	38	1,80	95	11,28
13	0,21	26	0,84	39	1,91	100	12,50

The first method of finding and marking two level points, is by a tangent whose point of contact is exactly in the middle of the level line; this method may be practised without regarding the difference between the apparent and true level, but to be used with success, it would be well to place your instrument as often as possible at an equal distance from the stations; for it is clear; that if from one and the same station, the instrument remaining at the same height, and used in the same manner, two or more points of sight be observed,

observed, equally distant from the eye of the observer, they will also be equi-distant from the center of the earth.

Thus let the instrument, *fig. 8, plate 23*, be placed at equal distances from C, D; E, F, the two points of sight marked upon the station staves C C, D H, will be the level points, and the difference in the height of the points E, F, will shew how much one place is higher than the other.

Second method. This consists in levelling from one point immediately to another, placing the instrument at the stations where the staves were fixed. This may also be performed without noticing the difference between the true and apparent level; but then it requires a double levelling, made from the first to the second station, and reciprocally from the second to the first.

Thus let the two stations be B, E, *fig. 9, plate 23*, the stations staves C B, D E, (which in the practice of levelling may be considered as parts of the radii, A B, A E, though they be really two perpendiculars parallel to each other, without the risk of any error.) In order to level by this method, plant the instrument at B, let the height of the eye, at the first observation, be at F, and the point of sight found be G; then remove the instrument to E, and fix it so that the eye may be at G; then if the line of sight cuts F, the points F and G are level, being equally distant from the center of the earth, as is evident from the figure.

But if the situation of the two stations be such, that the height of the eye at the second station could not be made to coincide with G, but only with H, *fig. 10, plate 23*; yet if the line of sight gives I as far distant from F as G is from H, the two lines F G, H I, will be parallel, and their extremities

extremities level points; but if that is not the case, the lines of sight are not parallel, and do not give level points, which however might still be obtained by further observations; but as this mode is not practised, we need not dwell further upon it.

The velocity of running water is proportional to the fall; where the fall is only 3 or 4 inches in a mile, the velocity is very small; some cuts have been made with a fall from 4 to 6 inches; a four inch fall in a strait line, is said to answer as well as one of 6 inches with many windings.

The distance from the telescope to the staff should not, if possible, exceed 100 yards, or five chains; but 50 or 60 yards are to be preferred.

OF THE PRACTICE OF LEVELLING.

1st. *Of simple Levelling.*

We term that simple levelling, when the level points are determined from one station, whether the level be fixed at one of the points or between them.

Thus let A, B, *fig. 1, plate 24*, be the station points of the level, C, D, the two points ascertained, and let the height

	Feet	Inches.
from A to C be	6	0 0
from B to D be	9	0 0
their difference is	3	0 0

shewing that B is three feet lower than A.

In this example, the station points of the level are below the line of sight and level points, as is generally the case; but if they were above, as in *fig. 2, plate 24*, and the distance of A to C be six feet,

C c

feet,

feet, and from B to D 9 feet, the difference will be still 3 feet, which B is higher than A.

2d. *Of compound Levelling.*

Compound levelling is nothing more than a collection of many single or simple operations, compared together. To render this subject clearer, we shall suppose, that for a particular purpose, it were necessary to know the difference in the level of the two points A and N, *fig. 3, plate 26*; A, on the river *Zome*, N, on the river *Belann*.

As I could find no satisfactory examples in any English writer that I was acquainted with, and not being myself in the habits of making actual surveys, or conducting water from one place to another, I was under the necessity of using those given by Mr. Le Febvre.

Stakes should be driven down at A and N, exactly level with the surface of the water, and these should be so fixed, that they may not be changed until the whole operation is finished; the ground between the two rivers should then be surveyed, and a plan thereof made, which will greatly assist the operator in the conducting of his business; by this he will discover that the shortest way from A to N, is by the dotted line A C, G H, H N, and he will from hence also determine the necessary number of stations, and distribute them properly; some longer than the other, according to the nature of the case and the situation of the ground. In this instance 12 stations are used; stakes should then be driven at the limits of each station, as at A, B, C, D, E, F, &c. they should be driven about 18 inches into the ground, and be about 2 or 3 inches above it's surface; stakes should also
be

be driven at each station of the instrument, as at 1, 2, 3, 4, &c.

Things being thus prepared, he may begin his work; the first station will be at 1, equidistant from the two limits A, B, the distance from A to B, 166 yards; and consequently; the distance on each side of the instrument, or from the station stake, will be 83 yards.

Write down in the first column, the first limit A; in the 2d, the number of feet, inches, and tenths, the points of sight, indicated on the station staff at A, namely, 7. 6. 0. In the third column, the second limit B; in the 4th, the height indicated at the station staff B, namely, 6. 0. 0. Lastly, in the fifth column, the distance from one station staff to the other, in this instance 166 yards.

Now remove the level to the point marked 2, which is in the middle between B and C, the two places where the station staves are to be held, observing that B, which was the second limit in the former operation, is the first in this; then write down the observed heights as before, in the 1st column B; in the 2d, 4. 6. 0; in the third, C; in the 4th, 5. 6. 2. in the 5th, 560, the distance between B and C.

As from the inequality of the ground at the third station, it is not possible to place the instrument in the middle between the two station staves; find the most convenient point for your station, as at 3; then measure exactly how far this is from each station staff, and you will find from 3 to C, 160 yards; from 3 to D, 80 yards: the remainder of the operations will be as in the preceding station.

In the fourth operation, it will be necessary to fix the station, so as to compensate for any error that might arise from the inequality of the last; therefore, mark out 80 yards from the station staff

C c 2

D,

D, to the point 4; and 160 yards from 4 to E; and this must be carefully attended to, as by such compensations the work may be much facilitated.

Proceed in the same manner with the eight remaining stations, as in the four former, observing to enter every thing in it's respective column; when the whole is finished, add the sums of each column together, and then subtract the less from the greater, thus from 82:2:5, take 76:9:7, and the remainder 5:4:8, is what the ground at N is lower than the ground at A.

	f.	in.		f.	in.	yards.
A	7	6	B	6	0 0	160
B	4	6	C	5	6 2	250
C	12	8 6	D	8	4 0	240
D	0	0 0	E	4	1 0	250
E	6	10 0	F	2	11 0	250
F	7	0 4	G	4	8 6	300
G	7	7 6	H	10	0 0	250
H	4	6 4	I	8	10 0	110
I	6	3 0	K	10	0 0	130
K	6	4 3	L	5	8 6	250
L	7	0 0	M	8	4 3	250
M	6	5 0	N	7	10 0	250
	76	9 7		82	2 5	2680
				82	2 5	
				76	9 7	
				5	4 6	

OF THE PROFILE OF THE LEVEL.

The next thing to be obtained is a section of this level; for this purpose draw a dotted line, as
 00,

o o, *fig. 4, plate 24*, either above or below the plan, which line may be taken for the level or horizontal line; then let fall perpendiculars upon this line from all the station points and places, where the station staves were fixed.

Beginning at A, set off 7 feet 6 inches upon this line from A to a; for the height of the level point determined on the staff at this place, draw a line through a, parallel to the dotted line o o, which will cut the third perpendicular at b, the 2nd station staff; set off from this point downwards 6 feet to B, which shews the second limit of the first operation, and that the ground at B, is 1 foot 6 inches higher than at A; place your instrument between these two lines at the height of the level line, and trace the ground according to it's different heights.

Now set off on the second station staff B, 4 feet 6 inches to C, the height determined by the level at the second station; and from c, draw a line parallel to o o, which will cut the fifth perpendicular at d, the 3d station staff from this point set off 5 feet; 6 inches $\frac{1}{10}$ downwards to C, which will be our second limit with respect to the preceding one, and the third with respect to the first; then draw your instrument in the middle between B and C, and delineate the ground with it's different inequalities. Proceed in the same manner from station to station to the last N, and you will have the profile of the ground, over which the level was taken.

To trace more particularly the profile of each station.

It is necessary to observe in this place, that if the object of the operation be only to know the reciprocal height of the two extreme terms, as in the preceding example, then the method there laid

down for the profile or section, will be sufficient; but if it be necessary to have an exact detail of the ground between the said limits, the foregoing method is too general; we shall, therefore, institute another example, in which we shall suppose the level to have been taken from A to N, by another route, but on more uniform ground, and less elevated above the level of the two rivers, in order to form a canal, marked O, P, Q, R, S, T, U, X, Y, for a communication from one river to the other.

Draw at pleasure a line Z Y, *fig. 5, plate 24*, to represent the level line and regulate the rest; then let fall on this line, perpendiculars to represent the staves at the limits of each station, taking care that they be fixed accurately at their respective distances from each other.

As the difference between the extreme limits ought to be the same as in the former example, is 5 feet 4 inches $\frac{4}{10}$, set off this measure upon the perpendicular at O the first limit, and from O, prolonging the perpendicular, mark off at *a*, the height determined at the first station staff, as in the preceding example; then do the same with the 2d and 3d, and so on with the following, till this part of the work is finished; there then remains only to delineate in detail, the ground between the station staves; the distances are assumed larger in this instance on account of the detail.

To get the section of the ground between O and P, place your instrument at one of the limits as P, fixing it so, that the cross hairs may answer to the point *b*; then look towards the first limit O, raising or depressing the vane, till it coincides with the intersection of the cross hairs, and the line of sight, from one point to the other, will mark the level or horizontal line.

Now

Now to set off the height of the brink of the river above the first limit, drive a stake down close to the ground at *a*, upon which place your station staff, and observe at what height the hairs intersect the vane, it will be at 4 10; then laying off upon the line *oz*, the distance from the first term to the first stake, from whence let fall a perpendicular, and set off thereon 4 10 to *a*, which gives the height at the first stake, or which is the same, the height from the edge of the river above the surface of the water, as is plain from the section.

Drive a second stake at *b*, in a line between the limits; place the station staff upon this stake, and observe the height 4. 6. intersected by the cross hairs, the instrument still remaining in the same situation; set off on the level line the distance from the first stake *a* to the second *b*, and then let fall a perpendicular, and mark upon it 4. 6. to *b*, which gives us the height of the ground at this place.

To mark out the small hollow *c*, drive down a third stake, even with the ground, in the middle of it at *c*; but as before in the station line, mark upon the level line the exact distance from the second stake *b*, to the third *c*; then let fall a perpendicular from *c*, and set off thereon 6:8:0 pointed out by the cross hairs on the staff, which determines the depth of the hollow, as may be seen by the section.

With respect to the ground between each stake, as the distances are now very short, it will be easily expressed by the operator, whose judgment will settle the small inequalities by a comparison with those already ascertained.

Proceed thus with the other stations, until you

arrive at the last, and you will obtain an accurate section in detail of your work; by such a section, it is easy to form a just estimation of the land to be dug away, in order to form the canal, by adding thereto the depth to be given to the canal.

Another example of compound levelling.

In the levelling exhibited in *plate 25*, we have an example taken where the situation was so steep and mountainous, that it was impossible to place the staves at equal distances from the instrument, or even to make a reciprocal levelling from one station to another.

Such is the case between the first point A, taken from the surface of a piece of water, which falls from the mountains, and the last point K at the bottom of a bason, where it is proposed to make a fountain, and the height is required, at which a jet d'eau will play by conducting the water from the reservoir A, to the point K of the bason, by tubes or pipes properly made, and disposed with all the usual precaution.

From the manner in which the operator must proceed in this instance, it is evident, that the instrument should be adjusted with more than ordinary care, as the true distance from one mountain to the other cannot be attained without much trouble.

The height is here so great, that it will be necessary to go by small ascents from A to D, and it will probably be commodious in some part of the work to use a smaller instrument; underneath is a table of the different level points as ascertained, which, with the profile and plan, render this part of the work sufficiently clear, without further explication.

A

f. in.			f. in.			yards.
A	21	6 0	C	0	9 0	90
C	4	3 0	D	0	3 0	40
D	3	9 0	E	16	3 0	350
E	5	0 0	F	17	9 0	250
F	10	6 0	G	5	0 0	375
G	5	8 0	H	19	0 0	300
H	5	0 0	K	47	3 0	1000
55 9 0			106 9 0			2405

	106	9	0
	55	0	0
Difference	51	9	0

Only two levellings are made here between A and D, though it is evident from the plan, that more would have been necessary; but as our design is only to shew the manner of proceeding in this particular case, and as more would have confused the plan and section, they are here neglected.

In the fourth, the height found was 16. 8. but as the distance 360 yards was considerable, it was necessary to reduce the apparent level to the true one; the same was also done in all other cases, where it was necessary; at the last limit, we first get the height from N to o, then from o to I, then from I to K, which added together, and then corrected for the curvature, gives 47. 3. 0. Now by adding each column together, and then subtracting one from the other, we obtain 51 feet 9 inches for the height, which the point A is above the bottom of the bason, and which will cause the jet d'eau to rise about 45 feet.

The

The general section of this operation is delineated under the plan at *fig. 2*, and is sufficiently plain from what has been already said in the preceding instances. But an exact profile of the mountain is not so easy, as it would require many operations, as may be seen in the section; some of these might however be obtained, by measuring from the level line already mentioned, without moving the instrument, as at *fig. 3*.

Example 3. *Fig. 1*, plate 26, Mr. Le Febvre gives us is that of a river, being one part of the river *Haynox*, from *Lignebruk* to *Villeburg*, and the mode he observed in taking this level.

The first operation was that of having stakes driven at several parts of the river, even with the water's edge; the first stake *A*, a little above the mills at *Lignebruk*, shews the upper water-mark, when the water is highest, and is our first limit; the stake *b* shews the low water-mark at the same mills, the stake *B* is the second limit.

The stakes *C* and *D* above and below the mills of *Mazurance*, shew the height of the waters when at the highest and lowest, and their difference; these stakes form our third and fourth limits. Lastly, the stakes at *E* and *F*, above and below the mills at *Villebourg*, mark as before the difference between the highest and lowest stations of the water, and are also the last limits of the operation.

Particular care was taken, that the marks should all be made exactly even with the edge of the water, and they were all made at the different parts of the river, as nearly as possible at the same instant of time.

The principal limits of the levelling being now determined and fixed, it only remains to bind the
level

level between the limits, according to the methods already pointed out, using every advantage that may contribute to the success of the work, and at the same time avoiding all obstacles and difficulties that might retard or injure the operations.

The first rule is always to take the shortest possible way from one limit to the other.

However, this rule must not be followed, if there are considerable obstacles in the way, as hills, woods, marshy ground, &c. or, if by going aside, any advantage can be obtained; thus, in the present instance, it was found most convenient, to go from A 2 to B, by the dotted line A c d e f g h i k B, which although it appears the longest, was in effect the shortest, as you have only to level from one pond to the other, at A c, d e, f g, h i, k B, (at the top of the plate) the distances c d, e f, g h, i k, the surfaces of the several ponds being assumed as level lines, thereby abridging the work without rendering it less exact; more so, as it was not the length of the river that was required, but only the declivity.

Having levelled from A to B, proceed from B to C, following the dotted line B l m n o C, whence we obtain the difference in height, between the surface of the water at A, and that at C.

The next step was to level between C and D, above and below the mills, to find the difference between the water when at it's highest and lowest situations. From D, levels were taken across the country to p, leaving p on account of the pond on lake which was assumed as level; we began at q, from thence to r, where we left off; beginning again at s, then levelling from thence to t, and so on to L, above the mills at Villebourg, and finished at F below them.

By

By these operations we obtained the knowledge of how much the waters above and below the mills of *Lignebruk* are higher than those of *Mazurance*, and these than those of *Villebourg*, with all the necessary consequences; from this example, the importance of a thorough knowledge of the ground, in order to carry on such a work, is very evident; this piece of levelling was near five German miles in length in a strait line, and 9 or 10, with the bendings of the river; for the profile or section of the foregoing operation, *fig. 2, plate 2*, first draw the dotted line *A G*, on which let fall perpendiculars from the principal limits *A B C D* produced; then beginning at the highest water mark at *Lignebruk*, set off 3 feet to *b*, for the difference between high and low water; from *b* draw the dotted line *b c*, parallel to *A G*. From the point set off on the perpendicular 4 feet to *B*, the difference found between *b* and *B*; from *B* draw *B d*, parallel also to *A G*; then set off 3 feet from *d* to *E* downwards, for the difference found between *B* and *C*, and $4\frac{1}{2}$ feet from *C* to *D*, for the difference in heights of the mills at *Mazurance*. From *D* draw the line *D e* parallel to *A G*, and from the point *e* to *L*, set off 3 feet for the difference of the level between *D* and *F*. And lastly, from *E* to *F*, set off 1 foot 6 inches for the difference between the higher and lower waters at the mills of *Villebourg*, shewing that there are nineteen feet difference which the upper waters at *Lignebruk* are higher than the lower waters of *Villebourg*.

ASTRONOMICAL OBSERVATIONS AT LAND,
WITH HADLEY'S OCTANT AND SEXTANT.

The portability of the sextant, it's cheapness compared with other astronomical instruments, the ease with which it may be used, the accuracy of the observations that may be made with it, strongly recommend it to the attention of the surveyor, &c. the only addition necessary, in order to employ it with success in astronomical observations at land, is an artificial or reflecting horizon.

The best artificial horizon that can be used at land, is naked quicksilver or water; but as these are always more or less affected by the air, the trough in which they are contained should be covered with a roof, consisting of two parallel planes of glass.

If these are parallel, it matters little at what angle they are set, and if any error be suspected, the frame may be placed sometimes with one side, sometimes with the other foremost, taking a mean of the observations. No observation should be made within half an inch of the sides of the basin, in which the mercury is contained.

All other reflecting surfaces, whether of glass or metal, circular levels, floating planes of glass, &c. are not to be depended upon, and will always give a different altitude or diameter from that observed from the surface of the mercury, or water, as any person may be readily convinced of, by taking the sun's meridian altitude, or it's diameter successively from these different surfaces; this arises from the imperfection of the surface of metal or glass, which has never been ground perfectly *flat*.

The parallelism of a glass may be readily examined, and it's defects easily discovered; whereas the want of a flat surface has scarce been suspected.

The

The parallelism of a glass may be discovered by looking at the moon with it, and receiving her rays in a very oblique manner, so as to make the angle between the direct and reflected rays as obtuse as possible; if it appears single and well defined, the glass is parallel. Or a glass may be accurately examined by laying it on a piece of paper, and viewing the top of a wall or chimney, &c. about 15 or 20 yards distant, for if the two surfaces be not parallel, the object will appear double and surrounded with a light fringe, and thus may every part of the glass be examined, and its defects discovered; the examination will be more perfect, if a small telescope be used.

To examine whether a surface be a perfect plane; take the sun's diameter very accurately with your sextant, when it's altitude is considerable; then examine, in the surface you wish to try, the two images, without altering the index; if it be concave, the two images will lap over, if convex, they will separate, and the quantity of this error may be found by the sextant. If, therefore, you use glass, &c. as a reflecting surface for an artificial horizon, you must either allow for the error, which makes the given altitude too great, if the glass be convex, and too small if concave; or you must make both your adjustments and observations from the same reflecting surface; but this will not entirely obviate the difficulty, as the surface is apt to vary from the sun's heat during a long course of observations.

The angle, observed by means of the artificial horizon, is always double the altitude of the star, &c. above the horizon; consequently, you cannot take an altitude of more than 45° with an octant, or of 60 by the sextant.

Every thing being ready, and the instrument properly

properly adjusted, move backward, till you see the reflected image of the sun in the water. If this image be bright, turn one or more of the dark glasses behind the horizon glass.

Hold now the sextant in a vertical plane, and direct the sight to the sun's image in the artificial horizon. Then move the index till you see the other image reflected from the mirrors come down to the sun's image seen in the horizon, so as to touch, but not pass it; then bring the edges in contact in the middle between the wires of the telescope, as before directed, and the divisions on the arch will shew the double altitude.

Correct the double altitude for the index error, before you halve it. Then to this half altitude add the sun's semi-diameter, and subtract the correction for refraction, and you will have the true altitude of the sun's center above the real horizon.

The altitude of a star must be taken in the same manner as that of the sun: the double altitude must be corrected for the index error, if any, then halved, and this half corrected for refraction, gives the true altitude above the real horizon.

In taking the sun's altitude, whether for the purpose of calculating time, or for double altitudes, it is best to fix the index to some particular division of the instrument with great nicety, and then wait till the sun is risen or fallen to that altitude.

This is much better than observing it's altitude and moving the screw to it, as the screw when thus suddenly moved is very apt to alter a small trifle by the inequality and pressure of the threads, after the hand is removed from it; whereas when it is fixed to some division previous to the observation, it may be repeatedly tried and examined before the observation is taken.

An

An accurate observer will find that the error of his sextant will vary according as he takes it, by moving the index backwards or forwards in taking the $\odot$'s diameter on the quadrantal arch of excess; this is owing to some spring in the index, or inequality in the adjusting screw, which it is very difficult totally to obviate.

The best way to correct this, is always to move the index the same way in making your observations, as you did in taking the error of adjustment; though where a great number of observations are taken, it were best both to settle the adjustment, and take the observations alternately by moving the index backwards and forwards, or by setting the objects open, and making them lap over alternately. A mean of all these will certainly be the most free from error, as the errors will counteract each other.

This may also correct a faulty habit, which an observer may have contracted in forming the contact between the two objects; and though there may seem to be some impropriety in the mode, yet a mean of them will be much nearer the truth than any single observation, where a person prefers seeming to real accuracy.

The lower limb of the $\odot$ or $\lrcorner$ always comes first into contact, when you move the index forward, and the index shews the double altitude of the $\odot$'s upper limb, if the moveable sun is uppermost, but of the lower limb when the moveable sun is lowermost.

At sea, they generally take the altitude of the $\odot$'s lower limb, because it is most natural to bring that to sweep the horizon. But by land, it is most correct to take the altitude of the $\odot$'s upper limb, 1. Because it is highest, and less liable to be affected by refraction. 2. Because the

femi-diameter and refraction are both subtractive; and the operation is more direct, than when one is plus the other minus.

TO TAKE THE ALTITUDE OF THE SUN BY THE
FORE OBSERVATION.

Observations taken by means of the fore horizon glass are called fore observations, because in them both objects are before the observer.

Previous to every observation, the instrument should be examined, in order to see whether the index or horizon glasses be firm, or whether any of the screws be loose; the horizon glass must also be adjusted.

One or two of the dark glasses should be placed before the horizon glass, always proportioning the strength of the shades to the brightness of the sun's rays, that the image may be looked at without injury to the eye.

Hold the quadrant (in a vertical position, the arch downwards) either by the braces or the radii, as may be most convenient, or still better, according to the foregoing directions. Let the eye be at the upper hole in the sight vane, and the lower part of the limb against the breast.

Turn yourself towards the sun, and direct the sight to that part of the horizon that lies directly under it, keeping the quadrant, as near as you can judge, in a plane passing through the sun's center and the nearest part of the horizon, moving at the same time the index with the left hand, so as to bring the image of the sun down toward the horizon; then swing the quadrant round in a line parallel to the line of sight; by this means the image of the sun may be made to describe the arch of a circle, with the convex side downwards. Now if

D d

that

that edge of the sun which is observed, just grazes upon the horizon, or if the horizon just touches it like a tangent, without cutting it, the observation is rightly made, and the degrees and minutes pointed out by the nonius on the arch, shew the apparent altitude of the sun. But if the sun's edge dip below, or cut the horizon, the index must be moved backward; if, on the contrary, it falls short of it, the index must be moved forward, until it just grazes the horizon.

Dr. Maskelyne gives the following advice; that in taking the sun's altitude, the observer should turn his quadrant round upon the axis of vision, and at the same time turn himself upon his heel, so as to keep the sun always in that part of the horizon glass which is at the same distance as the eye from the plane of the quadrant; and that unless care be taken to observe the objects in the proper part of the horizon glass, the measured angles cannot be true. In this method the reflected sun will describe an arch of a parallel circle round the true sun, whose convex side will be downwards, and consequently when by moving the index the lowest point of the arc is made to touch the horizon, the quadrant will stand in in a vertical plane, and the altitude above the visible horizon will be properly observed.

Great care should be taken that the situation of the index be not altered before the quantity it makes is read off.

The observed or apparent altitude of the sun requires three corrections, in order to obtain the true altitude of the sun's center above the horizon.

1. The first correction is to obtain the observed altitude of the sun's center.

All astronomical calculations, respecting the
heavenly

heavenly bodies, are adapted to their centers; but in taking altitudes of the sun, it is usual to bring his lower limb in apparent contact with the horizon. In this case it is evident, that a quantity equal to the semidiameter of the sun must be added to the observed altitude, to give the altitude of his center. But if on any occasion, as from clouds, the altitude of the upper limb be taken, the semidiameter of the sun must be subtracted.

The mean semidiameter of the sun is 16 minutes, which may be taken as a constant quantity in common observations, as the greatest variation from this quantity scarcely exceeds one quarter of a minute.

2. The second correction is, to rectify the errors arising from refraction.

One of the principal objects of astronomy is, to fix the situation of the several heavenly bodies. It is necessary, as a first step, to understand the causes which occasion a variation in the appearance of the place of those objects, and make us suppose them to be in a different situation from what they really are: among these causes is to be reckoned the following.*

The rays of light, in their passage from the celestial luminaries to our eyes, are bent from their true direction by the atmosphere; this bending is called refraction; and they are more or less refracted, according to the degree of obliquity with which they enter the atmosphere, that is, according to the altitude of the object; from this cause, their apparent altitude is always too great; the quantity to be subtracted from the observed altitude may be found in any treatise on navigation.

D d 2

The

* Adams's Astronomical and Geographical Essays.

The following pleasing and easy experiment will give the reader an idea of what is meant by the refraction of the rays of light; a wonderful property to which we are indebted for all the advantages of vision, and the assistance we receive from telescopes, &c.

Experiment. Into any shallow vessel (a basin) put a shilling, and retire to such a distance as that you can just see the farther edge of the shilling, but no more; let the vessel, the shilling, and your eye, remain in the same situation, while an assistant fills up the vessel with water, and the whole shilling will become visible, the rays coming from the shilling being lifted or bent upwards in their passage through the water. For the same reason, a strait stick put partly into water appears bent.

3. The third correction is for the dip or depression of the horizon.

The dip of the horizon is the quantity that the apparent horizon appears below the true horizon, and is principally occasioned by the height of the observer's eye above the water; for as he is elevated above the level of the sea, the horizon he views is below the true one, and the observed altitude is too great, by a quantity proportioned to the height of the eye above the sea: the quantity to be subtracted from the altitude will be found in a table subjoined to this tract.

Mr. Nicholson says, that observers at sea generally chuse to stand in the ship's waist when they take altitudes, because the height of the eye above the water is not so much altered by the motion of the ship; but this is of no consequence, for in rough weather the edge of the sea beheld from a small elevation is made uneven by waves whose altitudes

altitudes amount to 2 or 3 minutes, or more; which circumstance produces as great an uncertainty as the rise and fall of the object seen from the poop when the ship pitches. These are minute causes of error, but not to be disregarded by those who wish to obtain habits of accuracy and exactness.*

MERIDIONAL ALTITUDES.

The meridian altitude of the sun † is found by attending a few minutes before noon, and taking his altitude from time to time: when the sun's altitude remains for some time without any considerable increase, the observer must be attentive to mark the coincidence of the limb of the sun with the horizon, till it perceptibly dips below the edge of the sea. The quantity thus observed is the meridian altitude.

TO TAKE THE ALTITUDE OF A STAR.

Before an observer attempts to take the altitude of a star, it will be proper for him to exercise himself by viewing a star with the quadrant, and learning to follow the motion of the reflected image without losing it, lest he should take the image of some other star instead of that whose altitude he is desirous to obtain. His quadrant being properly adjusted, let him turn the dark glasses out of the way, and then,

1. Set the index of the nonius to the 0 line of the limb.
2. Hold the quadrant in a vertical position, agreeable to the foregoing directions.
3. Look through the sight vane and the trans-

D d 3

parent

* Nicholson's Navigator's Assistant.

† At places where it rises and sets.

parent part of the horizon glass, frant up to the star, which will coincide with the image seen in the silvered part, and form one star; but as soon as you move the index forwards, the reflected image will descend below the real star; you must follow this image, by moving the whole body of the quadrant downwards, so as to keep it in the silvered part of the horizon glass as the motion of the index depresses it, until it comes down exactly to the edge of the horizon.

It is reckoned better to observe close than open, that is, to be well assured that the objects touch each other; and this opinion is well founded, as many persons are near-sighted without knowing it, and see distant objects a little enlarged, by the addition of a kind of penumbra, or indistinct shading off into the adjacent air.

There are but two corrections to be made to the observed altitude of a star, the one for the dip of the horizon, the other for refraction.

RULES FOR FINDING THE LATITUDE; THE SUN'S ZENITH, DISTANCE, AND DECLINATION, AT NOON BEING GIVEN.

The first subject for consideration is, whether the sun's declination be north or south: and secondly, whether the required latitude be north or south. If the latitude or declination be both north or both south, they are said to be of the same denomination; but if one be north and the other south, they are said to be of different denominations. Thirdly, take the given altitude from 90 to obtain the zenith distance.

Rule 1. If the zenith distance and declination be of the same name, then their difference will give the latitude, whose denomination is the same with

with the declination, if it be greater than the zenith distance; but the latitude is of a contrary denomination, if the zenith distance be less than the declination.

Rule 2. If the zenith distance and declination have contrary names, their sum shews the required latitude, whose name will be the same as the declination.

Or: By the two following rules you may find the latitude of the place from the altitude and declination given.

Rule 1. If the altitude and declination are of different names, that is, the one north the other south, add 90 degrees to the declination; from that sum subtract the meridian altitude, the remainder is the latitude, and is of the same name as the declination.

Rule 2. If the meridian altitude and declination are of the same denomination, that is, both north or both south, then add the declination and altitude together, and subtract that sum from 90 degrees, if it be less, and the remainder will be the latitude, but of a contrary name. But if the sum exceeds 90 degrees, the excess will be the latitude, of the same name with the declination and altitude.

EXAMPLES FOR FINDING THE LATITUDE BY
MERIDIAN OBSERVATION.*

Example 1. Being at sea, July 29, 1779, the meridian altitude of the sun's lower limb was observed to be $34^{\circ} 10'$ N. the eye of the observer being 25 feet above the sea. The latitude of the place is required.

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* Nicholson's Navigator's Assistant.

The sun's declination for the third year * after leap year on July 29, is found in Table III. to be $18^{\circ} 46'$ N. the dip for 25 feet elevation is 5 minutes, the refraction for 34° is 1 minute; therefore

Altitude of sun's lower limb $34^{\circ} 10'$ N.

Add semidiameter 16

34 26

Subtract dip and refraction 6

Correct altitude 34 20

Zenith distance or co-alt. 55 40

Subtract declination 18 46 N.

Remains latitude 36 54 S. or of the
contrary name
to the declin.

Example 2. October 26, 1780, sun's meridional altitude, lower limb $62^{\circ} 09'$ S. required the latitude. Height of the eye 30 feet.

1780 is leap year, and the sun's declination in October 26, is $12^{\circ} 45'$ S. The dip for 30 feet elevation is 6 minutes, the refraction for 62° is $\frac{1}{2}$ minute. Therefore,

Sun's

* The annual course of the seasons, or the natural year, consisting of nearly 365 days 6 hours, and the current year being reckoned 365 days, it is evident that one whole day would be lost in four years if the six hours were constantly rejected. To avoid this inconvenience, which, if not attended to, would cause the seasons to shift in process of time through all the months of the year, an additional day is added to the month of February, every fourth year; this fourth year is termed leap year, and is found by dividing the year of our Lord by 4; leap year leaves no remainder; other years are called the first, second, or third years after leap year, according as the remainder is 1, 2, or 3.

Sun's apparent alt. l. l. $62^{\circ} 09' S.$

+ semidiam.—dip and refrac. $9 \frac{1}{2}$

Correct alt. $62 \ 18 \frac{1}{2}$

Zenith dif. $27 \ 41 \frac{1}{2} S.$

Sun's declination $12 \ 45 S.$

Difference is lat. $14 \ 56 \frac{1}{2} N.$ or of the
contrary
name.

Example 3. Jan. 7, 1776, altitude of sun's lower limb, at noon $87^{\circ} 10' S.$ height of the eye 30 feet, required the latitude.

Sun's apparent alt. l. l. $87^{\circ} 10'$

+ semidiam.—dip. 10 refraction being
inconsiderable

Correct alt. $87 \ 20$

Zenith distance $2 \ 40 S.$

Sun's declinat. $19 \ 13 N.$

Sum is latitude $21 \ 53 N.$

Example 4. In the year 1778, July 30, the sun's meridian altitude lower limb was $84^{\circ} 10' N.$ required the latitude: the height of the eye being 30 feet.

Sun's apparent alt. l. l. $84^{\circ} 10'$

+ Semidiam.— 10

Correct alt. $84 \ 20$

Zenith dist. $5 \ 40 N.$

Sun's declination $18 \ 28 N.$

Difference is lat. $12 \ 48 N.$

Example

Example 5. Being at sea in the year 1777. close weather prevented the meridian observation of the sun, but the night proving clear, the northernmost star in the square of the Great Bear was observed to come to it's least altitude $30^{\circ} 10'$.— Required the latitude; the height of the eye being 20 feet.

Star's altitude apparent	$30^{\circ} 10'$
— Dip and refraction	$6 \frac{1}{2}$
Correct altitude	$30^{\circ} 03 \frac{1}{2}'$
Star's co-declination	$27^{\circ} 03' N.$
Sum is latitude	$57^{\circ} 06 \frac{1}{2}' N. \text{ by Prob. II.}$

Example 6. June 11, 1770, the sun's meridian altitude of the upper limb below the pole was observed to be $2^{\circ} 08'$. The height of the observer's eye being 16 feet: required the latitude.

Sun's apparent altitude upper limb	$2^{\circ} 08'$
— dip, refraction, and semidian.	38
Correct alt.	$1^{\circ} 30'$
Sun's declin. $23^{\circ} 08' N.$ it's comp.	$66^{\circ} 52' N.$
Sum is latitude	$68^{\circ} 22' N.$

TO TAKE THE ALTITUDE OF THE MOON AT SEA.

The enlightened edge of the moon, or that edge which is round and well defined, must be brought in contact with the horizon, whether it be the upper or under edge; in other respects the same method is to be used in taking the altitude of the moon as was directed for the sun.

Between new and full moons the enlightened limb

limb is turned towards the west: and during the time from the full to the new moon, the enlightened limb is turned towards the east.

If that telescope which shews objects inverted be used, then the upper edge or limb of the moon will appear the lower, the left side will appear the right, and the contrary.

The wires of the telescope should be turned parallel to the plane of the instrument, as by keeping them in a perpendicular direction, they will serve at night as a guide for holding the sextant in a vertical position, which cannot otherwise at that time be readily ascertained.

The moon is generally bright enough to be seen by reflection from the unsilvered part of the glasses; if not, the telescope must be removed nearer to the plane of the instrument.

The observed altitude of the moon requires four corrections, in order to obtain the true altitude of her center above the horizon.

Correction 1. For the semi-diameter. This is to be found in the nautical almanack, page 7, of every month for every noon and midnight at Greenwich. If the lower limb was observed, add the semi-diameter thus found. If the upper limb was observed, it must be subtracted.

Correction 2. For the dip of the horizon to be subtracted.

Correction 3. For refraction. This is to be subtracted.

Correction 4. The moon's parallax in altitude. This is to be added in the observed altitude. It is to be found in the requisite tables to the nautical almanack.

To

TO DETERMINE THE LONGITUDE AT SEA, BY TAKING THE ANGULAR DISTANCE BETWEEN THE MOON AND ANY CELESTIAL OBJECT.

The latitude is obtained at sea without difficulty, and with as much accuracy as is requisite for nautical purposes; but the motion of the earth on it's axis prevents our ascertaining the longitude with the same facility: and hence it is that most methods of determining the longitude by celestial observation, consist in discovering the difference of apparent time between the two places under consideration.

The angular motion of the moon being much greater than that of any other celestial body, the observation of it's place is much better adapted to discover small differences of time, than similar observations made with any other instrument. The only practical method of observing it's place at sea, is that of measuring the angular distance between it and the sun and a fixed star.

GENERAL DIRECTIONS.

The most obscure, or rather, the least luminous of the two objects must be viewed directly, and the other must be brought by reflexion in apparent contact with it.

The well-defined image of the moon must be always made use of for the contact, even though it should be necessary for that purpose, to make the reflected image pass beyond the other.

In the night-time it is necessary to turn down one or more of the green screens, to take off the glare of the moon, which would otherwise prevent the star from being seen.

To

TO FIND THE DISTANCE BETWEEN THE MOON AND THE SUN.

The central distances of the sun and moon every three hours of time, at Greenwich, on such days as this method is practicable, are set down in the nautical almanack. From these distances you are to compute roughly the distance between their nearest limbs at the time of observation.

Mr. Ludlam says, the moon should be viewed directly through the unsilvered part of the horizon glass, but the sun by reflection, and if it be very bright, from the unsilvered part of the glass.

If the sun be to the left hand of the moon, the sextant must be held with it's face downwards; but with the face upwards, if the sun be to the right hand of the moon.

Set the index to the distance of the nearest limbs of the sun and moon, computed roughly as above; and placing the face of the sextant agreeable to the foregoing rules, direct the telescope to the moon, putting the sextant into such a position, that if you look edge-ways against it, it may seem to form a line passing through the sun and moon, a circumstance that can be only attained by practice, the parent of aptness; then give the sextant a sweep or swing round a line parallel to the axis of the telescope, and the reflected image of the sun will pass by the moon to and fro, so near, that you cannot fail of seeing it.

The nearest edges, or limbs, may now be brought into exact contact, by moving the index, and then using the adjusting screw; observing, first, that on giving the sextant a motion round the axis of the telescope, the images of the sun and moon only touch at their external edges, and that
the

the body of the sun must not pass over, or be upon the body of the moon. And secondly, that the edge of the sun touch the round or enlightened edge of the moon. Then will the index point out the observed or apparent distance of the nearest edges of the sun and moon.

But the observed distance requires several corrections, before the true distance of the centers of the objects, as seen from the center of the earth, can be found.

Correction 1. Is the sum of their semi-diameters, which is to be added, to give the apparent distance of the centers of the sun and moon.

The semi-diameter of the sun for every sixth day, and of the moon for every noon and midnight, at Greenwich, are to be found in the nautical ephemeris: from these their semi-diameters are to be computed at the time of observation, by the rules to be found in the same work.

Correction 2. Is to free the apparent distance of the effect of refraction and parallax, which will then be the true distance of the centers of the sun and moon, as seen from the earth.

For this purpose, two sets of tables, with directions how to use them, are to be found among the requisite tables of the nautical almanack; or by a set of tables, published for that purpose by the Board of Longitude.

TO TAKE THE DISTANCE BETWEEN THE MOON AND SUCH STARS AS ARE SELECTED IN THE NAUTICAL ALMANACK, FOR THE PURPOSE OF FINDING THE LONGITUDE AT SEA.

The distance of these stars from the moon's center for every three hours, at Greenwich, is given in the nautical almanack, from whence their
distance

distance from the enlightened edge may be roughly computed as before.

The star must be viewed directly; the moon is generally bright enough to be seen by reflection from the unsilvered part of the glass; the proper shade to take off the glare of the moon is soon found. When the star is to the left hand of the moon, the sextant must be held with it's face upwards; but if the star be to the right hand of the moon, with it's face downwards.

Set the index to the distance roughly computed, and placing the face of the octant by the foregoing rules, direct the telescope to the star. Then place the sextant so that if seen edge-ways it may seem to form a line passing through the moon and star, and give it a sweep round a line parallel to the axis of the telescope, and the reflected image of the moon will pass so near by the star, that you will see it in the field of the telescope; a proof that the sight is directed to the right star.

The enlightened edge of the moon, whether east or west, must then be brought into contact with the star, by moving the index. To know whether the contact is perfect, let the quadrant gently vibrate in a line parallel to the axis of vision, for the star should just graze the edge, without entering at all within the body of the moon; when this is the case, the index will shew the apparent distance of the moon from the star, which, when corrected, gives the true one.

Correction 1. For the semi-diameter of the moon. This may be found in the nautical almanack for every noon and midnight, at Greenwich; and from thence computed, by the rules there given, for the time of observation. If the observed or enlightened limb be nearest the star, the semi-diameter

diameter thus found is to be added; if the enlightened edge be the furthest from the star, then the semi-diameter is to be subtracted.

Correction 2. Is for refraction and parallax, to be found from the table as directed before for the sun and moon.

These corrections being properly made, you have the true distance of the moon's center from the star, as seen from the center of the earth. From this distance, and the time of observation, the longitude may be found.

The star to be observed is always one of the brightest, and lies in a line nearly perpendicular to the horns of the moon, or her longer axis; but if you have any doubt whether the sight be directed to the proper star, set the index to the supposed distance as before, hold the sextant as near as you can judge, so that it's plane, seen edgewise, may coincide with the line of the moon's shorter axis, and moving it in that plane, seek the reflected image of the moon through the telescope. Having found the reflected image of the moon, turn the sextant round the incident ray, that is, a line passing from the moon to the instrument, and you will perceive through the telescope all those stars which have the distance shewn by the index; but the star to be observed lies in a line nearly perpendicular to the horns of the moon, there will, therefore, be no danger of mistaking it.

TO OBSERVE CORRESPONDING ALTITUDES OF THE SUN.

The basis of all astronomical observation is the determination of the exact time of any appearance in the heavens. By corresponding altitudes this time may be determined, without the apparatus of

a fixed observatory; they are also useful in finding a meridian line, and may be easily and accurately made by a sextant.

For these observations, it is necessary to be provided with a clock. These altitudes should be observed (in our latitude) at least two hours distant from noon. The best time is when the sun is on or near the prime vertical, that is, the east or west points of the compass.

About these times in the forenoon, take several double altitudes of the sun, write down the degrees and minutes shewn on the arch, and also the exact time shewn by the clock at each observation; and let the different observations made in the forenoon be written one below the other in the order they are made.

In the afternoon let the index to the same degree and minute as the last morning observation; note very exactly the time shewn by the clock when the sun is come down to the same altitude, and write down the times on the right hand of the last morning observation; proceed in the same manner to find the time by the clock, of all the altitudes corresponding to those taken in the morning, and write down each opposite to that morning one to which it corresponds.

Take now the first pair of corresponding altitudes, add them together, and to half their sum add six hours: this being corrected for the change of the sun's declination between the morning and evening observations, you will have the time of solar noon derived from this pair of observations. Do the same for each pair, and take the mean of the times thus found (from each pair), and you will have the exact time shewn by the clock at solar noon.

The time by the clock of solar or apparent noon being thus obtained, the time of mean noon may be had by applying the proper equation of time.

Or thus. Add 12 hours to the time of the afternoon observation, from which subtract the time of the forenoon one,* and add half the difference to the time of the forenoon or morning observation; this will give the time of apparent noon nearly.

Having this time nearly, it must be corrected by the table of equations for equal altitudes, on account of the sun's change in declination, in the interval between the observations; and you must also apply the equation of time found in the nautical almanack with contrary signs, subtracting when it is +, and adding when it is —.

Example. Equal altitudes taken June 1782.

East Azimuth.			West Azimuth.					
h.	m.	f.	h.	m.	f.			
a	8	55 46	c	3	8 44			
b	8	57 41	b	3	6 48			
c	8	59 27	a	3	4 58			
h.	m.	f.	h.	m.	f.	h.	m.	f.
15	8	44	15	6	48	15	4	58
8	55	46	8	57	41	8	59	27
6	12	58	6	9	7	6	5	31
3	6	29	3	4	33½	3	2	45½
12	2	15	12	2	14½	12	2	12½

As

* This gives the whole interval between the observations, to which, if your clock is known to vary much in that time, you must add the clock's loss, or subtract it's gain, during that interval, and half this corrected (if necessary) interval must be added to the time of forenoon, &c.

As the seconds differ add them together, 15
and divide the sum by 3, by which you 14½
obtain a mean. 12½

3)42(14

Therefore, h. m. s. the mean.
12 2 14
0 0 0 8 equation for 6 hours.

12 2 14 8
1 55 1 equation of time sub.

12 0 19 7 clock near 20" too
fast for equal time.

MERIDIONAL PROBLEMS BY THE STARS; AND
VARIATION OF THE MAGNETIC NEEDLE BY
THE SUN.*

*How to fix a meridian line by a star, when it can be
seen at it's greatest elongation on each side of the pole.*

Provide two plummets. Let one hang from a
fixt point: let the other hang over a rod, sup-
ported horizontally about 6 or 8 feet above the
ground, or floor in a house, and so as to slide oc-
casionally along the rod: let the moveable plum-
met hang four or five feet northward of the fixt one.
Some time before the star is at it's greatest elonga-
tion, follow it, in it's motion, with the moveable
plummet, so as a person a little behind the fixt
plummet may always see both in one, and just
touching the star. When the star becomes sta-
tionary, or moves not beyond the moveable plum-
met, set up a light on a staff, by signals, about
half a mile off, precisely in the direction of both
the plummets. Near twelve hours afterwards,
when the star comes towards it's greatest elongation
on the other side of the pole, with your eye a foot

E c 2

or

* Mackenzie's Maritime Surveying, p. 47.

or two behind the fixt plummet, follow it with the moveable plummet, till you perceive it stationary as before: mark the direction of the plummets then by a light put up precisely in that line: take the angle between the places of the two lights with *Hadley's* quadrant, or a good theodolite; the center of the instrument at the fixt plummet, and a pole set up in day-light, bisecting that angle, will be exactly in the meridian seen from the fixt plummet.

This observation will be more accurate if the eye is steadied by viewing the plumb-lines and star through a small slit in a plate of brass stuck upright on a stool, or on the top of a chair-back.

This problem depends on no former observations whatever: and as there is nothing in the operation, or instruments, to affect it's accuracy, but what any one may easily guard against, it may be reckoned the surest foundation for all subsequent celestial observations that require an exact meridian line. The only disadvantage is, that it cannot be performed but in winter, and when the stars may be seen for 12 hours together; which requires the night to be about 15 hours long.

How to fix a meridian line by two circumpolar stars that have the same right ascension, or differ precisely 180 degrees.

Pitch on two stars that do not set, and whose right-ascensions are the same, or exactly 180 degrees different; take them in the same vertical circle by a plumb-line, and at the same time let a light be set up in that direction, half a mile, or a mile off, and the light and plummet will be exactly in the meridian.

In order to place a distant light exactly in the direction of the plumb-line and stars, proceed in the
the

the following manner. Any night before the observation is to be made, when the two stars appear to the eye to be in a vertical position, set up a staff in the place near which you would have the plummet to hang; and placing your eye at that staff, direct an assistant to set another staff upright in the ground, 30 or 40 yards off, as near in a line with the stars as you can. Next day, set the two staffs in the same places; and in their direction, at the distance of a mile, or half a mile northward, cut a small hole in the ground, for the place where the light is to stand at night; and mark the hole so, that a person sent there at night may find it.

Then chuse a calm night, if the observation is to be made without doors; (if it is moon-light, so much the better,) and half an hour before the stars appear near the same vertical, have a lighted lantern ready tied to the top of a pole, and set upright in the distant hole marked for it the night before; and the light will then be very near the meridian, seen from the place marked for the plummet. At the same time, let another pole, or rod, 6 or 8 feet long, be supported horizontally where the plummet is to hang, six or seven feet from the ground, and hang the plumb-line over it, so as to slip easily along it either to one hand or the other, as there may be occasion, (or tie a staff firmly across the top of a pole six or seven feet long; fix the pole in the ground, and make the plumb-line hang over the cross-staff). Let the weight at the end of the line be pretty heavy, and swing in a tub of water, so as it may not shake by a small motion of the air. Then shift the plumb-line to one hand or the other, till one side of the line, when at rest, cuts the star which is nearest the pole of the world, and the middle of the light together: as that star

moves, continue moving the plumb-line along the rod, so as to keep it always on the light and star, till the other star comes to the same side of the plumb-line also; and then the plummet and light will be exactly in the meridian.

There are not two remarkable stars near the north pole, with the same right-ascension precisely, or just a semicircle's difference: but there are three stars that are very nearly so, viz. the pole-star, ϵ in Ursa Major, and γ in Cassiopeia. If either of the two last, particularly ϵ , are taken in the same vertical with the pole-star, they will then be so very near the meridian, that no greater exactness need be desired for any purpose in surveying. At London γ is about $1'$ west of the meridian then; and ϵ much nearer it, on the west side likewise. Stars towards the S. pole proper for this observation are, γ , in the head of the cross; α , in the foot of the cross; and α , in the head of the phenix.

How to find a meridian-line by a circumpolar star, when it is at it's greatest azimuth from the pole.

By a circumpolar star is here meant, a star whose distance from the pole is less than either the latitude, or co-latitude, of the place of observation.

1. Find the latitude of the place in which you are to observe the star.
2. Pitch on a star whose declination is known, and calculate it's greatest azimuth from the north, or elevated pole.
3. Find at what time it will be on the meridian the afternoon you are to observe, so as to judge about what time to begin the observation.
4. Prepare two plummets, and follow the star with one of them, (as directed in page 419,) until the star is stationary; then set up a light half a mile,

mile, or a mile off, in the direction of the plumb-line and star, and mark the place of the light in the ground, and also of the plumb-line.

5. Next day set up a pole where the light stood, with a flag flying at it. Then with a theodolite, or *Hadley's* quadrant, set the index to the degree and minute of the star's azimuth from the north, found before; direct, by waving to one hand or the other, an assistant to set up a staff on the same side of the pole with a flag as the pole of the world was from it, so as at the plumb-line these two lines may make an angle equal to the azimuth of the star, and the plumb-line and staff will then be in the meridian.

When any star is descending, it is on the west-side of the pole of the world; while it ascends, it is on the east-side of it.

The pole of the world is always between the pole-star and *Urfa-Major*: so that when *Urfa-Major* is W. or E. of the pole-star, the pole of the world is W. or E. of it likewise.

To find the greatest azimuth of a circumpolar star from the meridian, use the following proportion.

Cosine of the lat. : R :: S. of the star's distance from the pole : S. of it's greatest azimuth.

The greatest azimuth is when the star is above the horizontal diameter of it's diurnal circle.

On the N. side of the equator, the pole star is the most convenient for this observation; for the time when it is at it's greatest elongation from the pole, may be known sufficiently near by the eye, by observing when ϵ in *Urfa-Major* and γ in *Cassiopeia* appear to be in a horizontal line, or parallel to the horizon; for that is nearly the time. Or, the time may be found more precisely by

making fast a small piece of wood along the plumb-line when extended, with a cross-piece at right angles to the top of the upright piece, like the letter T; when the plummet is at rest, and both stars are seen touching the upper edge of the cross-piece, then they are both horizontal. It is another conveniency in making use of this star, that it changes it's azimuth much slower than other stars, and therefore affords more time to take it's direction exact.

On the south-side of the equator, the head of the cross is the most convenient for this observation, being nearest to the S. pole; and the time of it's coming to the horizontal diameter of the diurnal circle, when it appears in a horizontal line with the foot of the cross, or the head of the phenix; and at the equator it's greatest azimuth is one hour sixteen minutes before or after that, as it is E. or W. of the meridian. The pole of the world is between the head of the cross and the last of these stars.

To find when any star will come to the meridian, either in the south or north. Find the star's right ascension, in time, from the most correct tables, also the sun's right ascension for the day and place proposed; their difference will shew the difference between their times of coming on the meridian in the south, or between the pole and zenith; which will be after noon if the sun's right ascension is least, but before noon if greatest. Eleven hours 58 minutes after the star has been on the meridian above the pole, it will come to the meridian north of it, or below the pole.

Given the latitude of the place, and the declinations and right ascensions of two stars in the same vertical line; to find the horizontal distance of that vertical
from

from the meridian; the time one of the stars will take to come from the vertical to the meridian; and the precise time of the observation.

CASE I. When the two stars are northward of the zenith. Plate 16, fig. 4.

Let ZPO be a meridian, Z the place of observation, HO it's horizon, and ZP a vertical circle passing through two known stars, s and S; P the pole, EQ the equator, PsD and PSd circles of declination (or right ascension) passing through these stars: then is ZP the co-latitude of the place, Ps and PS the co-declination of the two stars respectively, and the angle sPS the nearest distance of their circles of right ascension; VO the arc of the horizon between the vertical circle and the meridian; and dQ the arc of the equator between the star S and the meridian. When, in the triangle PZs, the angles PZs and sPZ (measured by the arcs VO and DE) are found, the problem is solved.

Solution. 1. Begin with the oblique-angled triangle P s S; in which are given two sides Ps and PS, the co-declinations of the two stars respectively, and the included angle S Ps; which angle is the difference of their right ascensions when it is less than 180 degrees; but if their difference is more than 180 degrees, then the angle S Ps is equal to the lesser right-ascension added to what the greater wants of 360 degrees. From hence find the angle S s P by the following proportions.

As radius

Is to the co-sine of the given angle (S P s),

So is the tan. of the side opposite the required ang. (Ps)

To the tan. of an angle, which call M.

M. is like the side opposite the angle sought, if the given

given angle is acute; but unlike that side, if the given angle is obtuse.*

Take the difference between the side (P S) adjacent the required angle and M; call it N. Then,

Sine N : sine M. :: tan. of the given angle (S P s) : tan. of the required angle (S s P) which is like the given angle if M is less than the side (P S) adjacent to the required angle; but unlike the given angle if M is greater than P S.

2. Next in the oblique-angled triangle P s Z there are given two sides, P Z the co-latitude of the place of observation; P s, the co-declination of the star s; and the angle P s Z opposite to one of them, which is the supplement of P s S (last found) to 180° : from thence find the angle P Z s opposite the other side, by the following proportion.

Sine of P Z : sine P s Z :: sine P Z s, either acute or obtuse.

This angle P Z s, measured by the arc of the horizon V O, is therefore equal to the horizontal distance of the vertical of the two stars from the meridian. Let the direction of the vertical be taken by a plumb-line and distant light, as before directed, or by two plumb-lines, and marked on the ground. Next day, the degrees and minutes in the arc V O may be added to it by *Hadley's* quadrant, and a pole set up there, which will be in the direction of the meridian from the plumb-line.

3. Last,

* Two arcs, or angles, are said to be like, or of the same kind, when both are less than 90° , or both more than 90° : but are said to be unlike, when one is greater and the other less than 90° : and are made like, or unlike, to another, by taking the supplement to 180° of the arc, or angle, produced in like proportion, in place of what the proportion brings out.

3. Last, in the same triangle $P s Z$, find the angle $s P Z$ between the given sides, by the following proportions.

As Radius

Is to the tan. of the given angle ($P s Z$),

So is the co. s. of the adjacent side ($P s$)

To the co. t. of M .

M is acute, if the given angle and it's adjacent side are like; but obtuse, if the given angle and the adjacent side are unlike.

As the co. t. of the side adjacent to the given angle ($P s$)

Is to the co. t. of the other side ($P Z$),

So is the co. s. M

To the co. s. of an angle, which call N .

N is like the side opposite the given angle, if that angle is acute; but unlike the side opposite the given angle, if that angle is obtuse.

Then the required angle $s P Z$ is either equal to the sum or difference of M and N , as the given sides are like or unlike.

The angle $s P Z$, thus found, added to $s P S$, and their sum subtracted from 180° , will leave the angle $d P Q$, or the arc $d Q$ that measures it, which is the arc of the equator the star S must pass over in coming from the vertical, $Z V$, to the meridian. Which converted into time, and measured by a clock, or watch, beginning to reckon the precise moment that a plumb-line cuts both stars, will shew the hour, minute, and second that S is on the meridian.

Find, by the right ascension of the star and sun, at what time that star should come to the meridian in the north the night of the observation; subtract from it the time the star takes from the vertical to the meridian, and the remainder, corrected by the sun's equation, will be the time when the two stars were in the same vertical.

The

The lowest of the two stars comes soonest to the meridian below the pole; the highest of them comes soonest to that part of the meridian which is above the pole.

The nearer in time one of the stars is to the meridian when the observation is made, it is the better: for then an ordinary watch will serve to measure the time sufficiently exact. It is still more advantageous if one of them is above the pole when the other is below it.

The nearer one of the stars is to the pole, and the farther the other is from it, the more exact will this observation be; because the change of the vertical will be the sooner perceived. For this reason, in north latitudes, stars northward of the zenith are preferable to those that are southward of it.

If the two stars are past the meridian when they are observed in the same vertical, then the arc dQ gives the time S took to come from the meridian to the vertical; and must be added to the time when that star was on the meridian to give the time of the observation: and the arc of the horizon VO must be marked on the ground on the side of the vertical, contrary to what it would have been in the foregoing supposition; that is, eastward *below* the pole, and westward *above* it.

When two stars come to the vertical line near the meridian, it may be difficult to judge on which side of it they are at that time; for determining this, the following rules may serve.

The right ascension of two stars may be either each *less*, or each *more* than 180 degrees, or *one more* and the *other less*.

When the right ascension of each of the stars is *either less, or more*, than 180 degrees, they will come to the same vertical on the *east-side* of the meridian

meridian when the star with the *greatest* right ascension is the *lowest*; but on the *west-side* of the meridian when it is *highest*.

When the right ascension of one of the stars is more than 180 degrees, and that of the other less.

If the right ascension of the highest is less than 180° , but greater than the excess of the other's right ascension above 180° , then they come to the vertical on the *east-side* of the meridian. But if the right ascension of the higher star is less than that excess, they come to the vertical on the *west-side* of the meridian.

If the right ascension of the higher star is *more* than 180 degrees, and that excess is *less* than the right ascension of the lower star, then they come to the vertical on the *west-side* of the meridian; but if the excess of the higher star's right ascension above 180° is *more* than the right ascension of the lower star, then they come to the same vertical on the *east-side* of the meridian.

CASE 2. *When the two stars are in the same vertical southward of the zenith. Plate 16, fig. 3.*

When two stars are observed in the same vertical line *southward* of the zenith (or toward the depressed pole), the operations and solutions are nearly the same as in case 1. For let PZO be a meridian, Z the place of observation, OH it's horizon, ZV a vertical circle passing through the two stars S and s, P the pole, QE the equator, PD and Pd two circles of declination (or right ascension) passing through the two stars respectively; then is ZP the co-latitude of the place, PS and Ps the co-declination of the two stars respectively, the angle SPs the difference of their right ascensions, the angle SZQ (or OV the arc of the horizon which measures it) the distance of the vertical from the meridian; the angle sPZ (or dQ which measures

tures it) the arc of the equator, which s must pass over from the vertical to the meridian, which converted into time, and measured by a watch, will shew when s is on the meridian; and subtracted from the calculated time that the star should come to the meridian, will (when corrected by the sun's equation) give the true time of the observation: $S P s$ is the triangle to begin the solution with; and in the triangle $s P Z$, the angles $s P Z$ and $s Z P$, when found, will give the solution of the problem, as in case 1. For the supplement $s Z P$ to 180° is the angle $V Z O$ (or it's measure $O V$) the horizontal distance of the vertical from the meridian; the angle $s P Z$ (or it's measure $d Q$) is the equatorial distance of the vertical from the meridian: for all which the solution in case 1, properly applied, will serve.

On the south side of the zenith, when the *highest* of the two stars has the *least* right ascension, they come to the same vertical on the *east-side* of the meridian; but when the highest star has the *greatest* right ascension, they come to the same vertical on the *west-side* of the meridian.

PROBLEM. *How to find the sun's amplitude at rising, or setting, and from thence the variation of the magnetic needle, with an azimuth compass.*

Make the needle level with the graduated circle in the box. Then, when the sun's lower edge is a semidiameter above the horizon, take the bearing of it's center (from the N. or S. whichever is nearest) through the sights, making the thread bisect the sun's disk, and that subtracted from 90° , will be the sun's magnetic amplitude, or distance from the E. or W. points by the needle.

Next, calculate the sun's true amplitude for that day, by the following proportion.

6

As

*As the co-sine of the latitude,
Is to radius;*

So is the sine of the sun's declination at setting or rising,

To the sine of his amplitude from the W. or E.
Which will be N. or S. as the sun's declination is N. or S: and the distance in degrees and minutes between the true E. or W. and the magnetic, is the variation of the needle.

An easy and sure way to prevent mistakes, which the unexperienced are liable to in this calculation, is, to draw a circle by hand, representing the visible boundary of the horizon, and on it to mark the several data by guess; then by inspecting the figure it will easily appear how the variation is to be found, whether by addition or subtraction, and on which side of the north it lies. For example:

Plate 16, fig. 5. Suppose the variation was sought at sun-setting. Draw by hand a circle N W S E, to represent your visible horizon: in the middle of it mark the point C, for your station: from C, draw the line C W, to represent the true west; then on the north or south side of that line, according as the sun set northward or southward of the true west, draw the line C $\odot$, representing the direction of the sun's center at setting, and another line C w, for the magnetic west, either on the north or south side of $\odot$, as it was observed to be, and at it's judged distance. Then by observing the situation of these lines, it will easily occur whether the magnetic amplitude and true amplitude are to be added, or subtracted, to give the variation; and on which side of the true north the variation lies. In the present supposition, w $\odot$ is the magnetic amplitude, and $\odot$ W the true amplitude;

amplitude; $\odot W$ therefore must be subtracted from $\odot w$ to give $w W$ the distance of the one from the other: and n , the magnetic N. 90 deg. from w , must be westward of N , the true north.

This method is sufficiently exact for finding the variation; but it is not exact enough for fixing a precise meridian line: because of the uncertainty of the refraction, and of the sun's center: but if the sun ascends, or descends, with little obliquity, the error then will be very little.

How to find the sun's azimuth, and from thence to find the variation of the needle.

First, let the latitude of the place be exactly found. Next let the quadrant be carefully adjusted for observation. Then, two or three hours before or after mid-day, take the altitude of the sun's center as exactly as possible, making the vertical wire of the telescope bisect the sun's disk; and, without altering the plane of the quadrant in the least, move the telescope vertically till you see some distant sharp object on the land, exactly at the vertical wire; and that object will be in the direction of the sun's azimuth when the altitude of its center was taken. If no such object is to be seen, let a pole be set up in that direction, about half a mile off, or as far as can be seen easily.

Next calculate the sun's azimuth by the following rule.

Add the complement of the latitude, the complement of the altitude, and the complement of the sun's declination to 90° together, and take the half of that sum, and note it down: subtract the complement of the declination from the half sum, and take the remainder; then take the complement

ment arithmetical * of the fines of the complement of the altitude, and of the complement of the latitude, and add them together, and to them add the fines of the fore-mentioned half sum and remainder: half the sum of these four logarithms is the co-sine of half the azimuth required. Therefore find by the tables what angle that co-sine answers to; double that angle, and that will be the sun's azimuth from the north.

If the sun's declination is S. in *north latitude*, or N. *south latitude*; in place of taking the complement of the declination to 90° , add 90° to it, and proceed as before.

In south latitudes the azimuth is found in the same manner; only, the sun's azimuth is found from the S.

Then, to find the variation, place your needle below the center of the quadrant, set it level, and find how many degrees the pole, or object, in the sun's azimuth, bears from the north by the needle; and the difference between that and the azimuth found by calculation, is the variation of the needle sought.

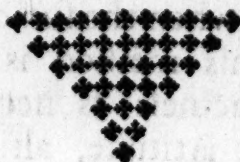
If the sun ascends, or descends with little obliquity, a meridian line may be fixed pretty exactly this way, because a small inaccuracy in the altitude of the sun's center, will not be sensible in the azimuth. But when the sun does not rise high on the meridian, this method is not to be relied on when great exactness is necessary; for then every inaccuracy in latitude, altitude, and refraction,

* The complement arithmetical of a logarithm is found thus: begin at the left-hand of the logarithm, and subtract each figure from 9, and the last figure from 10, setting down the several remainders in a line, and that number will be the arithmetical complement required.

tion, occasions severally a greater error in the azimuth. To mark the meridian line on the ground, place the center of a theodolite, or *Hadley's* quadrant, where the center of the quadrant was when the sun's altitude was taken, and putting the index to the degree and minute of the azimuth, direct (by waving your hat towards one side, or the other) a pole to be set up, making an angle with the former pole placed in the azimuth, equal to the sun's azimuth found; and that last-placed pole will be in the meridian, seen from the center of the quadrant.

N. B. The sun's declination in the tables must be corrected by the variation arising from the difference of the time between your meridian and that of the tables; and also for the variation of declination for the hours before, or after noon, at which the sun's altitude was taken.

The two last problems are constructed and explained in every treatise of navigation; it is therefore needless to be more particular here.



A COURSE OF
PRACTICAL GEOMETRY*

ON THE GROUND,

By ISAAC LANDMAN, PROFESSOR OF FORTIFICATION AND ARTILLERY, TO THE ROYAL MILITARY ACADEMY AT WOOLWICH.



IT is as easy to trace geometrical figures *on the ground*, as to describe them on paper; there is, however, some small difference in the mode of operation, because the instruments are different. A rod or chain is here used instead of a scale; the spade instead of a pencil; a cord fastened to two staves, and stretched between them, instead of a rule; the same cord by fixing one of the staves in the ground, and keeping the other moveable, answers the purpose of a pair of compasses; and with these few instruments, every geometrical figure necessary in practice may be easily traced on the ground.

PROBLEM I. *To draw upon the ground a strait line, through two given points, A, B, fig. 1, plate 27.*

Plant a picket or staff at each of the given points A, B, then fix another C between them, in

F f 2

such

* I have been obliged to omit in this course, so liberally communicated by Mr. Landman, the calculations that illustrate the examples; this, I hope, will not in the least lessen it's use, as this work will fall into the hands of very few who are ignorant of the nature and application of logarithms.

such a manner, that when the eye is placed so as to see the edge of the staff A, it may coincide with the edges of the staves B, and C.

The line may be prolonged by taking out the staff A and planting it at D, in the direction of B and C, and so on to any required length. The accuracy of this operation depends greatly upon fixing the staves upright, and not letting the eye be too near the staff, from whence the observation is made.

PROBLEM 2. *To measure a strait line.*

We have already observed, that there is no operation more difficult than that of measuring a strait line accurately; when the line is short, it is generally measured with a ten foot rod; for this purpose, let two men be furnished with each a ten foot rod, let the first man lay his rod down on the line, but not take it up till the second has laid his down on the line, so that the end may exactly coincide with that of the first rod; now let the first man lift up his, and count one, and then lay it down at the end of the second rod; the second man is now to lift up his and count two, and thus continue till the whole line is measured. Staves should be placed in a line at proper distances from each other, by problem 1, to prevent the operators from going out of the given line.

When the line is very long, a chain is generally used; the manner of using the chain has been already described, page 203.

PROBLEM 3. *To measure distances by pacing, and to make a scale of paces, which shall agree with another, containing fathoms, yards, or feet.*

In military concerns, it is often necessary to take plans, form maps, or procure the sketch of a field of battle, and villages of cantonment, or to reconnoitre

connoitre fortified places, where great accuracy is not required, or where circumstances will not allow the use of instruments; in this case, it is necessary to be well accustomed to measuring by the common pace, which is easily effected by a little practice; to this end measure, on the ground 300 feet, and as the common pace is $2\frac{1}{2}$ feet, or 120 paces in 300 feet, walk over the measured space till you can finish it in 120 paces, within a pace or two.

Example. Let us suppose that we have the map of a country containing the principal objects, as the villages, towns, and rivers, and it be necessary to finish it more minutely by laying down the roads, single houses, hills, rocks, marshes, &c. by measuring with the common pace: take the scale belonging to the map, and make another relative to it, (in the following manner,) whose parts are paces. Let the scale of the map be 200 fathoms, draw a line *AB*, *fig 3, plate 27*, equal to this scale, and divide it into 4 equal parts, *AC*, *CD*, &c. each of which will represent 50 fathoms; bisect *AC* at *E*, divide *AE* into 5 equal parts, *Ae*, *ef*, *fg*, &c. each of which will be 5 fathoms; draw *GH* parallel to *AB*, and at any distance therefrom; then through the points of division *A*, *ef*, *gh*, *E*, *C*, &c. draw lines perpendicular to *AB*, and cutting *GH*, which will be thereby divided into as many equal parts as *AB*. 200 fathoms at $2\frac{1}{2}$ feet per pace, is 480 paces; therefore write at *H*, the last division of *GH*, 480 paces, at *I* 360, at *K* 240, and so on. To lay down on the plan any distance measured in paces, take the number of fathoms from the line *AB*, corresponding to the number of paces in *GH*, which will be the distance to be laid down on the plan.

F f 3

When

When plans or maps are taken by the plain table or surveying compass, this is an expeditious method of throwing in the detail, and a little practice soon renders it easy.

A Table for reducing the common pace of $2\frac{1}{2}$ feet, into feet and inches.

Paces.	Feet.	In.	P.	Feet.	In.	P.	Feet.	In.	P.	Feet.	In.	P.	Feet.	In.
1	2	6	10	40	0	31	77	6	40	115	0	61	152	0
2	5	0	17	42	6	32	80	0	47	117	6	62	155	0
3	7	6	18	45	0	33	82	6	48	120	0	63	157	6
4	10	0	19	47	6	34	85	0	49	122	6	64	160	0
5	12	6	20	50	0	35	87	6	50	125	0	65	162	6
6	15	0	21	52	6	36	90	0	51	127	6	66	165	0
7	17	6	22	55	0	37	92	6	52	130	0	67	167	6
8	20	0	23	57	6	38	95	0	53	132	6	68	170	0
9	22	6	24	60	0	39	97	6	54	135	0	69	172	6
10	25	0	25	62	6	40	100	0	55	137	6	70	175	0
11	27	6	26	65	0	41	102	6	56	140	0	71	177	6
12	30	0	27	67	6	42	105	0	57	142	6	72	180	0
13	32	6	28	70	0	43	107	6	58	145	0	73	182	6
14	35	0	29	72	6	44	110	0	59	147	6	74	185	0
15	37	6	30	75	0	45	112	6	60	150	0	75	187	6

Distances of a certain extent may be measured by the time employed in pacing them; to do this, a person must accustom himself to pace a given extent in a given time, as 600 paces in five minutes, or 120 in one minute; being perfect in this exercise, let it be required to know how many paces it is from one place to another, which took up in pacing 1 hour and $\frac{1}{4}$, or 75 minutes. Multiply 75 by 120, and you obtain 120 the measure required.

This method is very useful in military operations; as for instance, when it is required to know the itinerary of a country for the march of an army, or to find the extent of a field of battle, an encampment, &c. It may be performed very well on horseback, having first exercised the horse so as to make him pace a given space in a determinate time,

time, and this may be effected in more or less time, according as he is trained to walk, trot, or gallop the original space.

PROBLEM 4. *To walk in a strait line from a proposed point to a given object.*

Suppose the point A, fig. 2, plate 27, to be the proposed point from whence you are to set out, B the given object, fix upon another point C, as a bush, a stone, or any mark that you can find in a line with B; then step on in the direction of the two objects B, C; when you are come within 10 or 15 paces of C, find another object D between C and B, but in a line with them; it is always necessary to have two points always in view, in order to walk in a strait line.

This problem is of great use in measuring distances, and surveying by the pace; particularly in reconnoitring either on foot or on horseback. It may also be used when a battalion is ordered to take a position at the distance of 3 or 400 paces, and parallel to that in which it is standing; to effect this, let two non-commissioned officers, who are well exercised in the practice of this problem, step out from the extremities or wings of the battalion, place themselves square with the front of the battalion, then fix upon two objects strait before them, and on a signal given, set out and step the required number of paces, then halt, thus becoming two guides for placing the battalion.

PROBLEM 5. *To place a troop E, F, between two given points A, B, fig. 4, plate 27.*

To perform this, let two persons, with each a pole in his hand, separate about 50 or 60 paces from each other, and then move on till the poles are situated in the direction A, B, i. e. so that the person at C may see the point D in a direction

F f 4

with

with B, at the same time that the person at D sees C in a direction with the object A.

Method 2, fig. 5. To perform the same with one person.

Take a staff OF, *fig. 5, plate 27*, 3 or 4 feet long, slit the top thereof, and place a strait rule, 6 or 7 inches long, in this slit; place this instrument between the two points, and see if A and B coincide with E, then view D from B, and if A coincides therewith, all is right; if not, move the rule forward or backward till it is in the direction of D, E.

PROBLEM 6. *To raise a perpendicular from a point C, to the given line A B, fig. 6, plate 27.*

Set off two points E, D, in A B, equally distant from C; fold a cord in two equal parts, place a staff in the middle at F, fasten the two ends to the staves E and D, then stretch the cord tight, and the point F will be the required point, and the line C F will be perpendicular to A B.

PROBLEM 7. *From a given point F out of the line A B, to let fall a perpendicular C I, fig. 7, plate 27.*

Fold your cord into two equal parts, and fix the middle thereof to the point F, stretch the two halves to A, B, and where they meet that line plant two staves as at E, G, divide the distance E G into two equal parts at C, and the line C F will be perpendicular to A B.

PROBLEM 8. *To raise a perpendicular B C, at the end B of the line B F, fig. 8, plate 27.*

From D taken at pleasure, and with the length D B, plant a staff at the point A, in the direction F B; with the same length set off from D towards C, plant a staff C in the direction D A, and B C will be perpendicular to F B.

Method 2, fig. 9, plate 27. By the numbers 3;

4, 5, or any multiple thereof. Set off from C to D, in the direction C A, 4 feet or 4 yards, and plant a staff at D, take a length of 3 feet and 5 feet, or yards, according as the former measure was feet or yards; fasten one end of the cord at C, and the other at D, then stretch the cord, so that three of these parts may be next to the point C, and five next to D; plant a staff at H, and CH will be perpendicular to CA.

PROBLEM 9. *To draw a line CF, parallel to the line AB, and at a given distance from it. Fig. 10, plate 27.*

At A taken at pleasure in AB, raise a perpendicular AC, then from B taken also at pleasure in AB, draw BF perpendicular to AB; make AC, BF, each equal to the given distance; plant staves at C and F, and in the direction of these two, plant a third E, and the line CEF will be the required line.

PROBLEM 10. *To make an angle abc on the ground, equal to a given angle ABC. Fig. 11 and 12, plate 27.*

Set off any number of equal parts from B to C, and from B to A, and with the same parts measure AC; describe on the ground with these three lengths a triangle abc, and the angle abc thereof will be equal to the angle ABC.

PROBLEM 11. *To prolong the line AB, fig. 13, plate 27, notwithstanding the obstacle G, and to obtain the length thereof.*

At B raise the perpendicular BC, draw CD perpendicular to BC, and ED to CD, make ED equal to BC; then raise a perpendicular EF to ED, and EF will be in a strait line with AB.

The measures of the several lines AB, CD, EF, added together, give the measure of the line AF.

This

This problem is particularly useful in mining.

PROBLEM 12. To draw a line BC , fig. 14, plate 27, parallel to the inaccessible face bc , of the bastion a, b, c, d, e , in order to place a battery at C , where it will produce the greatest effect.

Place a staff at the point A , out of the reach of musket shot, and in a line with bc ; from A draw AG , perpendicular to bA ; set off from A to G about 375 paces, and at G raise GB perpendicular to AG , and produce this line as far as is requisite for placing the battery, i. e. so that the direction of the fire may be nearly perpendicular to one third of the face bc .

PROBLEM 13. To make an angle upon paper equal to the given angle ABC , see fig. 15 and 16, plate 27.

Set off 36 feet from B to C , and plant a staff at C ; set off also 36 feet from B to A , and plant a staff at A , and measure the distance AC , which we will suppose to be equal to 52 feet. Let DC be a given line on the paper; then from C with 36 equal parts taken from any scale, describe the arch DH , then with 52 parts from the same scale, mark off the distance DH ; draw GH , and the angle DGH is equal to the angle ABC .

PROBLEM 14. To measure from the outside an angle ABC , fig. 17, plate 27, formed by two walls AB, CB .

Lay off 30 feet from B to E in the direction AB , and plant a staff at E , set off the same measure from B to F in the direction BC , and measure FE , and you may obtain the measure of your angle, either by laying it down on paper, as in the last problem, or by calculation.

PROBLEM 15. To ascertain the opening of an inaccessible flanked angle BCD , (fig. 18, plate 27,) of the bastion $ABCDE$.

Place

Place a staff at F in the direction of the face B C, but out of the reach of musket shot; make F G perpendicular to F B, and equal to about 250 paces; at G make G M perpendicular to G F, and produce it till it meets, at H, the direction of the face D C; place a staff at I in a line with D H, and the angle I H K will be equal to the required angle B C D, and the value of I H K may be obtained by problem 13.

PROBLEM 16. *To ascertain the length of the line AB, fig. 19, plate 27, accessible only at the two extremities A, B.*

Choose a point O accessible to A and B, draw A O, O B, prolong A O to D, and make O D equal to A O; prolong B O to C, and make O C equal to B O, and C D will be equal to A B, the required line.

PROBLEM 17. *To let fall a perpendicular from the inaccessible point D, upon the right line A B, fig. 21, plate 27.*

From A and B, draw A D, B D, let fall the perpendiculars A H, B F, upon the sides A D, B D, plant a staff at their intersection C, and another E, in the direction of the points C, D, and E C D will be the required perpendicular.

PROBLEM 18. *To find the breadth A B, of a river, see fig. 20, plate 27.*

Plant a staff at C in a line with A B, so that B C may be about $\frac{1}{2}$ of the length of A B, draw a line C E in any direction, (the longer the better). Set up a staff at D, the middle of C E, find the intersection F of E B, D A; draw D G parallel to B C; measure B F, F E; and you will find A B by the following proportion, as F E — B F : $\frac{1}{2}$ B C or D G :: B F : A B.

Inaccessible distances may be obtained in other modes,

modes, when circumstances and the ground will permit, as in the following problem.

PROBLEM 19. *Let it be required to measure the distance A B, of the produced capital of the ravelin, from the head of the trenches B, to the angle A, fig. 22, plate 27.*

Draw B E perpendicular to A B, and set off from B to E 100 feet, produce B E to C, and make E C equal one eighth or tenth part of B E; at C raise the perpendicular C D, plant staves at E and C, then move with another staff on C D, till this staff is in a line with E and A. Measure C D, and you will obtain the length of A B by the following proportion, as $EC : CD :: BE : AB$; thus if B E = 100 feet, E C = 10 feet, and C D = 38 feet, then as $10 : 100 :: 38 : 380$ feet, the distance of A B.

PROBLEM 20. *To measure the inaccessible distance A B, fig. 23, plate 27.*

Plant a staff at C, a point from whence you can readily see both A and B. By the preceding problems, find the length of C A, C B, make C D as many parts of C A, as you do C E of C B, and join D E; then as $CD : DE :: CA : AB$.

PROBLEM 21. *To determine the direction of the capital of a bastion produced. Fig. 24, plate 27.*

Upon the produced lines B D, B E, of the two faces C B, A B, place the staves E and D; find the lengths

* *An easy method of finding the distance of forts or other objects.*
—If you go off at any angle as 90, and continue in that direction until you bring the object and your 1st station under an angle of 63, the distance measured from the 1st station, is equal to half the distance the object is from your first station. But should the ground not admit of going off at an angle of 90, go off at an angle of 45, and whatever distance in that direction you find the object and the 1st station under an angle of $106\frac{1}{2}$, is half the distance of the object and 1st station. See also page 213 of these essays.

lengths of EB and DB , by problem 19, and measure DE . Then as the capital GB , divides the angle ABC , and it's opposite EBD , into two equal parts, we have as $DB:BE::DF:EF$, and as $DB+BE:BE::DE:EF$, and consequently the point F , which will be in the direction of the capital GB produced.

PROBLEM 22. *Through the point C, fig. 25, plate 27, to draw a line IK , parallel to the inaccessible line AB .*

Assume any point D at pleasure, find a point E in the line AD , which shall be at the same time in a line with CB ; from E draw EG , parallel to DB , and through G draw CGF , parallel to AD , meeting BD in F . Plant a staff at H in the line EG , so that H may be in a line with FA , and a line I, H, C, K , drawn through the points B, C , will be parallel to AB .

PROBLEM 23. *To take a plan of a place A, B, C, D, E , fig. 26 and 27, plate 27, by similar triangles.*

1. Make a sketch of the proposed place, as fig. 27. 2. Measure the lines AB, BC, CD, DE, EA , and write the measures obtained upon the corresponding lines a, b, c, d, e, a , of the sketch. 3. Measure the diagonal lines AC, AD , and write the length thereof on a, c, a, d , and the figure will be reduced to triangles whose sides are known. 4. To obtain a plan of the buildings, measure BG, BF, GH, IK, FK , &c. and write down the measures on the sketch. 5. Proceed in the same manner with the other buildings, &c. 6. Draw the figure neatly from a scale of equal parts.

PROBLEM 24. *To take a plan of a wood, or marshy ground, by measuring round about it, fig. 28 and 29, plate 27.*

Make

Make a rough sketch, *fig. 29*, of the wood, set up staves at the angular points, so as to form the circumscribing lines AB , BC , CD , and measure these lines, and set down the measures on the corresponding lines in the sketch, then find the value of the angles by problems 13 and 14.

PROBLEM 25. *To take a plan of a river, fig 30, and 31, plate 27.*

1. Draw a sketch of the river. 2. Mark out a line AB . 3. At C upon AB , raise the perpendicular CH . 4. Measure AC , CH . 5. Measure from C to D , and at D make DI perpendicular to AB , and measure DI . 6. Do the same the whole length of AB , till you have obtained the principal bendings of the river, writing down every measure when taken on it's corresponding line in the sketch, and you will thus obtain sufficient data for drawing the river according to any proportion.

PROBLEM 26. *To take a plan of the neck of land, A, B, C, D, E, F, G, fig. 32 and 33, plate 27.*

Take a sketch of the proposed spot, divide the figure into triangles ABC , ACD , ADE , AEG , by staves or poles placed at the points A , B , C , D , E , F , G , measure the sides AB , BC , AC , CD , AD , AE , AF , FG , AG of the triangles, writing down these measures upon the corresponding lines $a b$, $b c$, $a c$, &c. then measure AH on AB , and at H raise the perpendicular HI , and measure it's length; do the same at K , M , &c. writing down the measures obtained on their corresponding lines $a h$, $h i$, $i k$, $k l$, &c. Proceeding thus, you will ascertain a sufficient number of points for laying down your plan by a scale of equal parts.

PRACTICAL

PRACTICAL TRIGONOMETRY.

PROBLEM 27. *To ascertain the height of a building, fig. 34, plate 28.*

Measure a line F E from the foot of the building, (so that the angle C D A may be neither too acute nor obtuse,) thus, suppose E F = 130 feet, place your theodolet at D, and measure the angle A D C = $34^{\circ} 56'$.

Then as radius,

To tangent $34^{\circ} 56'$

So is E F = 130 feet,

To A C.

Which by working the proportion, you will find to be 89 feet 66 parts, or 89 feet 7.92 inches, to which adding 4 feet for D E, or it's equal C F, you obtain the whole height 93 feet 7.92 inches.

PROBLEM 28. *To find the angle formed by the line of aim, and the axis of a piece of ordnance produced, the calliber and dimensions of the piece being known. Fig. 35, plate 28.*

Suppose the line B I to be drawn through B, the summit of the swelling of the muzzle, and parallel to C D the axis of the piece; the angle A B I, will be equal to the angle A E C, formed by the line of aim A F, and the axis C D; then in the right-angled triangle A I B, we have the sides A I, and B I, to find the angle A B I, which we obtain by this proportion; as I B : A I :: radius, tangent of angle A B I, equal to A E C required.

PROBLEM 29. *The elevation (3 degrees) of a light twelve pounder being given, to find the height to which the line of aim rises at the distance of 1200 yards, which is about the range of a twelve pounder, with an elevation of 3 degrees. Fig. 35, plate 38.*

The line of aim (which we suppose to be found by

by the preceding problem) forming with the axis of a light twelve pounder, $1^{\circ} 24'$ will make with the horizon an angle of $1^{\circ} 36'$; thus the height at the horizontal distance of 1200 yards, will be the second side of a right-angled triangle, where the angle adjacent to the side of 1200 yards is $1^{\circ} 36'$, and may therefore be obtained by the following proportion; as radius, to the tangent of $1^{\circ} 36'$, so is 1200, to 33 yards 1 foot 6 inches = FG.

PROBLEM 30. *The first embrasure A, fig. 36, plate 28, of a ricochee battery being direct, to find the inclination of the seventh embrasure B, i. e. the angle formed by the line of direction BC, and the breast work AB, at the seventh embrasure AB; it is supposed that all the pieces are directed towards a point C, at the distances of 1500 feet.*

The line of direction AC of the first embrasure, is supposed to be perpendicular to the breast work AB; therefore we have to find the angle ABC, of the right-angled triangle BAC, in which the right angle is known AC = 1500 feet, and AB is determined by the size, the distance, and number of embrasures; thus suppose the distance from the middle of one embrasure to another be 20 feet, this multiplied by 6, will be 120 feet, and equal to AB; then as AB to AC, so is radius to the tangent of angle ABC, $85^{\circ} 26'$.

PROBLEM 31. *As the hurler DE, fig. 37, plate 28, is always perpendicular to the directing line of the gun, and as at least one end of it ought to be laid against the breast work, it will make an angle ADF, which was found by the preceding problem to be $85^{\circ} 26'$; therefore knowing the length DE of the hurler, and consequently it's half DF, it will be easy to calculate the distance BF from*

from the breast-work where the middle F of the hurler ought to be placed, upon the line of direction of the gun.

PROBLEM 32. *To ascertain the height of a building, fig. 38, plate 28, from a given point, from whence it is impossible to measure a base in any direction, the points A and F being supposed to be in the same horizontal line.*

Measure the angles CED, AED, let CED be equal $33^{\circ} 12'$, the angle AED equal $2^{\circ} 26'$, and the height EF from the center of the instrument to it's foot, 5 feet; then in the right-angled triangle ADE we have $AD = EF = 5$ feet, the angle DEA = $2^{\circ} 26'$ to find DE, which may be obtained by the following proportion, as radius : cotangent of angle DEA, so is AD to DE 118 feet; then in the right-angled triangle CDE, we have DE 118 feet, angle CED $33^{\circ} 12'$ to find CD, which is found by the following proportion, as radius to tangent of angle CED, join DE to DC, 77 feet, which added to EF 5 feet, is 82 feet, the height of the tower.

PROBLEM 33. *The distance AC, fig. 39, plate 28, (135 toises) from the point C, to the flanked angle of the bastion being given, and also AB (186 toises) the exterior side of the polygon, to find BC.*

1. Find the angle B by the following proportion; as AB is to the sine of angle C, so is AC to the sine of angle B, $39^{\circ} 8'$; because it is plain from the circumstances of the case, that B must be acute, and therefore angle BAC is also known. 2. BC is found by the following proportion, as sine angle C is to AB, so is sine of angle BAC : BC, 213 toises 3 feet.

PROBLEM 34. *To find the height of the building AC, fig. 40.*

G g

1. At

1. At B, measure the angle FBC in the direction FB . 2. Set off any distance BD as a base. 3. Measure the angle BCD , BCD is the supplement of FBC , and BCD is the supplement of $CBD + BDC$.

Then as $\sin. \angle BCD : BD :: CD : BC$, BC being found, we have in the right-angled triangle FBC , the side BC and angle FBC , to find FC , which is found by this proportion; as radius to $\sin.$ angle FBC , so is $BC : FC$, FC added to AF , the height of the instrument gives the height of the tower.

PROBLEM 35. *To ascertain the distance between two inaccessible objects, C, B, fig. 41, plate 28.*

1. Measure a base AB , from whose extremities you can see the two objects C, D . 2. Measure the angles CAB, DAB, DBA, CBA, CBD . 3. In the triangle BAC , we have the side AB , and angles ABC, BAC , to find BC , which is found by the following proportion, as $\sin.$ angle $ACB : AB :: \sin. BAC : BC$. 4. The angles CAB, CBA , added together and subtracted from 180 , gives the angle ACB . 5. In the triangle ABD , we have the angles DAB, ABD , and consequently ADB , and the side AB , to find BD , which is found by the following proportion, as the $\sin.$ of angle ADB is to AB , so is the $\sin.$ of angle $DAB : BD$. 6. In the triangle BCD , we have the two sides BC, BD , and the angle CBD , to find the angle CDB , and the side CD ; to find CD we use this proportion, as the sum of the two given sides is to their difference, so is the tangent of half the sum of the two unknown angles, to the tangent of half their difference; the angle CDB being found, the following proportion will give CD ;

as

as sine angle CDB to BC , so is sine angle CBD to CD .

PROBLEM 36. *To draw a line through the point B, parallel to the inaccessible line CD , fig. 41, plate 28.*

Find the angle BCD by the preceding problem, and then place your instrument at B , and with BC make an angle CBE equal BCD , and EB will be parallel to CD .

PROBLEM 37. *To ascertain several points in the same direction, though there are obstacles which prevent one extremity of the line being seen from the other, fig. 42, plate 28.*

1. Assume a point C at pleasure, from which the two extremities of the line AB may be seen.
 2. Measure the distances CB , CA , and the angle ACB , and find the value of the angle CAB .
 3. Measure the angle ACD , and then in the triangle ACD , we have the side AC , and the two angles $DA C$, ACD , and therefore the third to find CD , and as sine angle ADC to AC , so is sine angle CAD to CD .
 4. Set off CD (making with AC an angle equal the angle ACD) equal the measure thus found, and the point D will be in a line with AB ; and thus as many more points, as G , may be found as you please; in this manner a mortar battery may be placed behind an obstacle, so as to be in the direction of the line AB .

Thus also, you may fix the position of a ricochee battery, M , fig. 43, so as to be upon the curtain AD produced.

PROBLEM 38. *To measure the height of a hill, whose foot is inaccessible, fig. 44, plate 28.*

1. Measure a base FG , from whose extremities the point A is visible. 2. Measure the angles ABC , ACB , ACD . 3. In the triangle ABC , we have BC , and the angles ABC , ACB , to find

$Gg2$

AC ;

A C; but as sine angle B A C is to B C, so is sine angle A B C to A C. 4. In the right-angled triangle A D C, we have the side A C, and the angle A C D, to find A D; but as radius is to sine angle A C D, so is A C to A D, the height required.

PROBLEM 39. *To take the map of a country, fig. 45, plate 28.*

First, choose two places so remote from each other, that their distance may serve as a common base for the triangle to be observed, in order to form the map.

Let A, B, C, D, E, F, G, H, I, K, be several remarkable objects, whose situations are to be laid down in a map.

Make a rough sketch of these objects, according to their positions in regard to each other; on this sketch, the different measures taken in the course of the observations are to be set down.

Measure the base A B, whose length should be proportionate to the distance of the extreme objects from A and B; from A the extremity of the base, measure the angles E A B, F A B, G A B, C A B, D A B, formed at A with the base A B.

From B, the other extremity of the base, observe the angles E B A, F B A, G B A, C B A, D B A.

If any object cannot be seen from the points A and B, another point must be found, or the base changed, so that it may be seen, it being necessary for the same object to be seen at both stations, because it's position can only be ascertained by the intersection of the lines from the ends of the base, with which they form a triangle.

It is evident from what has been already said, that having the base A B given, and the angles observed, it will be easy to find the sides, and from them

them lay down, with a scale of equal parts, the several triangles on your map, and thus fix with accuracy the position of the different places.

In forming maps or plans where the chief points are at a great distance from each other, trigonometrical calculations are absolutely necessary.

But where the distance is moderate, after having measured a base and observed the angles, instead of calculating the sides, the situation of the points may be found by laying down the angles with a protractor; this method, though not so exact as the preceding, answers sufficiently for most military operations.

PROBLEM 40. *The use of the surveying compass, (fig. 3, plate 15,) in determining the particulars of various objects to be inserted in a plan; see fig. 46, plate 28.*

The first station being at A, plant staves at the requisite places, and then place the compass at A, directing the telescope to C; observe the number of degrees north A C made by the needle and the telescope, (or it's parallel line drawn through the center of the compass) and mark this angle in your sketch, see fig. 46; observe and mark in the same manner the other angles, north A O, north A P, north A Q; then measure A C, and at the second station C, observe the angles north C M, north C Q, north C O, north C P, north C D, and so on at the other stations; in observing the angles, attention must be paid when the degrees pass 180, to mark them properly in the sketch, in order to avoid mistakes in protracting.

PROBLEM 41. *To raise perpendiculars, and form angles equal to given angles by the surveying compass, fig. 47, plate 28.*

Let it be required to trace out the field work A D C B, at the head of a bridge.

The direction of the capital F E being given, describe the square A B C D in the following manner; place your compass at F, and as A B is to be perpendicular to F E, direct the telescope to E, and observe, when the needle is at rest, the number of degrees it points to; then turn the compass box upon it's center, till the needle has described an arc of 90° , and place a staff in the direction of the telescope towards A, and A F will be perpendicular to F E; continue A F towards B, and make A F, B F, each equal 30 toises at E; in the same manner raise the perpendicular D C, and make E D, E C, each equal 30 toises; join D A, C B, and you have the square A B C D.

Make C G, D K, A O, each equal to $\frac{1}{4}$ of A B, through G and K draw the line G K, set off G H, K I, each equal to $\frac{1}{4}$ of A B or C D, draw the lines of defence I C, H D, place the compass at I, and the telescope in the direction I H; observe where the needle points when at rest, and turn the compass till the needle has described an arc of 100, the value of the angle L I H of the flank, place a staff at L in the direction of the telescope, and at the same time in the line I H, which gives the length of the face D L and the flank I L; the face C M and flank H M are ascertained in the same manner.

Make B Q equal to $\frac{1}{6}$ of A B, and A R equal to $\frac{A B}{10}$, and draw D R, C Q, through Z the middle of A D, draw O Z, make R I equal to $\frac{1}{4}$ A B; at T and with T D, form an angle D T U, equal to 105° ; plant a staff at U, so that it may be in the line Z O, and at the same time in the direction T U, which gives the flank T U and face O U.

Make Q S equal $\frac{1}{3}$ A B, at S form an angle C S P equal to 120° , draw the line S P, meeting the river,
and

and the lines O, U, T, D, L, I, H, will be the line of the tete de pont required.

THE USE OF THE PLANE TABLE IN MILITARY OPERATIONS.

PROBLEM 42. *To take the plan of a camp, fig. 1, plate 29.*

Place the table at A, where you can conveniently see the greater part of the field, and having made a scale on it, fix a fine needle perpendicular to the table at the place that you fix upon to represent the point A; the fiducial edge of the index is always to be applied against the needle.

Turn the plain table so that the index may point to the object B, and be so situated as to take in the field; then plant a staff at C, in a line with B, point the index to the windmill F, and draw the indefinite line A F, then point it to K, the right wing of the cavalry K L, and draw A K, then to L, and draw A L, afterwards point the index to the steeple I, and then to the points M, O, P, N, H, E, G, then draw a line on the table parallel to the north and south of your compass, to represent the magnetic meridian.

Remove the plane table to C, planting a staff at C, measure AC, and set off that measure by your scale from A upon the line A C, and fix the needle at C; then set the index upon the line A C, and turn the table till the line of sight coincides with A, fasten the table, point the index to F, and draw C F, intersecting A F in F, and determining the position of the windmill F; from C draw the indefinite lines C M, C K, C L, &c. which will determine the points M K L, &c. draw K L and m n parallel thereto, to represent the line of cavalry.

G g 4

Remove

Remove the table from C to D, setting up a staff at C, measure C D, and set off the distance on C B from your scale, place the needle at D, the index on C D, and turn the table till C coincides with the sights, and take the remarkable objects which could not be seen from the other stations.

Staves should be placed at the sinuosities of the river, and lines drawn at the stations A, C, D, B, to these staves, which will give the windings of the river.

Having thus determined the main objects of the field, sketch on it the roads, hills, &c.

PROBLEM 43. *To take a plan of the trenches of an attack, fig. 2, plate 29.*

The plan of trenches taken with accuracy, gives a just idea of the objects, and shews how you may close more and more upon the enemy and be covered from their enfilade fire, and also how to proceed in the attack without multiplying useless works, which increase expence, augment the labour, and occasion a great loss of men.

Measure a long line A B, parallel to the front of the attack D F, place the table at A, and set up a staff at B, point the index to B, and draw a line to represent A B, fasten the table, and fix a needle at the point A, direct the line of sights to the flanked angle of the ravelin C, and draw A C; proceed in the same manner with the flanked angles D, E, F, G, to draw the lines at the opening of the trenches, plant staves at H and R, from A draw a line on the table in the direction A H, measure A H, and set off that measure from your scale upon the line A H on the table.

Remove the plane table from A to B, set up a staff at A, lay the fiducial edge of the index against the line A B, and turn the table about till the staff
at

at A coincides with the line of sight, then fasten the table, direct the sights to C, and draw BC, intersecting A at C; in the same manner ascertain the flanked angles D, E, F, G, draw a line in the direction BR, and set off the measure thereof from your scale.

Remove the table from B to R, and set up a staff at B where the plain table stood, lay the index upon the line corresponding with R B, then turn the table about till the line of sight is in the direction B R, screw the table, direct the sights towards P, and draw on the table the line R P, and by the scale lay off it's measure on that line.

Remove the plain table from R to P, lay the index upon the line P R, and turn the table about till the line of sight coincides with R, screw the table fast, and draw a line upon it in the direction P Q; measure P Q, and take the same number of parts from your scale, and set it off on the line, and so on with the other station Q.

Having removed the table to the station S, and duly placed it with regard to Q; from the point S, draw the lines S V, S T, S U, setting off from your scale their lengths, corresponding to their measures on the ground.

Having thus taken the zigzags R P Q, S T W, and the parts U U W Z of the parallels, you remove the plain table to H, and proceed in a like manner to take the zigzags H I K, &c. as above, which will represent on your plain table, the plan of the attack required.

OF LEVELLING, *fig. 48, plate 28.*

Levelling is an operation that shews the height of one place in respect to another; one place is said to be higher than another, when it is more distant from

from the center of the earth than the other; when a line has all it's points equally distant from the center, it is called the line of true level, whence because the earth is round, that line must be a curve, and make a part of the earth's circumference, as the line A B E D, all the points of which are equally distant from the center C of the earth; but the line of sight A G, which the operation of levelling gives, is a right line perpendicular to the semi-diameter of the earth C A, raised above the true level, denoted by the curvature of the earth, and this in proportion as it is more extended; for which reason, the operations which we shall give, are only of an apparent level, which must be corrected to have the true level, when the line of sight exceeds 300 feet.

Suppose, for example, that A B was measured upon the surface of the earth to be 6000 feet, as the diameter of the earth is 41825024 feet, you will find B F by the following proportions.

* $41825024 : 6000 :: 6000 : B F$ equal to 0,86096 which is 10 in. 33152, that is to say, between two objects A and F, 6000 feet distant from each other, and in the same horizontal line, the difference B F of the true level, or that of their distance from the center of the earth, is 10 in. 32874.

When the difference between the true and apparent level, as of B F, has been calculated, it will be easy to calculate those which answer to a less distance, for we may consider the distances B F, b f, as almost equal to the lines A I, A i, which are

* As the arc A B = 6000 feet is but very small, it may be considered equal to the tangent A F, and in this respect, A F is a mean proportional between the whole secant B C, and the exterior part B F.

are to each other as the squares of the chords, or of the arcs AB , ab , because in this case the chords and the arcs may be taken one for the other.

Thus to find the difference fb of level, which answers to 5000 feet, make the following proportion; $6000 \text{ f.} : 5000 \text{ f.} :: 0,86096 : fb$, which will be equal to 0,59789, or 7 in. 17468.

The point F which is in the same horizontal line with A , is said to be in the apparent level of A , and the point B is the true level of F ; so that BF is the difference of the true level from the apparent.

PROBLEM 44. *Fig. 49, plate 28.*

The above notions being supposed to know the difference of level between points B and A , which are not in the same horizontal line; then at A , make use of an instrument proper to take the angle BCD , and having measured the distance CD , or CI , by a chain which must be kept horizontal in different parts of it on the ground $ALVB$, you may in the triangle CDB , considered as rectangular in D , calculate BD , to which add the height CA of the instrument, and calculate the difference of level DI , as we have shewn above.

But as this method requires great accuracy in measuring the angle BCD , and an instrument very exact, it is often better to get at the same end with a little more trouble, which is shewn by the following method.

PROBLEM 45. *Use of the spirit or water level, fig. 50, plate 28.*

Place the level at E , at equal distances from B and G , fix one station staff at B , the other at G ; your instrument being adjusted, look at B , and let the vane be moved till it coincides with the line of sight; then look at the staff G , and let the vane be

be moved till it coincides with the line of sight H, and the difference in height shewn by the vanes on the two staves, will be the difference in the level between the two points B and G. Thus, suppose the vane at G was at 4 f. 8 in. and at B 3 f. 9 in. subtract one from the other, and the remainder 11 inches, will be the difference in the level between the two points B and G; you may proceed in the same manner with the other points, but more need not be said on this head, as I have already treated this subject very fully in the foregoing part of this work.

DEFINITIONS.

Definition 1. Perspective is the art of delineating the representations of bodies upon a plane, and has two distinct branches, linear and aerial.

Definition 2. Linear perspective shews the method of drawing the visible boundary lines of objects upon the plane of the picture, exactly where those lines would appear if the objects were transparent; this drawing is called the outline of those objects in perspective.

Definition 3. Aerial perspective gives rules for drawing the objects with colours and shades, so that they may appear to recede into the distance, and to be affected by the atmosphere.



AN
ESSAY ON PERSPECTIVE,
 AND A DESCRIPTION OF SOME INSTRUMENTS, FOR
 FACILITATING THE PRACTICE OF THAT USEFUL
 ART.

DEFINITIONS.

Definition 1. Perspective is the art of delineating the representations of bodies upon a plane, and has two distinct branches, linear and aërial.

Definition 2. Linear perspective shews the method of drawing the visible boundary lines of objects upon the plane of the picture, exactly where those lines would appear if the picture were transparent ; this drawing is called the outline of those objects it represents.

Definition 3. Aërial perspective gives rules to fill up this drawing with colours, lights and shades, such as the objects themselves appear to have, when viewed at that point where the eye of the spectator is placed.

To illustrate these definitions, suppose the picture to be a plate of glass inclosed in a frame PLN , *fig. 1, plate 30*, through which let the eye of the spectator, placed at E , view the object $QRST$; from the given point E , let the visual lines EQ , ER , ES , &c. be drawn, cutting the glass (or picture) in q , r , s , &c. these points of intersection, will be the perspective representations of

of the original $QRST$, &c. and if they are joined, $qrst$ will be the true perspective delineation of the original figure. And lastly, if this out-line be so coloured in every part, as to deceive the eye of a spectator, viewing the same at E , in such a manner, that he cannot tell whether he views the real object itself, or it's representation, it may be truly called the picture of the object it is designed for.

Definition 4. When the eye, or projecting point, is supposed at an indefinitely great distance, compared with the distance of the picture and the object to be represented, the projecting lines being then supposed parallel, the delineation is called a parallel one, and by the corps of engineers, *military perspective*.

Definition 5. If this system of parallel rays be perpendicular to the horizon and to the picture, the projection is called a plane.

Definition 6. If the parallel rays be horizontal, and the picture upright, it is called an elevation.

Definition 7. A right line Ee , *fig. 2, plate 30*, from the eye (E) cutting the plane of the picture at right angles, and terminating therein (at e) is called the distance of the picture.

Definition 8. The point e , where this central ray cuts the picture, is called the center of the picture.

Definition 9. The point X , *fig. 2, plate 30*, where any original line cuts the picture, is called the intersection of that line.

Definition 10. The seat (F , *fig. 2, plate 30*,) of any point E upon a plane, is where a perpendicular from that point cuts the plane; thus, if a perpendicular EF , be drawn from any elevated point E , to the ground plane FGN at F , this point

point F is called the seat of the point E upon the ground plane.

Definition 11. Apparent magnitude is measured by the degree of opening of two radials passing through the extremes of bodies, whose apparent magnitudes are compared; thus, the apparent magnitude of Q R, *fig. 3, plate 30*, to an eye at E, is measured by the optic angle Q E R. Hence it is evident, that all objects viewed under the same angle, have the same apparent magnitude.

Definition 12. The interfection G N, *fig. 2, plate 30*, of the picture with the ground plane, is called the ground line.

Definition 13. A plane passing through the eye, and every where parallel to the ground, (or ground plane, as it is commonly called, because it is supposed every where flat and level,) is called the horizontal plane, as E h n, *fig. 2, plate 30*.

Definition 14. The interfection of the horizontal plane with the picture, is called the *horizontal line*; thus h e n, *fig. 2, plate 30*, is called the horizontal line, being the interfection of the horizontal plane E h e n, parallel to the ground plane F G N.

Definition 15. The center of any line is where a perpendicular from the eye cuts it.

Definition 16. A plane i E F g b, *fig. 2, no. 2, plate 30*, passing through the eye E, perpendicular to the ground, is called the *vertical plane*; and that part of the ground plan g f N which lies to the left hand, is called amplitudes to the left, and all that lying on the other side, amplitudes to the right, their measures being taken on the base line G F N, or it's parallels, as depths are by fg or it's parallels.

Proposition 1. Parallel and equal strait lines

appear less, as they are farther removed from the eye.

Let OP , QR , *fig. 3, plate 30*, be two equal and parallel strait lines viewed at the point E ; QR being the farthest off, will appear the least.

For draw EP , and ER , cutting OP in r ; then QR and Or have the same apparent magnitude, (*definition 11*,) but Or is only a part of OP .

Corollary. Similar figures parallelly situated, appear less, the farther they are removed from the eye.

Proposition 2. All original parallel strait lines, PR , *fig. 3, plate 30*, which cut the picture (at P), appear to converge to the same point O therein; viz. that point which is the intersection with the picture, and a line passing through the eye parallel to the original parallel lines.

For since PR is parallel to EOQ , let RQ be parallel to OP , and therefore equal to it. By the last proposition, the farther RQ is taken from OP , the nearer it (the representation of the point R) approaches to the fixt point O , and the case is the same with any other line parallel to PR ; and therefore, if all the parallels be indefinitely produced, that is at least till the strait lines measuring their distance become invisible to the eye, they will all appear to vanish together in this point, which, in consequence thereof, is called the *vanishing point* of all those parallels.

Corollary 1. Hence all original parallel planes ($PRSP$) intersecting the picture, (as in Pp), seem to converge to a right line therein, viz. that line Oo , which is the intersection of the picture, with a plane $QOEoq$, parallel to them all passing through the eye.

For since pS , *fig. 3, plate 30*, is parallel to oq ,
therefore

therefore S will appear to converge to it's vanishing point o , (by the foregoing proposition;) and since both the points R, S , equally appear to tend to the respective points $O o$, therefore the line RS also appears to approach to the line $O o$; and this is the case with all planes parallel to $PRSp$, and therefore if they are produced till they become invisible, they will appear to meet in $O o$, whence this line is called the vanishing of all those parallel planes.

Corollary 2. The representations of original lines pass through their intersections and vanishing points, and of original planes through their intersections and vanishing lines.

Corollary 3. Lines parallel to the picture, have parallel representations.

For in this case, the line EO , which should produce the vanishing point, never cuts the picture.

Corollary 4. Hence the representations of plane figures parallel to the picture, are similar to their originals.

Let $qrst$, fig. 4, plate 30, be the representation of the original figure $QRSTU$, to an eye at E ; then all the lines qr, rs, st , &c. being respectively parallel to their originals QR, RS, ST , &c. as well as the diagonals, tq, TQ , &c. therefore the inscribed triangle qtu , and QTU , are similar to each other, (and so of all the other triangles,) and therefore the whole figure $qrst$, to $QRST$.

Corollary 5. The length of any line in the representation is to that of it's respective original, as the distance of the picture to that of the original plane; for all the triangles EQU, Equ, EQR, Eqr , &c. are similar; therefore as Eq is to qu , so is EQ to QU ; that is, as qu is to QU , so is

H h

the

the distance of the picture, to the distance from the plane QU .

Problem 1. Having the center and distance of the picture given, to find the representation of any given point thereon.

From the eye E , *fig. 5, plate 30*, to the center of the picture e , draw Ee , and parallel to it Qd from the given point Q , cutting the picture in d ; join ed , and draw EQ , intersecting ed in q , the perspective place of the original point Q .

For ed is the representation of the original line dQ , indefinitely produced from it's intersection d , till it appears to vanish in e ; therefore q must be somewhere in this line, *pr. 2, cor. 2*; it must also be somewhere in EQ , and therefore in the point q , where they intersect.

Problem 2. The center and distance of the picture being given, to find the representation of a given line QR , *fig. 6, plate 30*.

Produce RQ , to intersect the picture in X , draw EN parallel to RQX , cutting the picture in V , the vanishing point of the line RX ; draw ER and EQ , cutting VX the indefinite representation of RX , in r and q , and rq is the representation of RQ .

This needs no demonstration.

Problem 3. Having the center and distance of the picture to find, the representation of an original plane, whose position with respect to the picture is given; draw any two lines (except parallel ones) upon the plane, and find their vanishing points by the last problem; through these points a line being drawn, will be the vanishing line required.

This is evident from *prop. 2, cor. 1*.

Problem 4. Having the center and distance of the

the picture given, to find the vanishing point of lines, perpendicular to a plane whose representation is given.

Let PL , *fig. 7, plate 30*, be the vanishing line of the given representation, X it's center, e that of the picture, and eE it's distance; join EX , and perpendicular to it draw Ef , cutting xe in f .

For EPL is the original plane, producing the vanishing line PXL , and EF being a visual ray, parallel to all the original lines that are perpendicular to the plane EPL , or it's parallels; f is therefore the vanishing point of all of them.

Corollary 1. When the vanishing line passes through the center of the picture, that is, when the parallel planes are perpendicular to that of the picture, the points X and e coinciding with Ef , become perpendicular to Ee , or parallel to the picture, and therefore the lines will have parallel representations.

Corollary 2. If the original lines were desired to make any other angle with the original plane than a right one, it is only making xEf equal to it.

Problem 5. The center e , and distance Ee , *fig. 7, plate 30*, of the picture, being given, and the vanishing point f of a line parallel to Ef , to find the vanishing line of planes, perpendicular to that line whose parallel (Ef) produces the vanishing point f .

Join fe , and make EX perpendicular to, cutting fe , produced in X , draw PL perpendicular to ex , and PL will be the vanishing line required.

The planes may form any angle instead of a right one, if that fEX be made equal to the same.

Problem 6. The center and distance of the picture being given, and the vanishing point of a line,

to find the vanishing line of planes, perpendicular to the line whose vanishing point is given.

From f the given vanishing point, through e , *fig. 7, plate 30*, the center of the picture, draw feX , and through the eye E , draw Ef , perpendicular to which draw eX , cutting fe in X ; make PXC perpendicular to Xef , and PL will be the vanishing line.

Problem 7. Having given the center and distance of the picture, the inclination of two planes, the vanishing line of one of them, and the vanishing point of their common intersection, to find the vanishing line of the other plane.

Case 1. Let the inclination of the planes be a right angle, and let PL be the vanishing line, P the vanishing point of their common intersection. By the 5th problem, find f the vanishing point of lines perpendicular to the plane, whose vanishing line is PL ; join Pf , which is the vanishing line required.

For since this last plane is perpendicular to the former, f will be the vanishing point of one line in it, (prop. 2,) and P being the vanishing point of another, therefore Pf is the vanishing line.

Case 2. When the inclination nem , *fig. 8, plate 30*, is greater or less than a right angle.

Let Nx be the vanishing line of one plane, x the vanishing point of it's intersection with the other plane; from the eye E draw Ex , and perpendicular thereto, the plane NEM , intersecting xN in N , and the picture in NfM ; make the angle NEM equal to nem the given inclination, join xM , which will be the required vanishing line.

For the planes being parallel to the original planes

planes by construction, Mx , Nx , are their vanishing lines, (cor. 2, pr. 2.)

Problem 8. To find the projection of any solid figure.

Find the representation of any one of it's faces, by prob. 3, and of the others by the last; if any side be convex or concave, a number of points may be found therein, by pr. 1, and curves drawn evenly through them, will represent the curved superficies required; or squares, or other regular figures may be inscribed or circumscribed about the original figures, and these plane figures being projected by the foregoing methods, together with a few points therein, by which means the curves may be similarly drawn about or within these projected squares, &c. by this means also may tangents be drawn to the representations of all kinds of curve lines, in all kinds of situations.

GENERAL REMARK.

So far I have endeavoured to render the principles of perspective obvious by a mere inspection of the figures. For the young artist, whose mind seldom condescends to mathematical reasoning, may, in all the foregoing problems, suppose the plane of the picture placed upright upon that of the paper, (which paper he may consider as the ground-plane,) the operator's eye (E) being always in it's proper situation with respect to the picture, as well as to the planes of the original objects. The data or things required to be known before objects can be put into perspective, are their plans and elevations, (see defin. 5 and 6,) which must be actually laid down by a convenient scale adapted to the size you mean your picture should be, which may be very easily accomplished, for you must remember,

H h 3

that

that the two extreme visual rays, that is, those which pass from the eye to the two opposite borders of the picture, must not make an angle less than *two-thirds*, or greater than *three-fourths* of a right one. With respect to the distance of the picture, it must be remembered, that objects cannot be seen distinctly nearer to a common eye than six inches, and therefore in the smallest miniature pieces, the distance must exceed that quantity; the height of the eye should be about half the distance of the picture, and about the third part of the picture's whole height. - For example, if the whole height of the picture be 3, the distance from it should be 2, and the height of the eye 1. If the plan, when laid down by it's proper scale, be bounded by a quadrangle G N c d, *fig. 9, no. 2, plate 30*; then the ground line of the picture is supposed to be placed upon the shortest side of it, and upright to it's plane; therefore this side of the figure should be made exactly equal to the breadth of the picture, and the other two adjacent sides should, if produced, meet at the distance of the picture, as at F the foot of the observer, as is represented in *fig. 9, no. 3*. The problems already given are sufficient for all cases that can happen in putting objects in whatsoever position into perspective; and though they are perhaps solved in such a manner, as to give the clearest ideas of the genuine practice of perspective, yet others may possibly prefer some of the following methods, which are however easily deduced from the preceding system.

Example 1. Having given the center e of the picture P L N f, *fig. 2, no. 2, plate 30*, it's distance e E, (12 inches) height of the eye E F, 6 inches, to find the representation of a point T, whose

5
depth

depth in the plan is 8 inches, and amplitude to the left 5 inches.

Method 1. From f in fN , take $fn = 5$ inches, and having draw nT perpendicular to fN , make it equal to 8 inches, then will T be placed in it's proper situation, (and n will be it's seat on the picture) join TF , (F as usual being the foot of the observer's eye upon the ground plane, that is, in the present example, 6 inches perpendicularly below the eye E ;) cutting fN in t , draw to perpendicular to fN , and join TE , intersecting to in o , the representation of the point T required.

Otherwise, if the point T be situated on the ground plane, draw FT , cutting fN in t , and ts perpendicular to fN , cutting the visual ET in o , the point required.

If the point T has any elevation perpendicular over the same point as at S , make TS equal to the height it should have, and draw ES , cutting the perpendicular ots in s , the perspective point of S required.

Remark. This method is very convenient in some cases, as when there are many windows, &c. perpendicularly over one another, &c.

Method 2. In the horizontal line eE , *fig. 2, no. 3, plate 30*, from the center of the picture e , take eE equal to the distance of the picture; from the given point Q draw Qd perpendicular to the ground line GN , make dQ^* equal to dQ , and join Q^*E and ed , intersecting each other in q , the representation of the point Q .

Method 3. Perpendicularly over the center e , *fig. 2, no. 4, plate 30*, of the picture, take Ee , equal to the distance of the picture, draw Qd perpendicular to GN , join EQ and ed , intersecting each other in q , the point as before,

H h 4

Remark.

Remark. Both these methods are in fact the same as that in prob. 1, as may be seen by comparing the figures with each other, being marked with the same letters for that purpose. It may be farther remarked, that both may be alternately used in the same piece, remembering to use that which you judge will give the bluntest intersection, that is, whichever makes the angle $e q e$ the greatest.

And now supposing a clear knowledge both of the theory and practice to be obtained, it may not be amiss to shew how naturally a practice more elegant and simple may be deduced, viz. by supposing all the planes to coincide with the paper, or plane of the picture.

Thus, *fig. 5, plate 30* if the triangles $q e E$, $q d Q$ revolve round $d q e$ till they fall into the plane $P L N$, no change takes place in any of the comparative distances, $E e$, $E Q$, &c, and the point q preserves it's situation on the picture, as in *fig. 2, no. 4*; and again, if in *fig. 2, no. 4*, $e E$ be moved round on the point e , till it falls into the horizontal line at n , and $d Q$ be moved similarly round, till it becomes (as in *fig. 3, no. 3*) parallel to $e E$, the point q still remains unaltered.

Method 4. Let $m N f s r$, *fig. 2, no. 5, plate 30*, be the ground line, $f e E$ perpendicular to it, passing through the center of the picture e , and let $f E^2$ equal to $f N$, be the distance of the picture, $N P$ being perpendicular to $N f$, and the representation of the point Q , whose distance from $N r$ is $r Q$.

Draw $E^2 Q$, cutting $N r$ in s , taking $N m$ equal to $Q r$, and draw $e m$, cutting $N P$ in o , draw $o q$ parallel to $N r$, and $f q$ to $N o$, intersecting each other in q , the point required.

Remark. The lines $E^2 Q e m$, need not be drawn,

drawn, but a dot made at s where the ruler crosses $n r$, and $e m$; a tee square may also be applied to the line $N s r$, and a dot made on it's fiducial edge at s ; then if the square be slid up till the fiducial edge crosses the mark at o , the point s will be transferred to q , the representation of the given point, obtained without drawing any lines over the picture. (This method was first discovered by Mr. Beek, an ingenious artist, well known for many useful contrivances.) It is also more accurate than the last method in general, as the intersections are more obtruse.

But nevertheless, when the two points in a representation are projected very near together, it is the best way to work by the intersections and vanishing points of the original lines; for then having the full extent of the representation, it's true directions may be ascertained very correctly. The vanishing points by this method are thus found; (suppose for instance of the line $I Q$, intersecting the ground line in I , in $f e E^2$, take $e E$ equal to $E^2 f$, that is, equal to the distance of the picture,) draw $E V$ parallel to $I Q$, intersecting the horizontal line $h o$ in V , the vanishing point of $I Q$ and all it's parallels.

Example 2. To put any plane figure, as $QRST$, or $M N O P$, into perspective, *plate 30, fig. 9.*

This is in fact only a repetition of the last; for by finding the projections of the several points as before, nothing remains but to join them properly, and the thing is done; thus the points m, n, o, p , projected from M, N, O, P , being joined, gives $m p o n$ the representation of the quadrangle $M P O N$, &c. But if any line as $p o$ happens to be very short, (so that a small error in point o , would considerably alter the direction of it.)

it,) find the intersection and vanishing point of it's original; or if the vanishing point falls at too great a distance, as it very frequently does in practice, find the intersection E^2 , and vanishing point Z , of any other line passing through the point O , as the diagonal OM , and the truth of the projection may be depended upon. All the three last methods may be made use of in projecting the same figure; thus the point p , by the second method, that is, make eE in the horizontal line, equal to E^2k , the distance of the picture, draw Py perpendicular to SX , and take yk equal to yP , join Ek and ey , intersecting in p , which is a better intersection than the third method would give, viz. by drawing EP , but yet inferior to that in the fourth method.

Example 3. To put any solid body into perspective. Find the seats of all the points upon the ground plane, and project them as before; let G be the seat of one of the points, g it's projection, produce eg to the ground line aE^2 , and make E^2H perpendicular, and equal to the height of the given point, from it's seat join He , and it will cut the perpendicular gg^2 in g^2 , the representation of the required point.

Or, by the *fourth method*, (G^2S being equal to the distance of G from SX .)

Make G^2G^3 perpendicular and equal to the given height, draw G^2e , cutting SU in e^2 , then e^2g^2 parallel to SX , will cut the perpendicular gg^2 , in the point g^2 required; or the point g may be transferred to g^2 by the tee square, without drawing gg^2 or e^2g^2 .

Proceed in this manner, till you have obtained all the requisite points in the figure, marking or numbering them as you proceed, or else correct them

them by strait or curved lines according to your original, by which means you may instantly see the connection of your work at all times without confusion.

Remarks. 1. Sometimes when there a great number of small parts in a body to be perfectly made out, it may not be amiss to draw squares over the ground plane of the object, and find the seats of it's several elevated points; then by throwing these squares into perspective, the positions of the several points will likewise be found by inspection, and the horizontal row of squares will serve as a scale for the altitudes of bodies, whose seats lie in that row, or both plans and elevations may be used as in *fig. 10, plate 30.*

2. Perspective may also be practised without having any recourse to ground plans, for by taking the horizontal angles, the amplitudes of objects are to be ascertained, and by vertical ones their heights and depths in the picture; the angles thus taken, may be entered in a table of this form,

Vert. ∠	Horizontal.		Vert. ∠
Up.	Left.	Right.	Down.

In protracting which angles you must make the distance of the picture radius, and lay down the angles by a line of tangents adapted to that radius,

radius, which is therefore best done by means of the sector; or more expeditiously by the fourth method, (page 472,) for if a protractor be fixt at E^2 , *fig. 9, plate 30*, horizontal angles will cut the ground line SX in the same places as if the lines forming them pass'd over the original points in the plan; and if the protractor be fixt at e , the same may be said of SU with respect to elevations.

Solid bodies may also be put into perspective, by drawing lines in particular directions, as from the center of a circle, or of concentric ones, and finding the representations of the same, raise perpendiculars of the proper heights, always supposing the bodies to be transparent, *fig. 10, plate 30.*

OF SHADOWS.

Luminous bodies, as the sun, moon, lamps, &c. are generally considered as points; but the artist takes the advantage of their not being perfectly so, by softening the extremities of his shadows, which are so softened in nature, and for this reason, which may be thus explained.

Let RS , *fig. 11, no. 2, plate 31*, be the radius of a luminous body whose seat is r , and center is S ; from the opaque body O , draw xr , touching the extremities of both the bodies on both sides, by which means a penumbra or semi-shade, $qx n$, will be formed on each side of the main shadow, which becomes extremely tender towards the outer extremity, and from thence gradually strengthens, till it blends with the uniform shadow, which will, if the diameter of the luminous be greater than that of the opaque one, (measured in the direction of their centers,) converge to a point, as in *fig. 11, plate 31.*

N. B.

N. B. The opaque body which casts the shadow, is called the shading body ; and those that are immersed in the shadow, are called shadowed bodies.

Problem 9. To find the shadow of any object upon a plane.

Through the luminous body, draw planes touching all the illumined planes of the object, and the interfections of these planes with the given plane, will give the boundary of the shadow required.

Example 1. Let the luminous point be the sun, the plane of projection $A R S B$, *fig. 12, plate 31*, and the object the parallelogram $A B C D$.

The sun's rays on account of his distance, may be supposed parallel, therefore the planes $O R A$, $O S B$, are parallel, and therefore since $C D$ is parallel to $B A$, $R S$ is parallel to it also, and the shadow a parallelogram, whose length is to the height of the object (supposed upright) as radius to the tangent of the sun's altitude; and it's breadth as radius to the sine of the inclination of it's rays, with $B A$ the base of the parallelogram.

Therefore, find the seat of one of the rays, as $O A R$, and make the angle $A D R$ equal to the complement of the sun's altitude, and make $B R$, and the thing is done.

Example 2. Let the luminous body be a lamp placed at O , *fig. 13, plate 31*, the object, a parallelogram $A B C D$ standing upon the plane $A R S D$.

Having drawn $O G$, perpendicular to the plane $A R S D$, and the rays $O B R$, $O C S$; from G , (the seat of the lamp on the plane $A S$.) draw $G A R$, $G D S$, intersecting the rays $O B$, $O C$ in R and S , and $A R S D$ will be the shade required.

Example

Example 3. Where the shadow from the sun, &c. passes over different objects.

Continue the sides $C A D B$, to A', A'', B', B'' , &c. *fig. 14, plate 31*, and where $A G, B H$, cut the body $G H I K$, draw $K G A', I H B'$, making the angles $A G A', B H B'$, equal to the inclination of the plane $G I$ to $G B$, proceed in the same manner with every new plane, or if the object is curvilinear, tangents will always pass through the lines $C A'$ and $D B$ respectively, except when they are perpendicular.

Example 4. Let a lamp O , *fig. 15, plate 31*, throw a shadow on the body $R S U T$, and let $Q T U$ be the central line of the shadow from the parallelogram, $A B$, upon the ground; draw $O B, G P$, parallel to $Q T$, and from T to $O P$, draw $T P$, touching the plane $U s$ of the body; from P draw $P S u, P w U$, cutting the extremities of the shadow in U and u , and $w U u S$ will be the shadow of $A B$ upon the face $U S$; proceed in like manner with all the other illumined faces.

N. B. If $T P$ never meets $O P$, it denotes the face of the body to be parallel to $A B$, and the shadow on that face to be a parallelogram.

The shadows being thus ascertained, may be put into perspective by the foregoing rules.

Problem 10. To find the reflexions of objects upon polished surfaces.

Let fall a perpendicular upon the reflecting plane, to which draw a radial from the eye, as much below the horizontal line, as the real object appears to be above it.

Example 1. Let $A B$, *fig. 18, plate 31*, be any object placed on the water; from B draw $B b$, perpendicular to the surface $A b$, which continue till the angles $B E b$ and $C E b$ are equal; that is, ($E b$ being

being a horizontal line) till bc is equal to bB , and bC will be the reflexion of AB .

Example 2. When objects are upright, the lines may be produced below the horizontal line, as much as the real ones are above it, see *fig. 19*.

OF PARALLEL OR MILITARY PERSPECTIVE.

In this kind of projections, the eye is supposed to be placed at an indefinite distance from the object in the diagonal, and looking down upon it in an angle of 45° , so that the top, one side, and one end, are seen under the same angle, and therefore appear in their true proportions with respect to each other; and therefore heights, lengths, and breadths, must be laid down by the same scale, and all parallel lines made parallel, see *fig. A, plate 30*.

OF AËREAL PERSPECTIVE.

Before we can give rules for regulating the force of lights and shades in a picture, we must consider what degree of it the bodies themselves are endued with, according to their several positions, with respect to the illuminating body.

Proposition 3. The intensity of light upon any plane, is reciprocally as the square of the distance of that plane, from the illuminating body.

Let $ABCD$, *fig. 16, plate 31*, be the shadow of the square $abcd$ upon a plane parallel to it, which projection will therefore be a square, and in proportion to $abcd$ as the square upon OA , to that upon Oa ; therefore since the real quantity of light is the same as would be received upon $ABCD$, the intensity of it is reciprocally as the square upon AB ,

AB, to the square upon ab, or as square OA, to square Oa.

For example, if parallel planes are at the distance of 1, 2, and 3 feet from a luminous point, the intensity of light upon them would be 1, $\frac{1}{4}$, $\frac{1}{9}$, &c.

Corollary. All parallel planes are equally illuminated by the sun at the same moment.

For his rays being parallel, the squares a b c d, and A B C D are equal.

Proposition 4. If the sun's beams fall perpendicular upon one face A B, *fig. 17, plate 31*, of an object, and inclined upon another A C, the intensity of light on the faces, is as radius to the sine of the angle of incidence.

Produce A B to b, the quantity of light A b receives, is the same as would be received on A C if b A were away; therefore the brightness is as A C to A b, that is, the brightness of A b, or A B is to that of A C, as A C to A b, or as radius to sine of the angle of incidence A C b.

Proposition 5. A plane uniformly enlightened, does not appear so to an eye in different situations.

For as all bodies are porous, the little exuberances will have their light and dark sides, and the eye will view more of the former, as it is more nearly situated in a line with the rays of light, and more of the latter, the more it looks against them.

The subject of this proposition is one great cause of the graduation of light upon faces of buildings and other plains, and not altogether owing to a greater teint of air (as the artists call it) on that part, which is the farthest off.

Remark. It is very necessary to observe, that transparent and polished bodies are not included amongst those mentioned in this proposition, for
they

they seem most illuminated in that part which makes the angle of reflexion equal to that of incidence; but if bodies of this kind are not flat, as water when just broken by small rippling waves, then the light is reflected from some part of almost every wave, and so is extended to a great space, but is strongest perpendicular under the luminary, and gradually decreases on each side.

The case is the same in the sky, which is brightest near the sun's apparent place, and graduates into a deeper azure as it retires farther off, and for a reason nearly the same; for the pellucid particles floating above us, having large interstices between them, act in the same manner as the rippling waves in disturbed water; and therefore the more obliquely the light strikes upon them, the more united their force will be to an eye situated in the proper angle of reflexion.

Proposition 6. All shaded and shadowing objects would be equally dark and indistinguishable, if they received no secondary or reflected light.

For light is not visible of itself, but by striking upon other bodies renders them so, and these enlightened bodies serve as lights to bodies, otherwise in shade, and such lights are called secondary or reflected ones, the chief of which is the sky.

Proposition 7. Every body participates of the colour of the light by which it is illumined; for blue rays thrown upon a yellow body, will produce a green; red rays, purple; and purple rays, (that is, blue and red,) black.

Corollary. Hence shadows are often observed green in the morning or evening, for the sky is always very green at those times compared with other times of the day, owing to the warm rays being more copiously reflected downwards by the

sun's beams striking more obliquely on the atmosphere, which partly acts as a prism, and the shadows become more and more blue, as the sky becomes so; but clouds are of all colours, and as they are denser than the blue part of the sky, they throw stronger reflections, and cause many accidental tints in the shadows of bodies; therefore as the shadow of every body is partially enlightened by all the bodies surrounding it, it must partake of the colours of all of them, and this is the grand source of harmony in painting; of which system the colour of the original light serves as a key, and is to be attended as nicely to in painting, as in music.

Proposition 8. Bodies partake more of the colour of the sky, as they are farther off.

For the sky itself is nothing but a body of air, which every where surrounds us, the natural colour of which is supposed to be blue; therefore the farther off any body is, the more of this blue air is intercepted between us and the body, and therefore the bluer it is, and that in proportion to its distance.

OF INSTRUMENTS FOR DRAWING IN PERSPECTIVE.

Various have been the methods used to facilitate the practice of perspective, as well for those who understand, as those who are ignorant of that art; and though some have supposed that the warmth of imagination and luxuriance of fancy, which impels the mind to the cultivation of the fine arts, is not to be confined to mechanical modes, yet upon inquiry they will find, that the most able and accomplished artists are often obliged to have recourse to some rules, and to use some mechanical contrivances to guide and correct their pencil. So

great is the difficulty, and so tedious the operation of putting objects in true perspective, that they trust mostly to their eye and habit for success; how well they succeed, we may decide from the portraits drawn by the best artists, and the different judgments formed concerning them. Mr. Eckhardt has well observed, that there is no artist who will be hardy enough to say, that he can delineate (by the eye) the same object twice with exactness, and preserve a just and similar proportion of parts in each. In one of the figures, we shall find some of the parts larger than in the other: both cannot be right: yet supposing them perfectly the same, neither may be conformable to nature. Add to this, many situations of an object occur, which no eye, however habituated, can represent with accuracy.

On this account, I have a long time endeavoured to complete an instrument that should give the outline of an object with accuracy. An history of my various contrivances for this purpose, though it might not be entertaining, would not be unprofitable. These essays have, however, now swelled so far beyond my intentions, that I must be as concise as possible. I must however acknowledge the valuable hints I have received from various ingenious men, but more particularly from Mr. Heywood, whose name has been already mentioned in this work, and which can never be mentioned without praise.

The methods most generally in use are, 1. The camera obscura. 2. The glass medium or plane. 3. A frame of squares. The inconveniences and inaccuracies which attend these expedients, induced Sir Christopher Wren, Mr. Ferguson, Mr. Hirst, My Father, Mr. Watt, Mr. Eckhardt, Pere Toussaint, and others, to have recourse to different contrivances to remedy their defects; of which,

those by the Rev. Mr. Hirst, My Father, Mr. Watt, and Mr. Eckhardt, are undoubtedly the best; Mr. Eckhardt's and Mr. Hirst's vary but little from each other.

Those represented at *fig. 1* and *2*, *plate 32*, appear to me far superior to any that have been hitherto contrived: the object is delineated on a horizontal plane, the pencil may be moved in any direction, whether curved or strait, with the utmost freedom. By either, the artist may be sure of obtaining the measure of every part of the object with mathematical exactness; and this is performed without any loss of time. The instrument may be moved from any place, and brought back to the same with great exactness; and the outline may be formed either of a number of points, or one continued line, at the pleasure of the draftsman.

That represented at *fig. 2*, is the simplest of the two instruments: *fig. 1*, though more complex, merits, for its contrivances and motions, the attention of the mechanic, as well as the draftsman. They both move with facility in every direction, and the whole operation consists in looking through the sight, (which may be placed in any convenient situation,) and moving the pencil, so that the apex of the triangle may go over the object, whose outline will be delineated at the same time.

To lessen the expence, and render the instrument more portable, I have constructed an instrument somewhat similar to that represented at *fig. 2*, *plate 32*, but which moves only in a vertical plane, the board on which the drawing is made being in the same plane with the triangle.

There is another simple contrivance, by which any number of points may be ascertained with ease and accuracy.

To

To these we may add the parallel rule, and the perspective compasses, either of which are exceeding useful for ascertaining the relative measures of distant objects. The compasses have a fixed sight; but as there is no sight to the parallel rule, it's distance from the eye must be regulated by a piece of thread tied to it, and held between the teeth.

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Plotting scales

Sets of plotting scales

A new-invented parallel rule, with plotting scales

A parallel rule and protractor

A plotting scale, in form of a beam compass

Protractors for laying down angles

Ditto with a nonius and moveable limb — 2 12 6

Ditto rendered more accurate, as the limb is made
to move with a tooth and pinion — 4 14 6

Measuring wheels — 6 16 6

Pedometers for measuring a given distance in walking

Measuring chains of 50 and 100 feet

Gunter's chain — 0 7 6

Tape boxes, according to the length

Surveying cross or square — 1 11 6

Plain tables, with an index and sights; by these
the plan is taken on the spot, and does not require
a future protraction — 3 13 6Beighton's improved plain table, with an index of a
peculiar make; in this instrument the line of sight
is always over the center of the table, the station
lines are also drawn parallel to those measured
on the ground — 14 14 0Theodolites, or instruments for measuring angles,
distances, &c. are made in various ways, some
being more simple and portable, others more ac-
curate, and with a greater number of adjustmentsTheodolites, with four plain sights, and a com-
pass box, from 5l. 5s. to — 5 15 6A small theodolite, with telescopic sights and ver-
tical arch — 9 9 0

An improved small theodolite with rack motions

A larger ditto, in which the limb of the theodolite
is made to serve occasionally the purpose of a
protractor — 14 14 0A ditto of the same size, on a different construc-
tion, the vertical arch being fixed to a long
axis; the limb does not serve for a protractor 14 14 0A larger ditto, like the preceding, only moving
with rack-work — 21 0 0

The

	£.	s.	d.
The latest improved theodolet, with double telescope, and every requisite adjustment	31	10	0
Circumferenters, the principal instrument used in America, from 2l. 2s. to ———	4	14	6
An improved circumferenter, so contrived, that the operator need not rest the truth of his work entirely on the needle; it may also be used to take altitudes			
Air or spirit levels with telescopic sights, from 4l. 14s. 6d. to ———	12	12	0
Air or spirit levels with plain sights ———	1	11	6
Station staves with sliding vanes for levelling	1	16	0
A small surveying compass, with sights, a nonius division, and three-legged staff ———	4	14	6
A small surveying compass and single stick	2	2	0
[The two foregoing instruments are portable and light, and may be put with ease in the pocket.]			
Miners compasses, used for carrying on works under-ground, from 10s. 6d. to ———	1	1	0
Ditto with a small telescope fixed to one side	1	11	6
Green's telescope and tangent board, for measuring distances at one station: (it is obvious, that on the measuring of a strait line with accuracy, the whole business of surveying depends; Mr. Green's method is certainly more expeditious than any other, and is not liable to objections which may be made to them)			
Optical square, a small instrument for surveying by right angles; it requires no staff, and may be easily corrected or adjusted ———	1	16	0
An optical instrument for determining with accuracy when objects are in a strait line ———	1	7	0

Military Instruments.

Gunners levels or perpendiculars
 Gunners callipers
 Beam callipers
 Shot gauges
 Shell ditto
 Gunners quadrants with a plummet
 Ditto with a level

Ditto

	£.	s.	d.
Ditto with an adjusting screw			
Ditto and perpendicular combined together			
General Williamson's instruments for howitzers, mortars, &c.			
Surveying compasses, surveying crosses, cases of in- struments, telescopes, plain tables, theodolites, &c.			
A complete apparatus for an officer, in a box	21	0	0

Instruments for Navigation.

Cases of instruments, and telescopes of different kinds, sizes, and prices			
Night telescopes, from 1l. 11s. 6d. to —	2	2	0
Opera glafs for the same purpose —	1	11	6
A telescope with an eye-glass micrometer, for de- termining the distance of a ship at sea			
Hadley's quadrants in mahogany frames —	2	2	0
Ditto in black ebony frames —	3	3	0
Hadley's sextant in wood —	6	16	6
Ditto in brass, on the most improved plan, from 11l. 11s. to —	15	15	0
Knight's steering compass, with improvements	2	12	6
Knight's azimuth ditto —	5	15	6
Ditto on friction wheels —	10	10	0
Marine barometers; by these storms have been foretold at sea some hours before they happened			
Circular instruments, to answer the purpose of the sextant			
Dipping needles, from 12l. 12s. to —	31	10	0

Optical Instruments.

The best double-jointed silver spectacles, with glasses —	1	1	0
The best ditto, with Brazil pebbles —	1	16	0
Single joint silver spectacles, with glasses —	0	15	0
Ditto, with Brazil pebbles —	1	10	0
Double-joint steel ditto, with glasses —	0	7	6
Best single-joint spectacles —	0	5	0
Ditto, inferior frames, from 2s. 6d. to —	0	3	0
			Note

	£.	s.	d.
Nose spectacles, mounted in silver	—	7	0
Ditto in tortoiseshell and silver	—	4	0
Ditto in horn and steel	—	1	0
Spectacles for couched eyes			
Spectacles with shades			
Concave glasses in horn boxes, for short-sighted eyes			
Ditto, mounted in tortoiseshell and silver, pearl and silver, in various manners, and at different prices			
Reading glasses, from 2s. 6d. to	2	2	0
Opera glasses, from 10s. 6d. to	2	2	0
Ditto on an improved construction, 1l. 7s. and	1	11	6
Ditto to be used at sea by night	1	11	6
Diagonal operas of a new construction	1	1	0
Telescopes of various lengths, sizes, and prices			
Acromatic telescopes, portable and convenient for the pocket; the sliding tubes are of brass, and therefore not subject to the inconveniences of those that are made with vellum drawers, from 1l. 11s. 6d. to	8	8	0
Telescopes to be used at sea by night			
Acromatic perspective glasses for the pocket, from 10s. 6d. to	2	12	6
An optical vade mecum, or portable acromatic telescope and microscope, from 3l. 13s. 6d. to	4	14	6
A thirty-inch acromatic telescope, with different eye-pieces for terrestrial and celestial objects; this is one of the most pleasant telescopes that is made for general purposes, from 8l. 8s. to	11	11	0
An acromatic telescope, about three feet and an half long, with different eye-pieces	18	18	0
A three-feet reflecting telescope, with four magnifying powers, with rack-work	36	15	0
A ditto, two feet long, with ditto	21	0	0
A two-feet reflecting telescope, with two powers	12	12	0
An eighteen-inch ditto	8	8	0
A twelve-inch ditto	5	5	0
Adams's LUCERNAL MICROSCOPE, for opaque and transparent objects; it does not fatigue the eye, is in all cases a proper substitute for the solar microscope, and on many occasions superior to it	21	0	0
A small double-reflecting microscope	2	12	6
A larger ditto	3	13	6
An improved universal double microscope	6	6	0
Ditto fitted up in a different form, from 8l. 8s. to	14	14	0

Ellis's

	£.	s.	d.
Ellis's aquatic microscope — — —	2	2	0
Ditto with an adjusting serew — — —	2	12	6
Withering's and other botanical microscopes, from 10s. 6d. to — — —	3	3	0
Small pocket microscopes, from 6s. to — — —	3	13	6
Solar microscopes — — —	5	5	0
Ditto — — —	6	6	0
Solar microscopes for opaque and transparent objects, from 16l. 16s. to — — —	21	0	0
A microscope and lanthorn to imitate the solar microscope			
Curious collections of objects for the microscope, either opaque or transparent			
Collections of salts, properly prepared for the microscope			
Magnifying glasses for botanical, anatomical, and other purposes, from 2s. to — — —	1	11	6
Small magic lanthorns, with twelve small glass sliders, from 10s. 6d. to — — —	1	1	0
Ditto with the sliders better painted			
Large magic lanthorns, from 1l. 5s. to — — —	1	11	6
Optical machines for viewing perspective prints, from 18s. to — — —	1	16	0
Scioptric balls — — —	0	10	6
Small camera obscuras, from 10s. 6d. to — — —	3	3	0
Book and pyramidal camera obscuras, from 3l. 3s. to — — —	7	7	0
An artificial eye for illustrating the principles of vision			
Prisms, mounted in various manners			
Concave and convex mirrors, from 7s. 6d. to — — —	18	18	0
Cylindrical ditto, from 1l. 1s. to — — —	13	13	0

Geographical and Astronomical Instruments.

PRICES OF GLOBES,

28 inches diameter, mounted in the common manner — — —	35	0	0
28 ditto mounted in mahogany frames — — —	50	0	0
28 ditto frames carved and ornamented — — —			
18 ditto mounted in the common manner — — —	6	6	0
18 inches			

		£.	s.	d.
18 inches diameter, mounted in the common manner in mahogany frames		8	8	0
18 ditto mounted in the best manner, in stained frames		10	10	0
18 ditto mounted in the best manner, in mahogany frames		12	12	0
18 ditto mounted in the best manner, in carved frames		16	16	0
16 ditto mounted in the common manner		6	6	0
12 ditto mounted in the common manner		3	3	0
12 ditto mounted in the common manner, in mahogany frames		4	4	0
12 ditto mounted in the best manner, in stained frames		5	15	6
12 ditto mounted in the best manner, in mahogany frames		7	7	0
9 ditto mounted in the common manner		2	2	0
9 ditto mounted in the best manner		4	4	0
6 ditto mounted in the best manner		3	3	0
3 ditto mounted in the best manner		1	11	6
3 ditto for the pocket		0	10	6
An armillary dialling sphere		40	0	0
An armillary sphere, shewing at one view the real and apparent motion of the heavens		31	10	0
An armillary sphere, with a planetarium within it		31	10	0
Plain armillary spheres				
A manual planetarium and tellurian		3	13	6
A new instrument for illustrating the phenomena of the seasons		4	10	0
A ditto for illustrating the phases of the moon, and the causes of eclipses		4	10	0
A manual planetarium		2	2	0
(The three last-mentioned instruments are those represented in plates V. XV. and XVI. of my Astronomical Essays.)				
A planetarium with wheel-work, by which the order and motion of the planets, their situation with respect to the earth at different times, and the reason of their appearing to be sometimes stationary, and to move at other times in contrary directions, are rendered obvious to the eye		7	17	6
A tellurian and lunarium for illustrating the phenomena of the earth and moon, and forming a proper companion for the former				
(The tellurian and lunarium may be combined together, or separate.)				

A correct

	£.	s.	d.
A correct and elegant planetarium, lunarium, and tellurian, (see plate XIX. of my Astronomical Essays)	18	18	0
Ditto with the diurnal motion to the earth	24	0	0
The most complete planetarium, tellurian, and lunarium, plates XVII. and XVIII.	36	15	0
Orreries, from 18l. 18s. to	1000	0	0
A small quadrant, with an horizontal and vertical motion, for the instruction and amusement of young people	0	16	0
A ditto framed in brass, more accurately made and graduated	2	12	6
A small equatorial, being at once an accurate universal dial, and a portable observatory; with it a great number of curious and interesting problems may be solved, and any person soon rendered master of the elements of practical astronomy	8	8	0
Ditto on a larger scale, with a telescope, and more adjustments	26	5	0
Complete equatorial instruments, from 50 gs. to 500	0	0	0
Transit instruments, astronomical quadrants, circular instruments, according to their sizes, the variety and accuracy of their adjustments.			

Instruments for Electricity.

A small electrical machine, with a select apparatus	8	8	0
An electrical machine and medical apparatus in a box; the machine is mounted in a plain but strong manner, and so as to act with power; the apparatus is the most convenient and simple hitherto contrived for medico-electrical purposes	6	16	6
Ditto with an additional apparatus lately contrived for more easily giving partial shocks	7	17	6
Improved electrical machines, from 3l. 13s. 6d. to	40	0	0
Electrical batteries, from 2l. 12s 6d. to			
Electrical jars of different sizes			
Ditto with an electrometer affixed to them	0	18	0
Medical bottles, with a tube for qualifying the shock	0	7	6
			Ditto

	£.	s.	d.
Ditto mounted with a discharging rod, and an electrometer, on an improved plan, for giving the electric shock	1	1	0
Directors with glass handles for medical purposes	0	10	6
Jointed dischargers with glass handles	0	10	6
Plain discharging rods	0	8	0
An universal discharger and press	1	11	6
Kinnerly's electrical thermometer	1	1	0
Quadrant electrometer	0	10	6
Cavallo's atmospheric electrometer, from 15s. to	1	1	0
Ditto with additions by De Sauffure	1	11	6
Bennet's gold leaf electrometer	0	18	0
An apparatus for making Canton's and Wilson's experiments on electric attraction, &c.			
Compound apparatus, fig. 49, plate III. of my Essays on Electricity; by this apparatus a great number of neat and satisfactory experiments may be performed	3	3	0
Ditto without the exhausted flask and conductor	1	11	6
Leyden vacuum	0	10	6
Luminous conductors	1	1	0
Spiral tubes, from 4s. 6d. to	0	10	6
Coloured spirals	0	9	0
Sets of spirals, see plate V. fig. 98. of my Essays on Electricity	1	11	6
Luminous words, from 10s. 6d. to	1	11	6
Spotted bottle	0	9	0
Belted bottle, plate III. fig. 46.	1	1	0
Double bottle, a pleasing and useful part of an electrical apparatus, to gain a clear idea of the Franklinian theory	0	18	0
Plates and stand for dancing images, pith images, and pith balls			
An artificial spider			
A small head with hair			
An electrical pistol for inflammable air	0	7	6
Ditto mounted in sets			
An electrical cannon for gunpowder			
A thunder house	0	6	6
Ditto with a drawer	0	8	6
A powder house, fig. 89, plate V.	0	16	0
A pyramid, fig. 90, plate V.	0	15	0
Nicholson's revolving doubler	3	3	0
A bone ball and a ball of box wood fitted on brass wires			
Electric flyer and points			

A plain

	£.	s.	d.
A plain set of bells, fig. 17, plate II.	—	0	9 0
Five bells mounted on a stand, fig. 18, plate II.	—	0	18 0
A set of musical bells, fig. 19, plate II.	—	1	7 0
Magic picture	—	0	10 6
Electrical stools, from 8s. 6d. to	—		
An electrophorus, from 10s 6d. to	—		

Apparatus for Experiments on Magnetism.

An apparatus for explaining the principal phenomena of magnetism, from 3l. 3s. to	—	15	15	0
Magnets				
Small compound magnets				
Horseshoe magnets				
Compound ditto, from 15s. to	—	21	0	0
Dipping needles, from 12l. 12s. to	—	31	10	0
Variation compasses, from 2l. 12s. 6d. to	—	21	0	0

Instruments for Experiments on Pneumatics.

A small single-barrel air-pump	—	2	12	6
A small double-barrel ditto	—	4	14	6
A larger ditto	—	6	16	6
A table air pump	—	10	10	0
The American double barrelled air pump, the latest improvement on this instrument, in which the air receives no impediment from the action of valves or cocks, exceeding Smeaton's in accuracy and simplicity, and far superior in both respects to several later contrivances				
A condensing engine; this may be, if desired, combined with the former, but the rational and practical experimentalist will find many advantages in having them detached from one another				

Apparatus

Apparatus for an Air Pump.

	£.	s.	d.
The madgeburg hemispheres, from 12s. to	1	11	6
A flat plate and collar of leathers for placing on open receivers	0	15	6
Guinea and feather apparatus, for experiments on the resistance of the air, from 18s. to	1	11	6
A set of mills for ditto	1	11	6
A ditto on a better construction	4	4	0
Bell apparatus, for shewing that a vacuum does not communicate sound	0	5	6
Ditto on a better construction			
Ditto with wheel-work, by which the bell may be put in motion or stopped at pleasure	3	13	6
A new apparatus for striking flint and steel in vacuo			
An apparatus for firing gunpowder in vacuo			
A copper bottle, beam and stand, for weighing of air	2	16	0
A box bladder and lead weights, to shew the elastic power of the air	0	15	6
Ditto on an improved plan	0	18	0
A model of a pump, illustrating at the same time the nature of pumps, and proving that there is no such thing as suction	1	5	0
A small receiver and plate, which clearly evinces that receivers are kept on the pump by pressure, not suction	0	12	0
A filtering cup	0	5	6
A plate and piece of wood	0	4	6
(The two last articles are for shewing the porosity of vegetables)			
The torricellian experiment			
Fountain in vacuo	0	5	6
Ditto on a different construction	0	18	0
Lungs glass	0	5	6
Ditto on a different construction			
A single transferer plate and pipe for a fountain	0	15	6
A double transferer, for communicating a vacuum from one receiver to another	3	3	0
A burnt air pipe, for experiments on infected air	0	18	0
An apparatus to illustrate the lateral pressure of the air			

£. s. d.

Breaking square and cage
 A small bladder and lead weight
 A small ballance beam and stand
 Ditto on an improved construction
 Receivers of different sizes

— 0 7 6

Meteorological Instruments.

A plain portable barometer	—	2	2	0
Ditto with a thermometer	—	3	3	0
A plain barometer, covered frame and glass door		2	12	6
Ditto with a thermometer	—	3	13	6
A barometer with a long cylindric thermometer		4	4	0
A ditto with ditto, and De Luc's hygrometer		7	7	0
A barometer and thermometer, with a guage, the indexes moving by rack-work	—	5	15	6
A barometer for measuring the altitude of mountains, &c.				
Marine barometers				
Diagonal, wheel, and statical barometers				
Fahrenheit's thermometers, from 11. 1s. to		2	12	6
Ditto for botanic purposes	—	0	18	0
Ditto for the brewery	—	1	1	0
De Luc's hygrometer; these are the only instruments by which comparative observations can be made on the dryness and moisture of the air, from 31. 3s. to	—	7	7	0
Rain guages				
Dr. Lind's wind guage				
Hygrometers with the beard of the wild oat		0	10	6
Fontana's eudiometer for ascertaining the purity of the air	—	2	5	0

Instruments for illustrating the Mechanic Powers, the Laws of Motion, &c.

A concise apparatus for illustrating the nature of the ballance, the pulley, the different kinds of

levers,

	£.	s.	d.
levers, the inclined plane, the wheel and axle, the screw, a compound engine, and a compound lever; also, a double cone to move up an inclined plane, and other pieces to shew the properties of the center of gravity, from 21l. to	26	5	0
A ditto on a more enlarged scale			
Atwood's apparatus for demonstrating with accuracy the laws of accelerated and retarded motion. It is one of the most pleasing and scientific instruments in mechanics, as well from the variety of experiments that may be made with it, as the accuracy with which they are performed	26	5	0
A machine for illustrating the theory of central forces; in this machine the times are marked by sound, the spaces are shewn by an index, the errors arising from friction are so far lessened as to be scarcely sensible	31	10	0
Pullies of various combinations and constructions			
A small carriage, inclined plane, wheels of different sizes, &c. for experiments on wheel carriages			
Roberval's paradoxical balance			
Cycloidal lever, see Emerson's Mechanics, prop. 20, 25.			
Compound steelyard			
An apparatus for experiments on collision			
Ditto for illustrating the composition and resolution of motion			
Pyrometers on various constructions			
With many other articles and models for experiments on friction, pendulums, &c. too numerous to be comprized in a small catalogue			

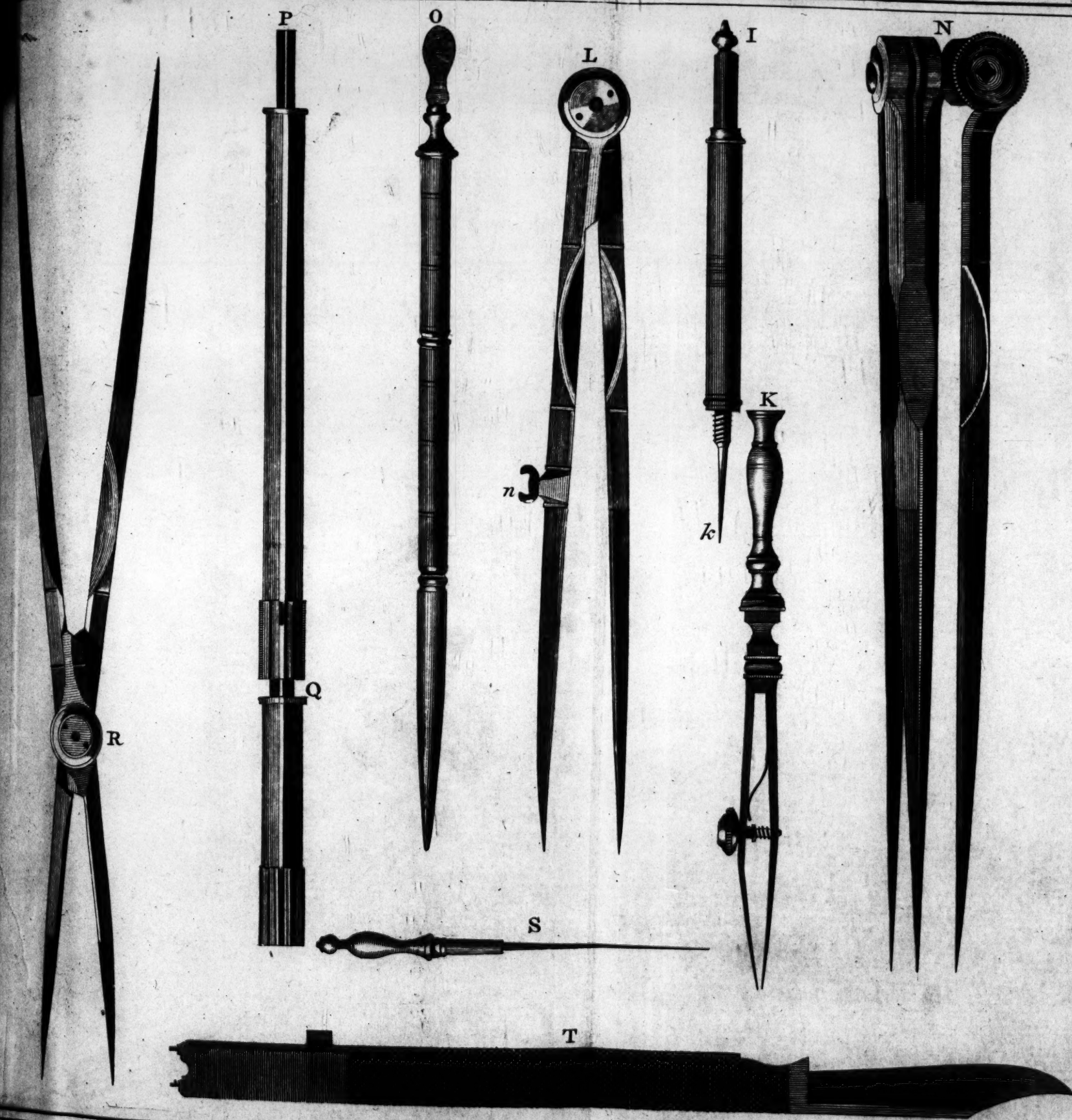
Instruments for Experiments in Hydrostatics and Hydraulics.

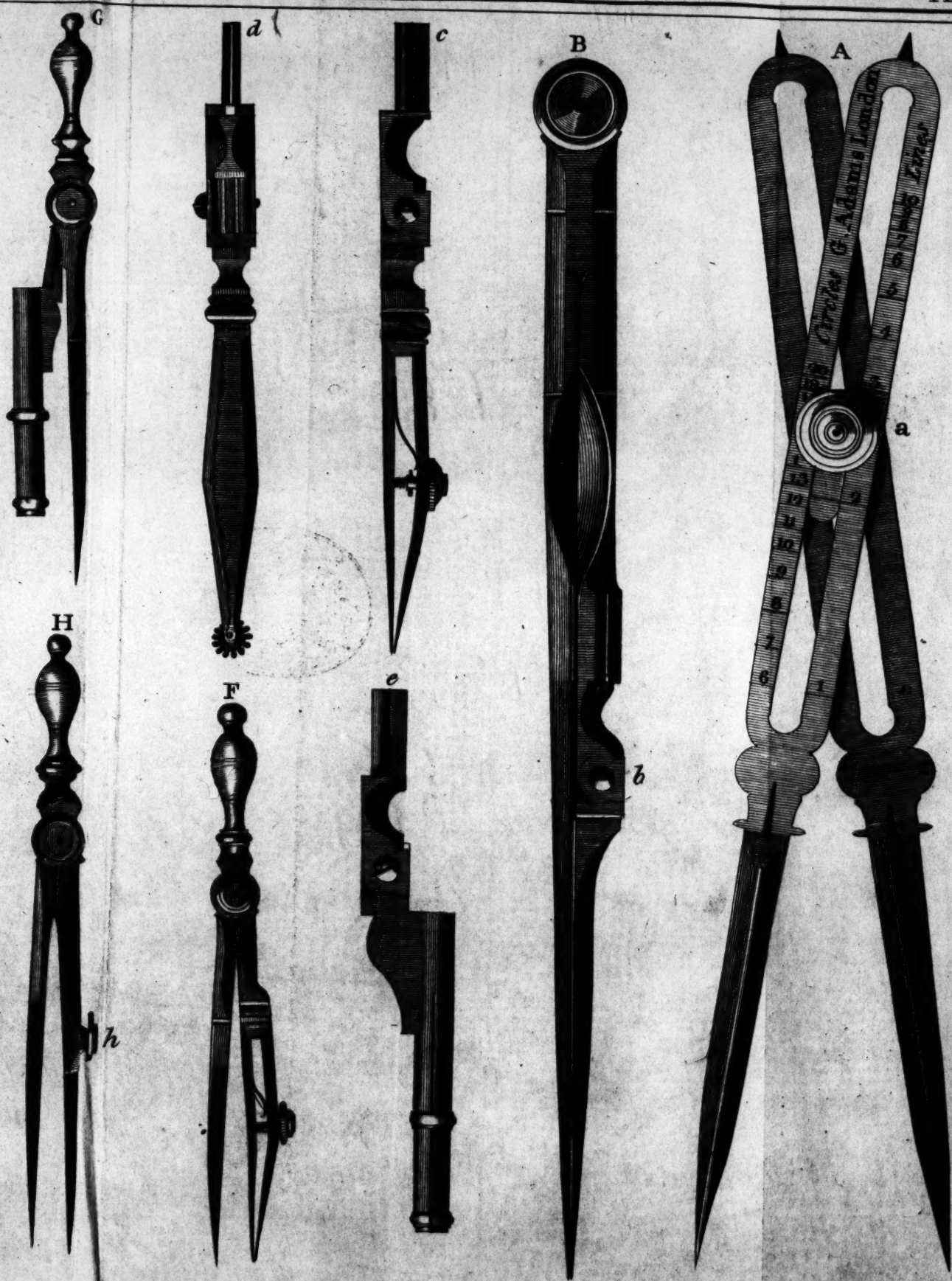
Hydrostatic ballances, from 2l. 2s. to	—	10	10	0
Nicholson's improved hydrometers, for ascertaining the specific gravity of bodies				
Ditto for examining of coin				
A concise apparatus for experiments on hydrostatics	21	10	0	
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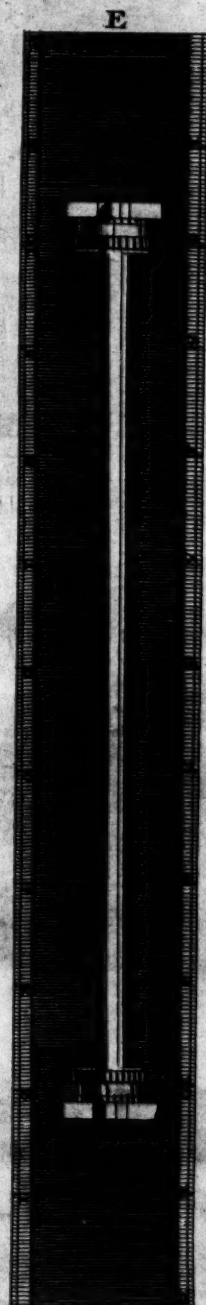
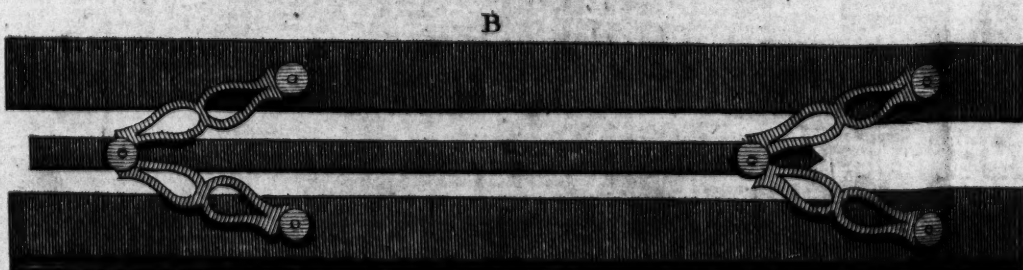
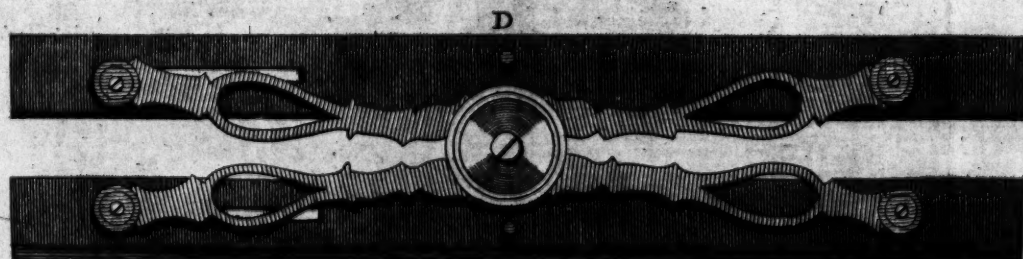
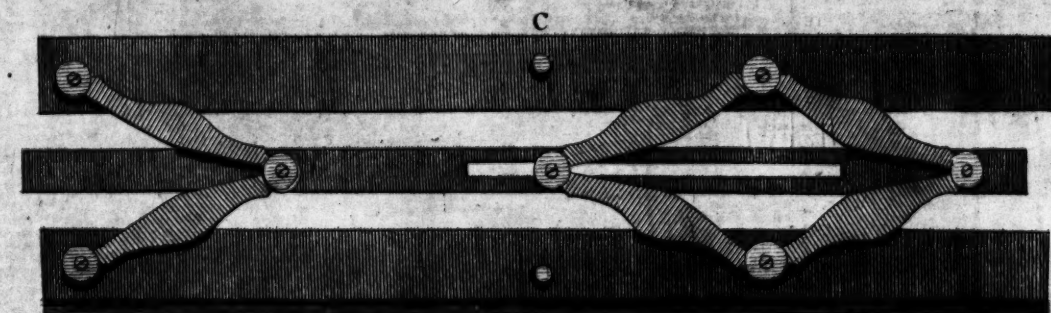
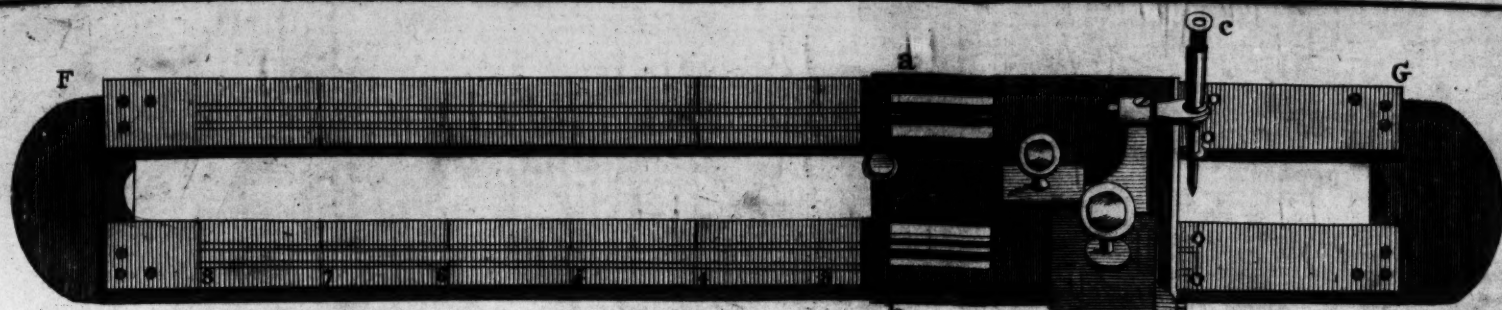
£. s. d.

- An apparatus for shewing that fluids have weight
 Ditto for shewing that the particles of fluids exercise their pressure independently one of the other
 Ditto to shew that fluids press in every direction
 Ditto to demonstrate the lateral pressure of fluids
 Ditto to shew that, *cæteris paribus*, the pressure of fluids is as their perpendicular height
 The hydrostatic paradox
 The hydrostatic bellows
 Apparatus for illustrating the laws of pressure and equilibrium between heterogeneous fluids
 Ditto for illustrating the action of fluids upon bodies immersed in them
 An apparatus for experiments on spouting fluids
 Hydrometers for proving spirits, from 1l. 11s. 6d. to 4 14 6
 An apparatus for making experiments on capillary tubes
 The model of the diving bell
 A glass model of the lifting pump
 A ditto of the lifting and forcing pump
 Hiero's fountain in copper japanned
 Ditto double
 Fountain of command
 A japanned copper fountain to act by condensed air with a variety of jets
 Apparatus for experiments on syphons.









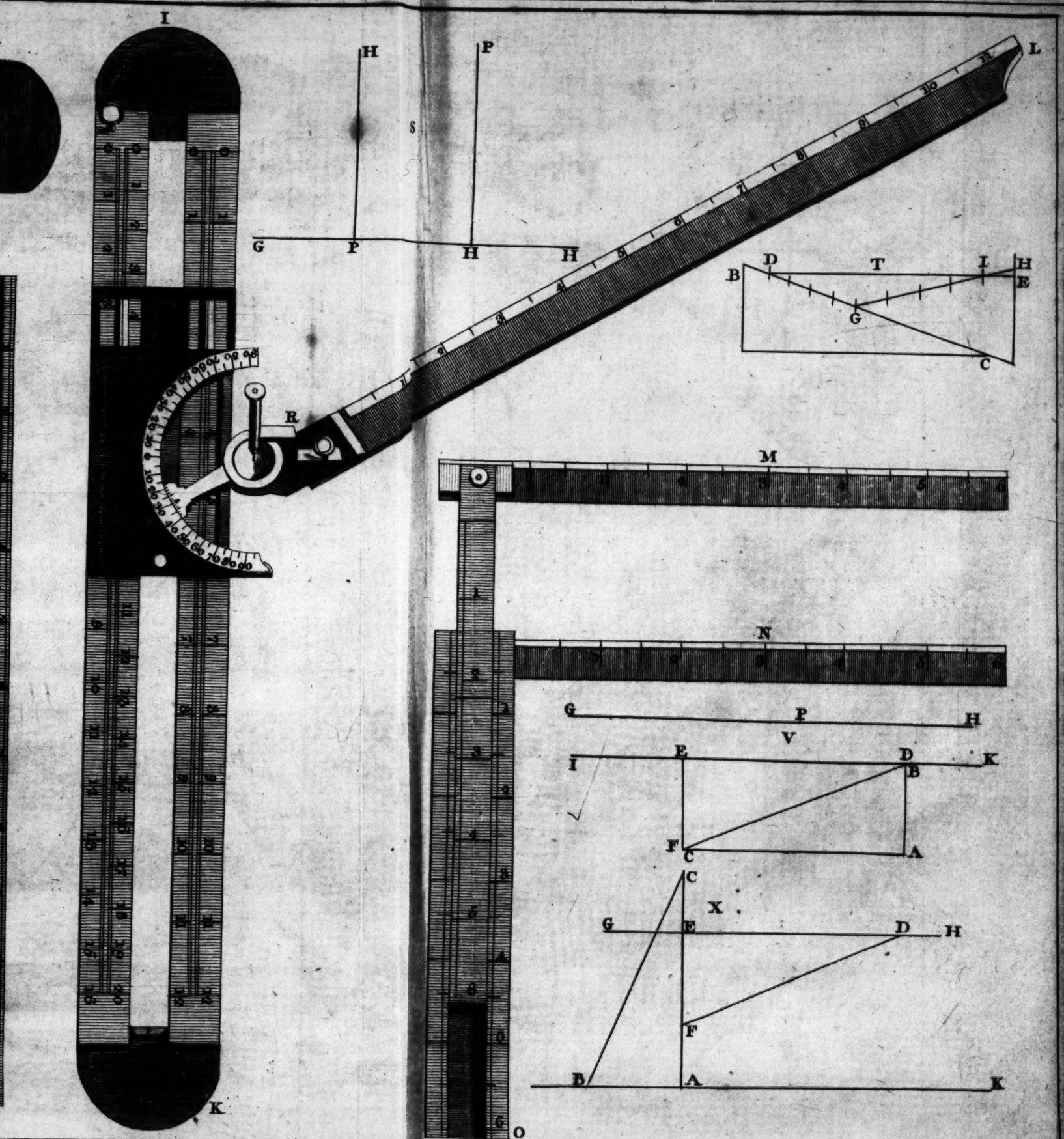


Fig. 1.

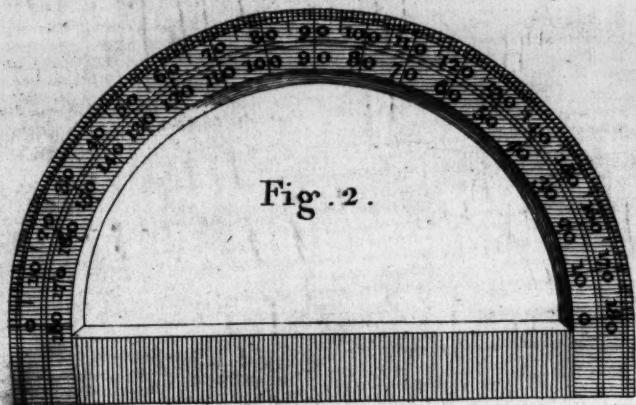


Fig. 2.

Fig. 6.

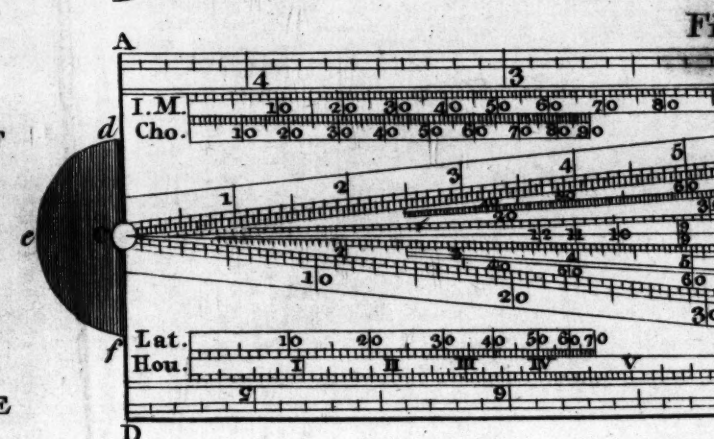
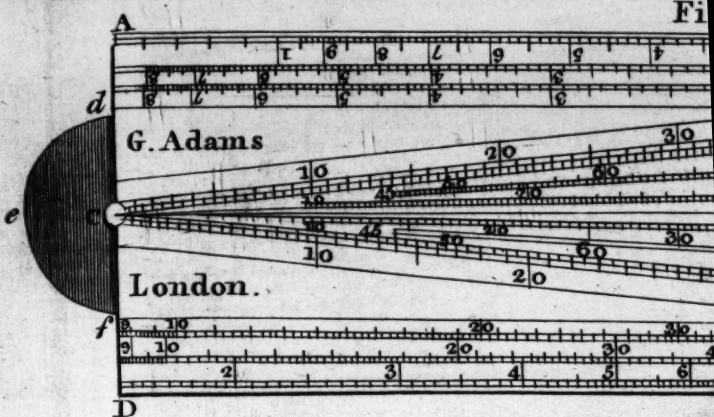
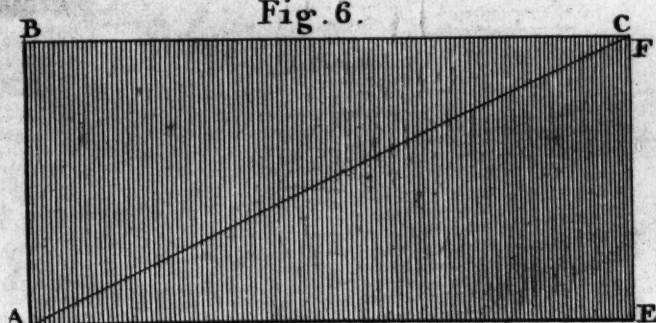


Fig. 14.

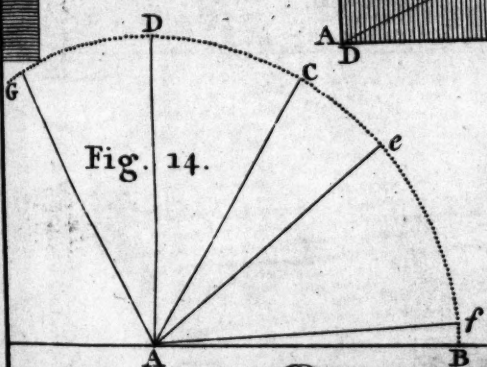


Fig. 15.

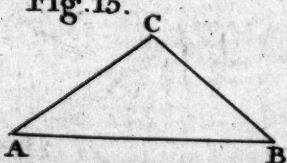


Fig. 16.

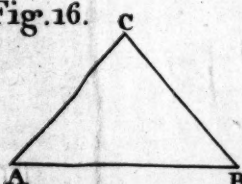


Fig. 17.

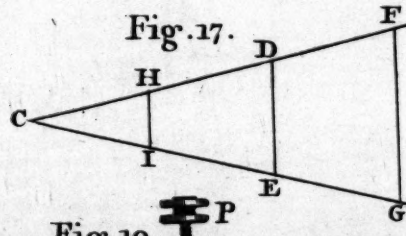


Fig. 10.

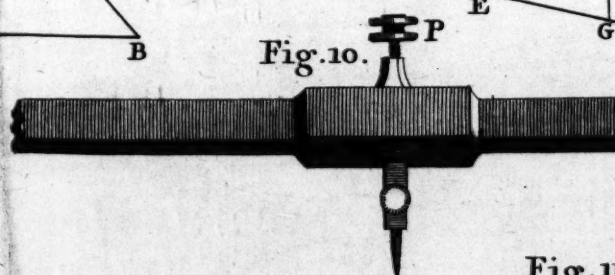


Fig. 13.

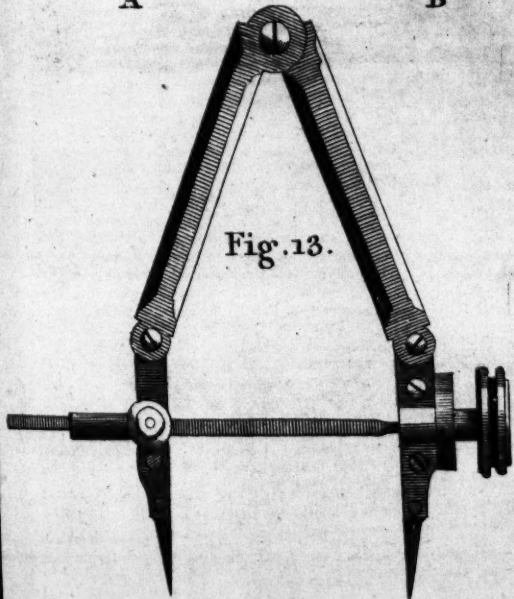


Fig. 12.

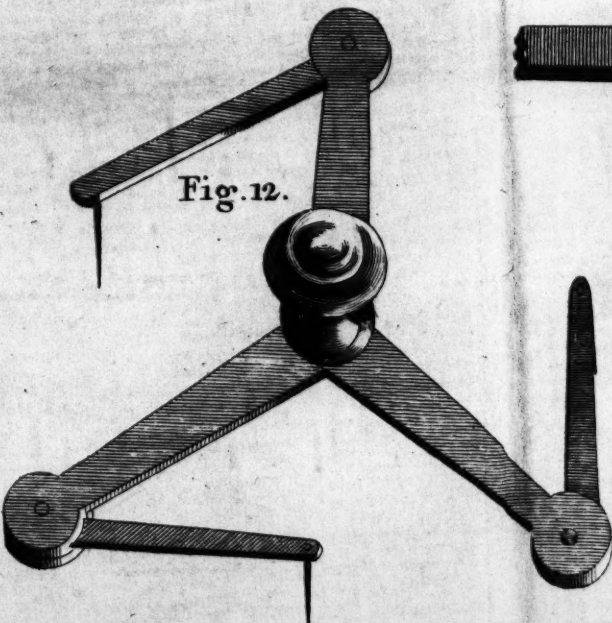
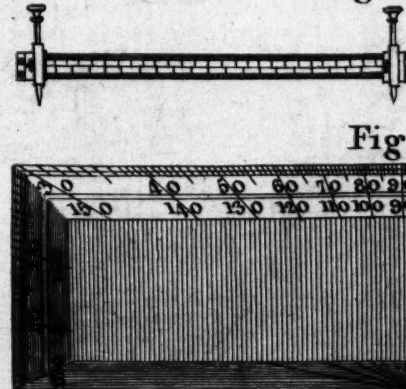


Fig. 11.



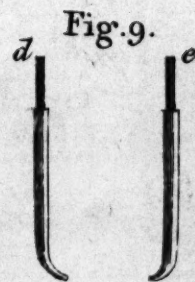
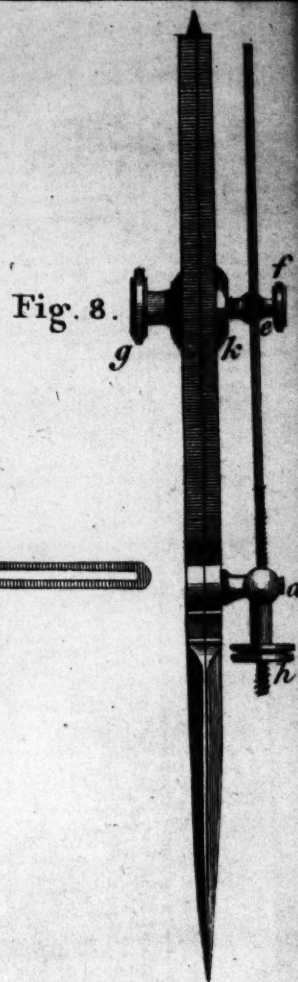
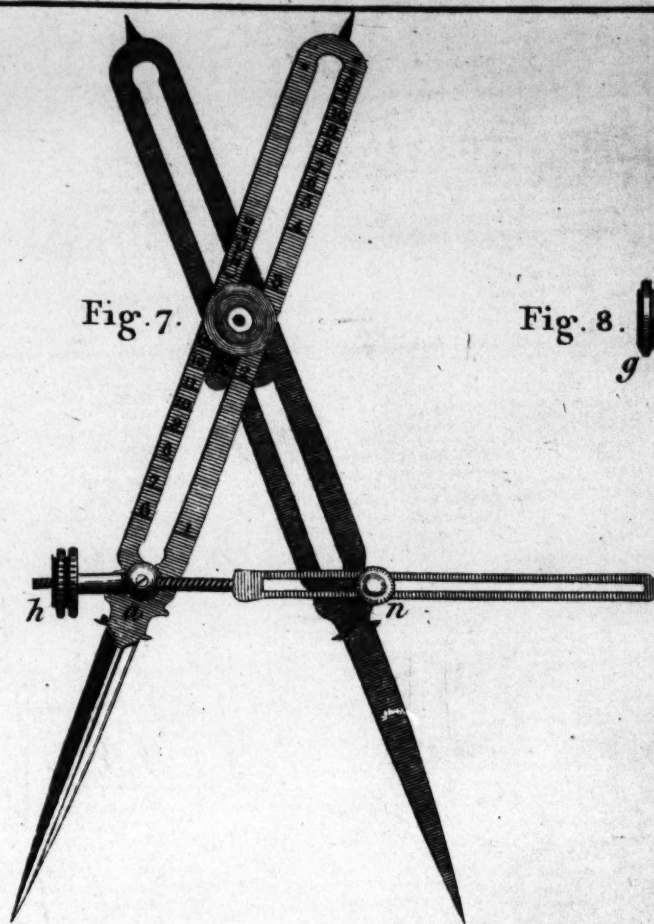
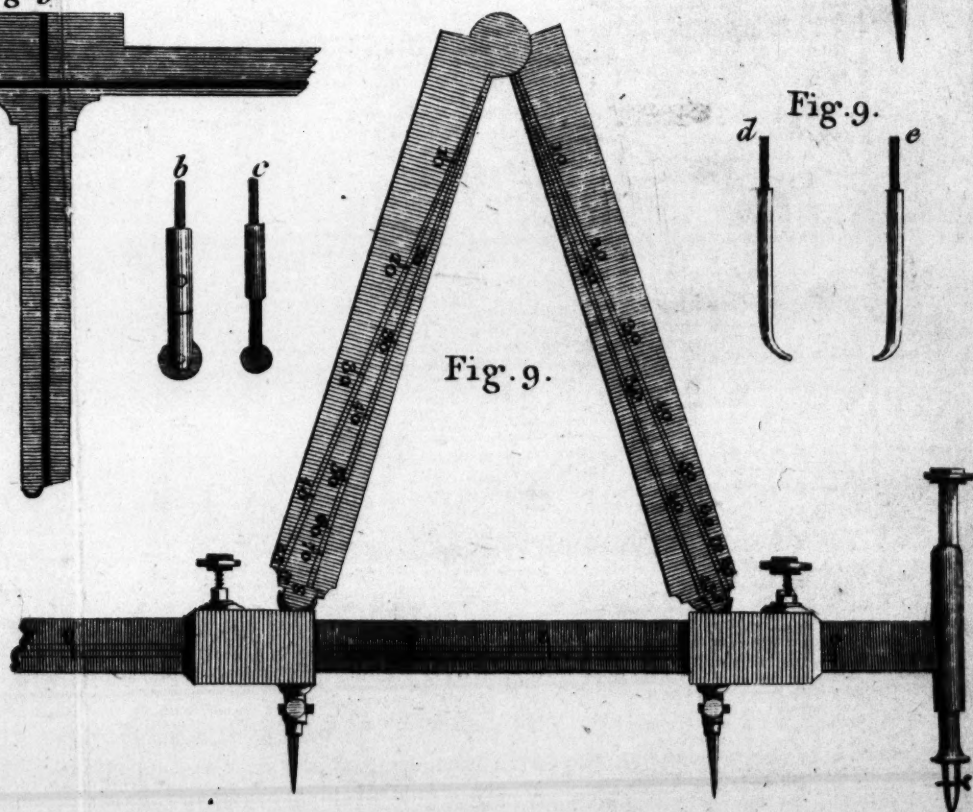
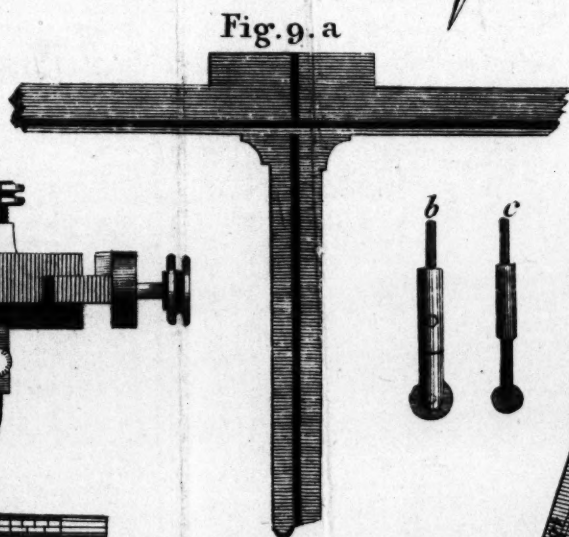
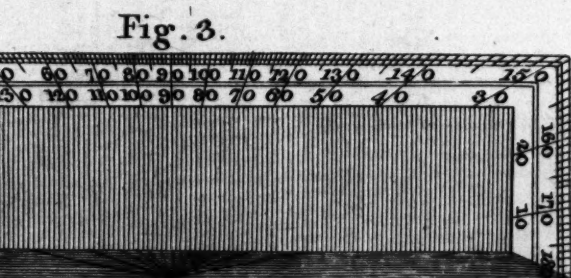
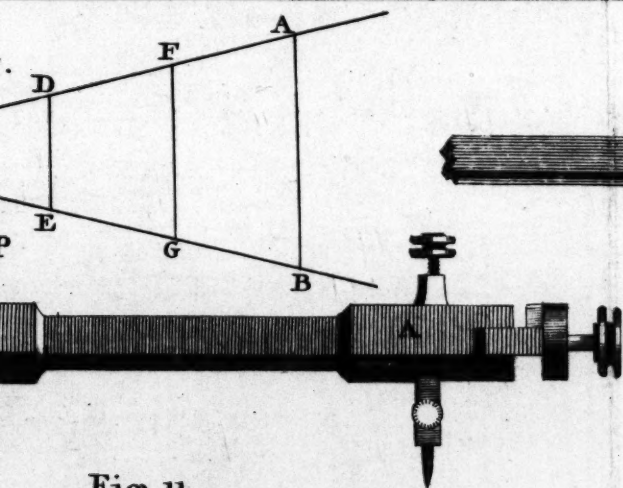
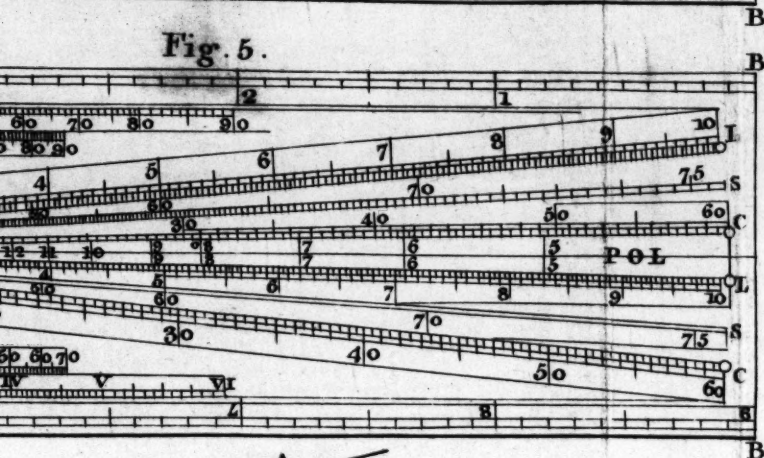
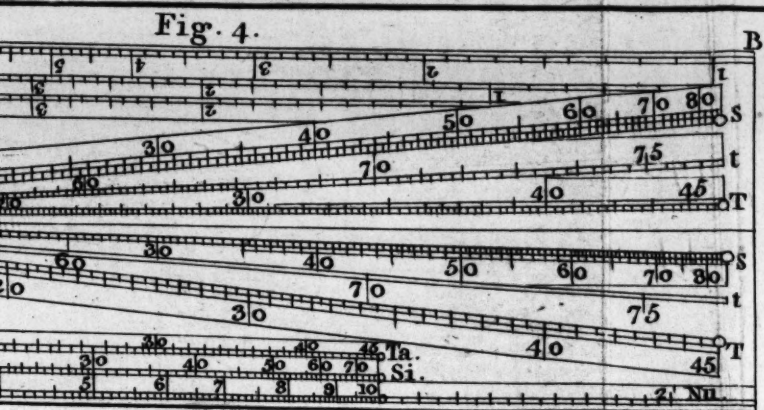


Fig. 1.

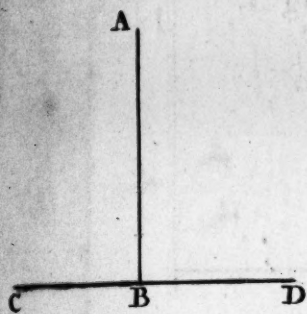


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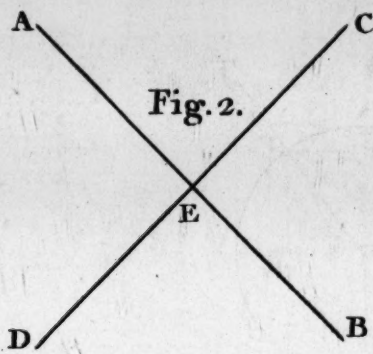


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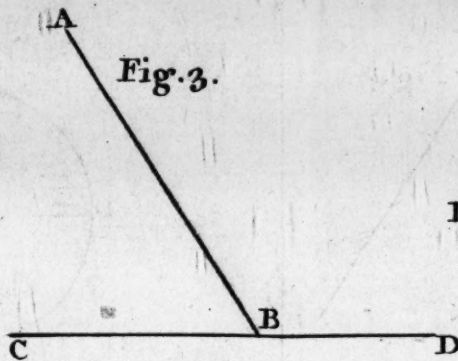


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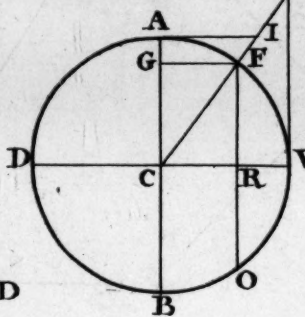


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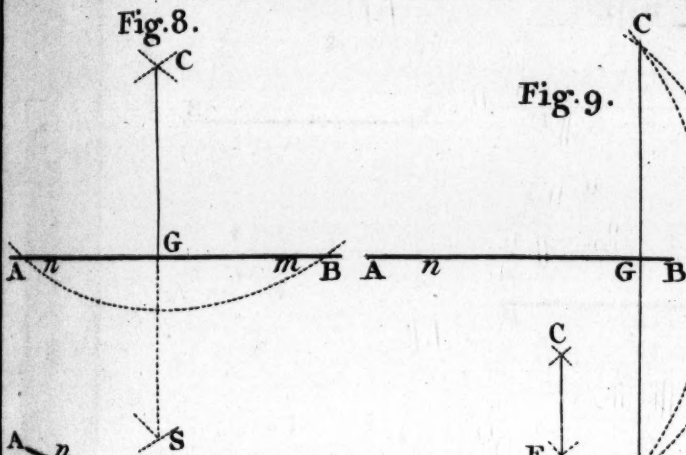


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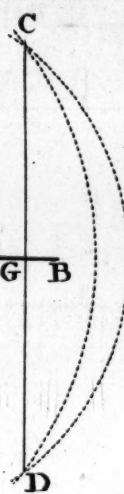


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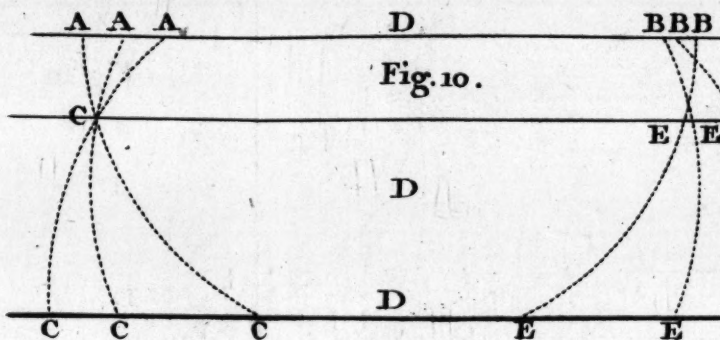


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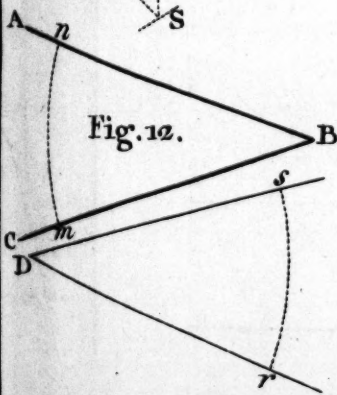


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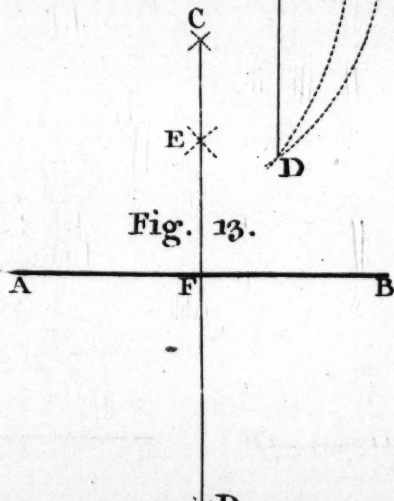


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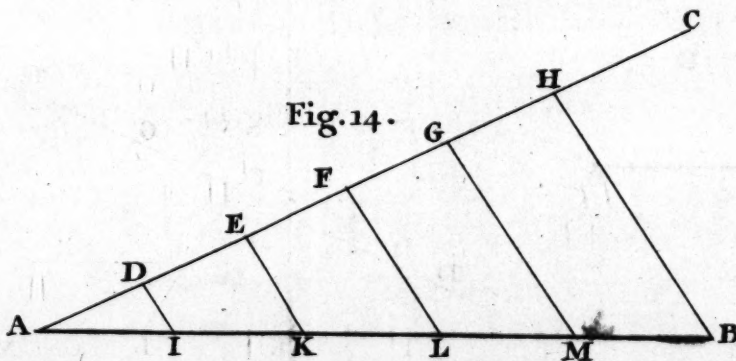


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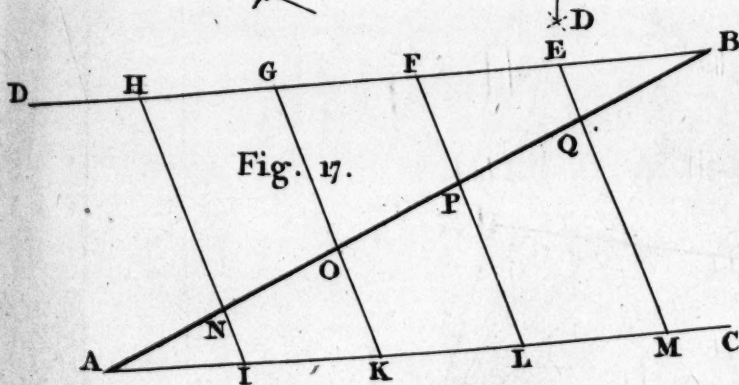
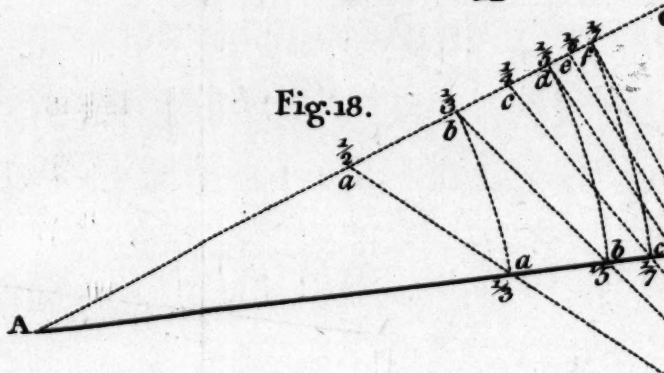
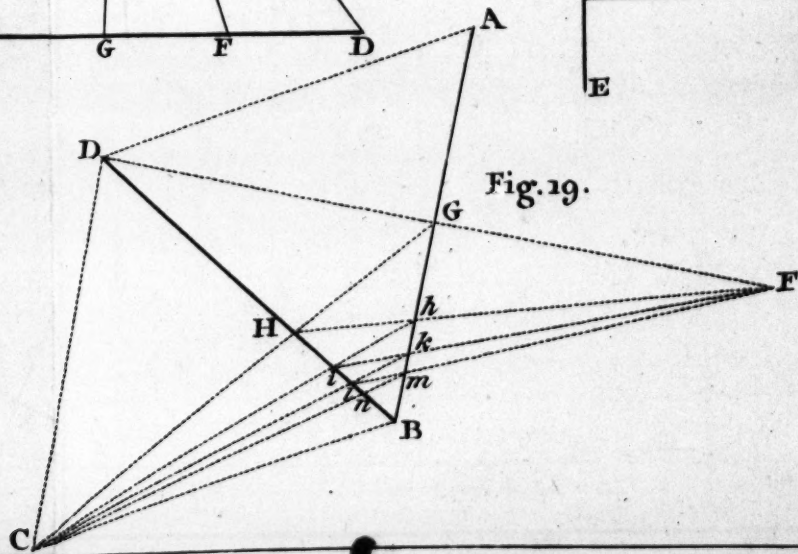
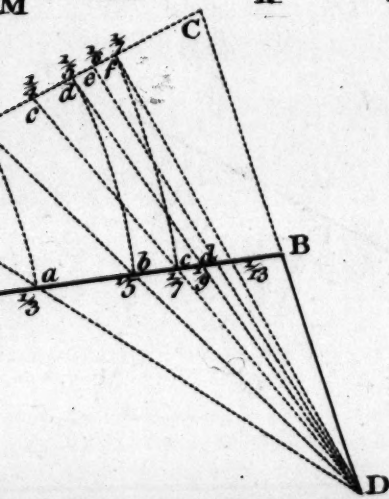
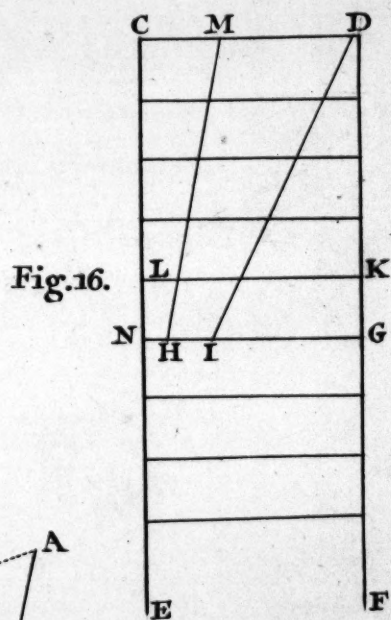
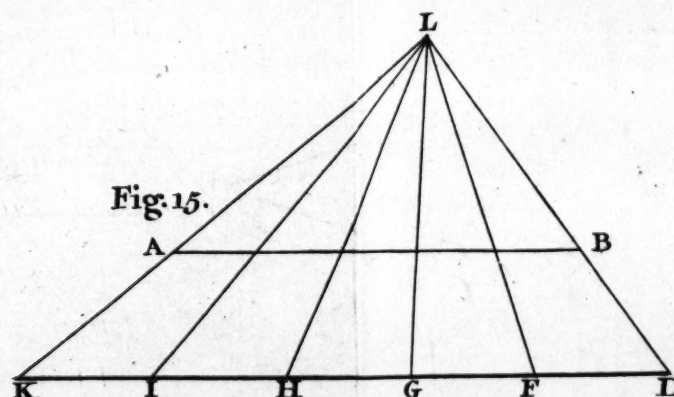
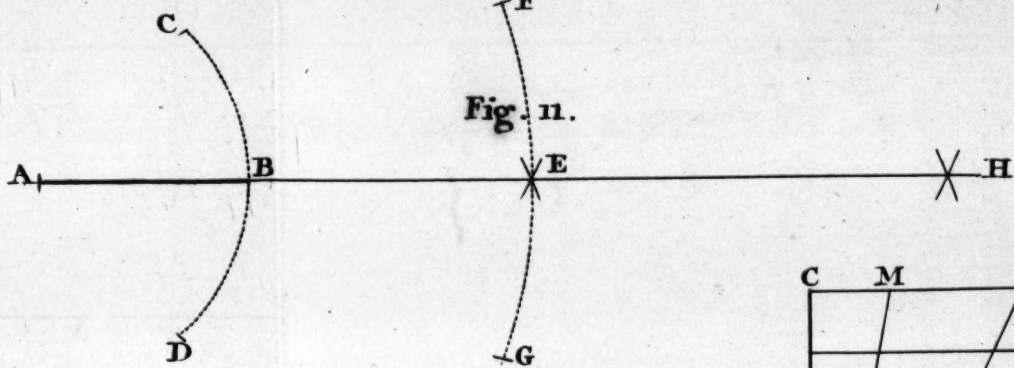
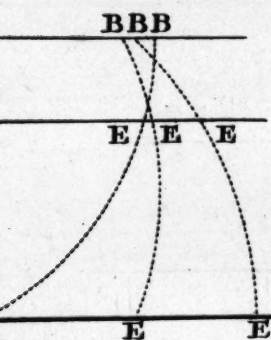
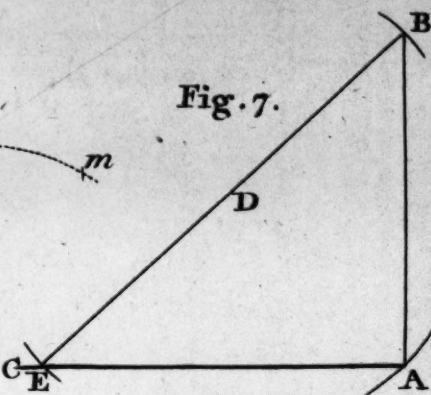
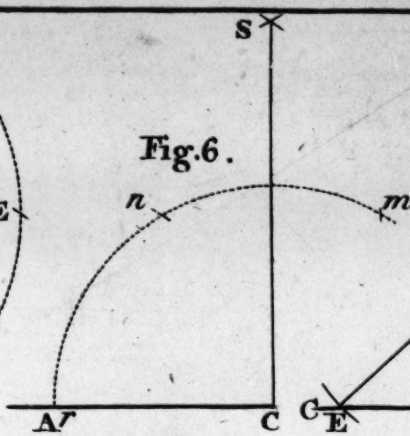
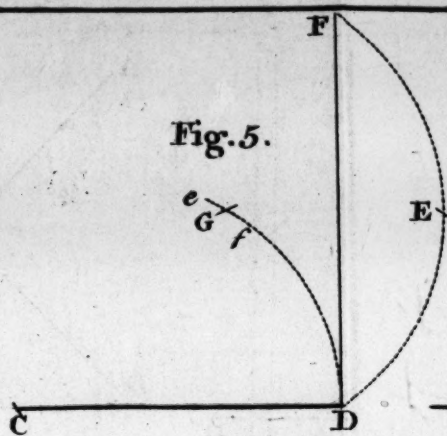
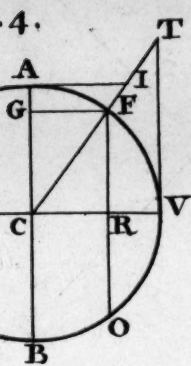
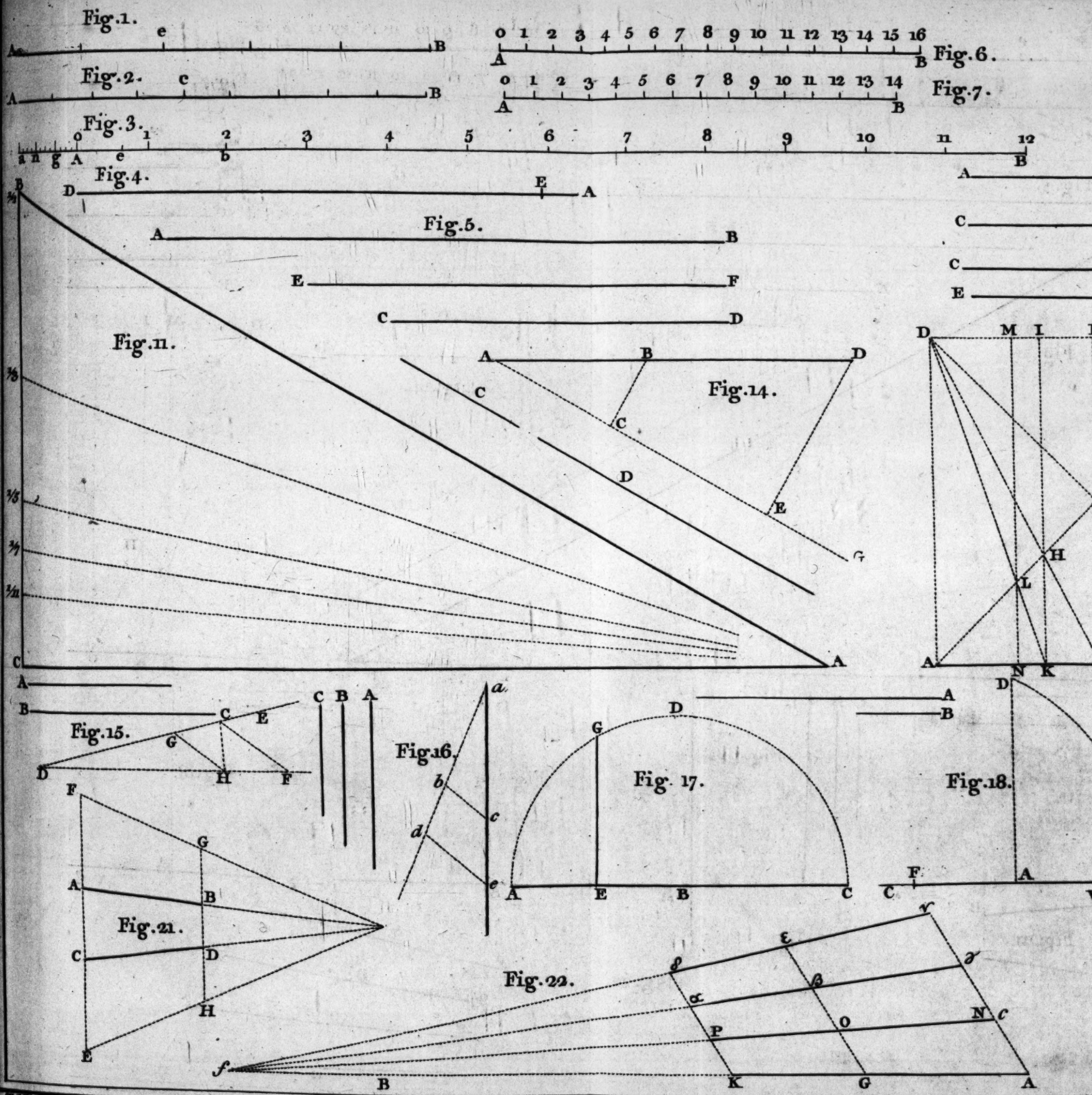


Fig. 18.







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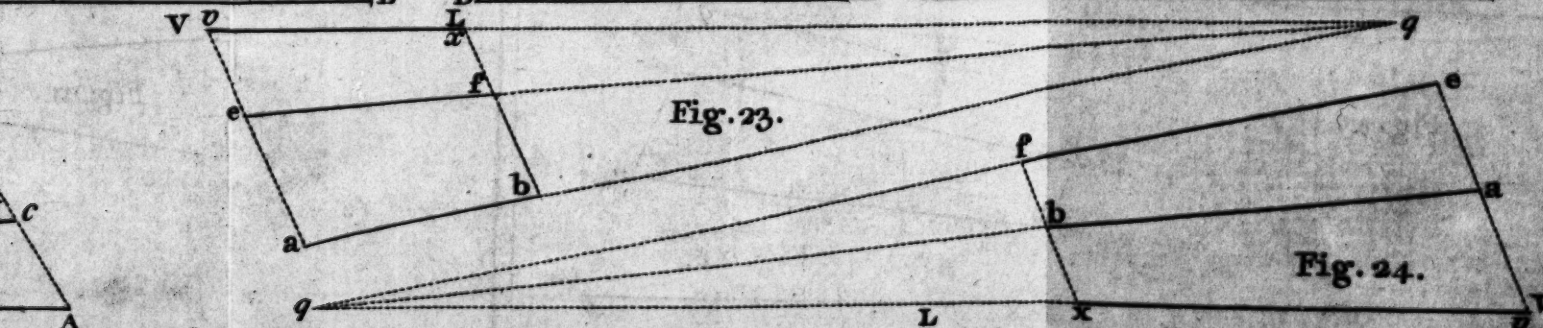
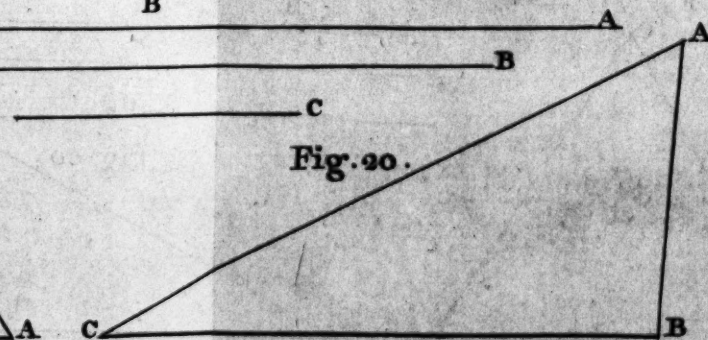
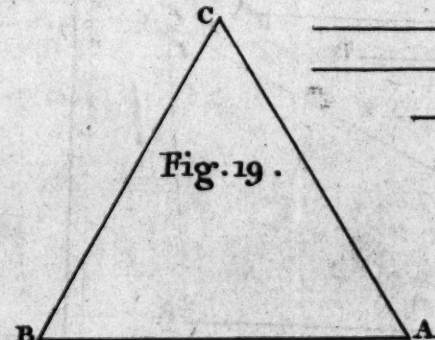
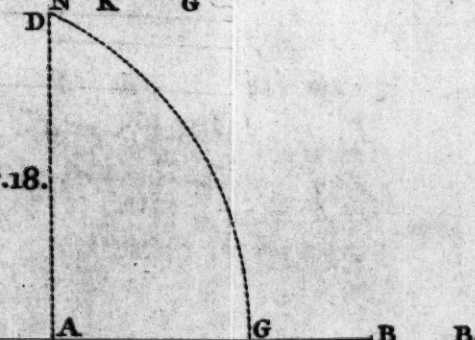
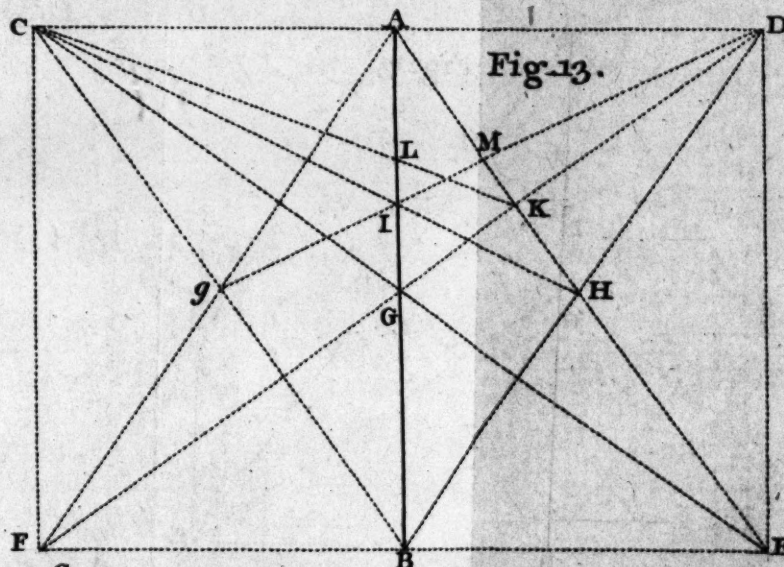
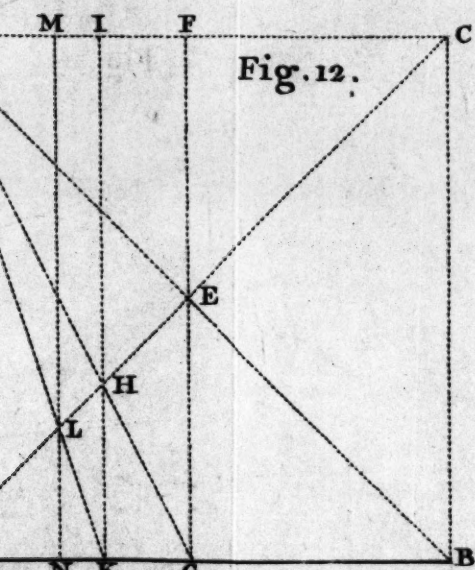
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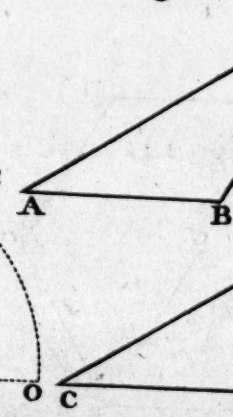
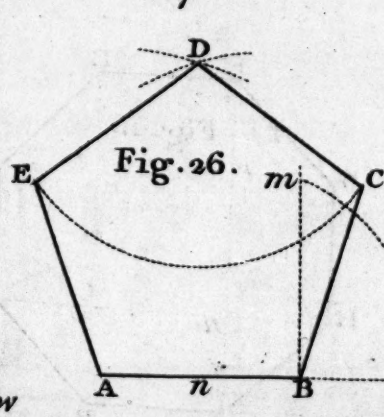
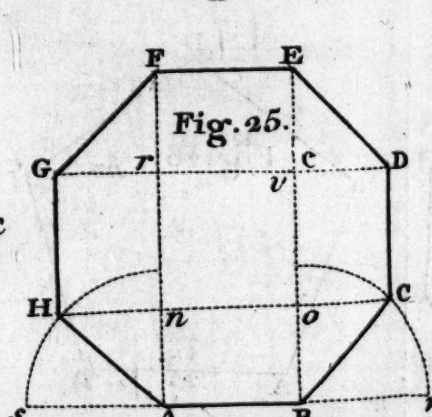
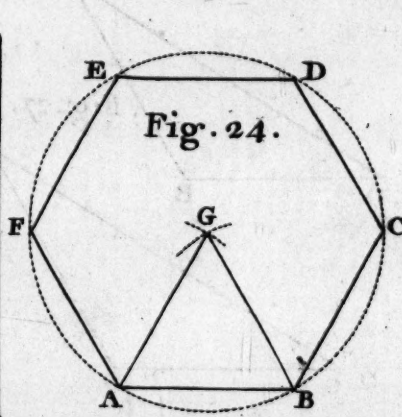
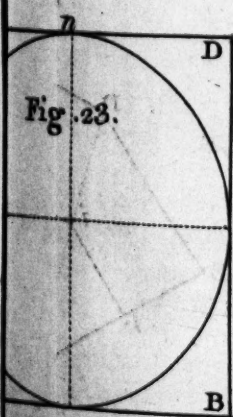
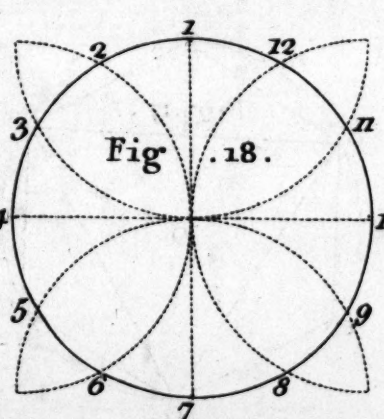
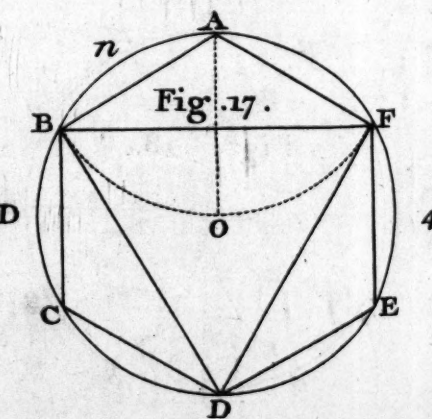
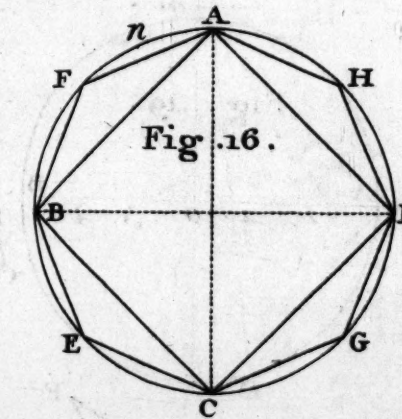
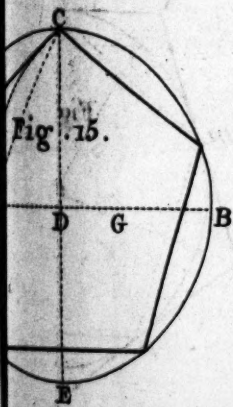
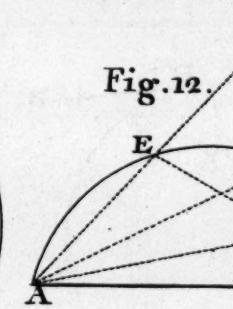
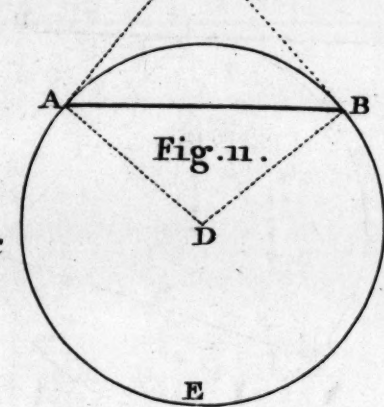
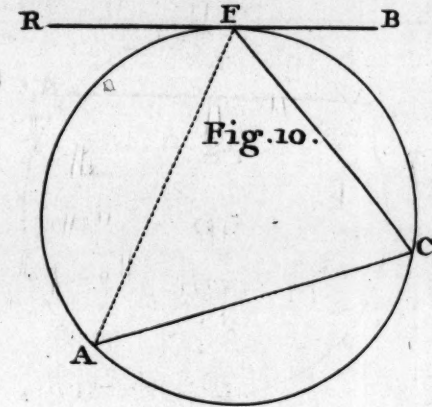
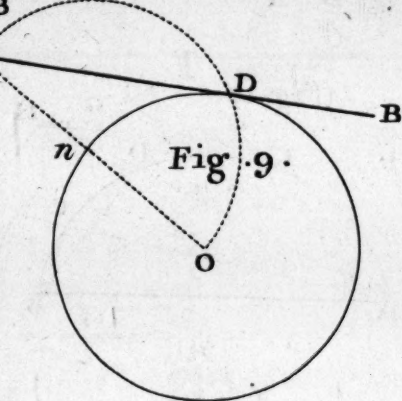
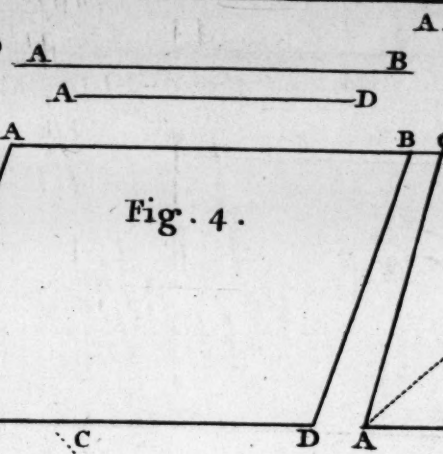
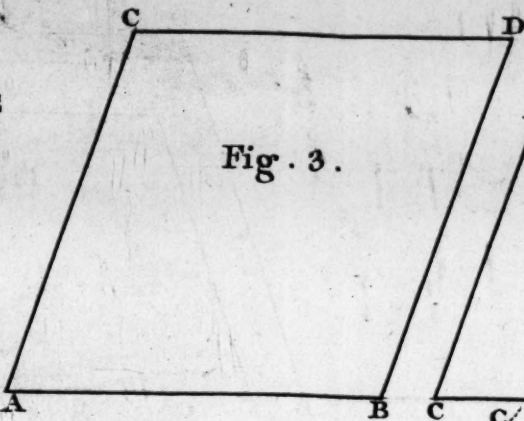
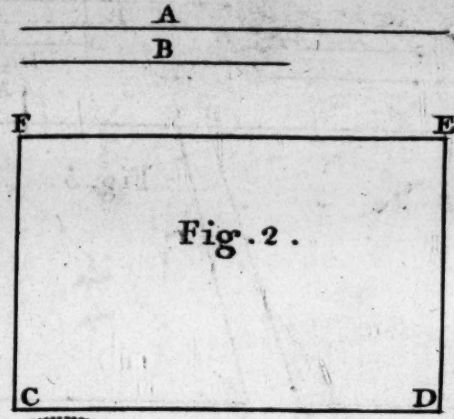
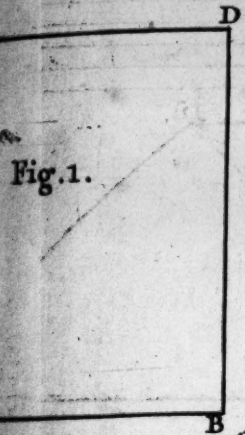
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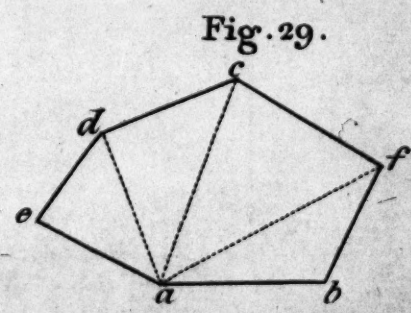
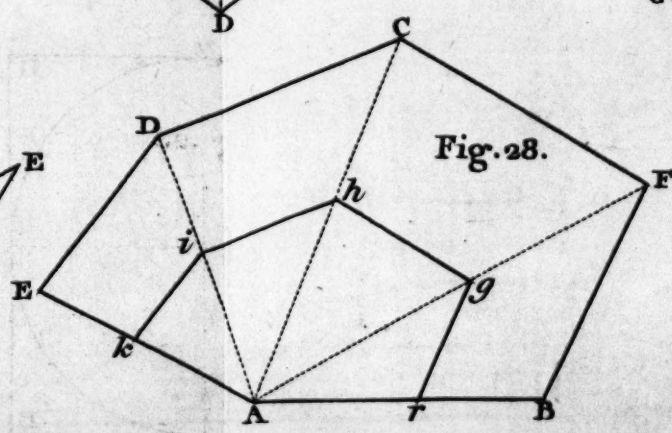
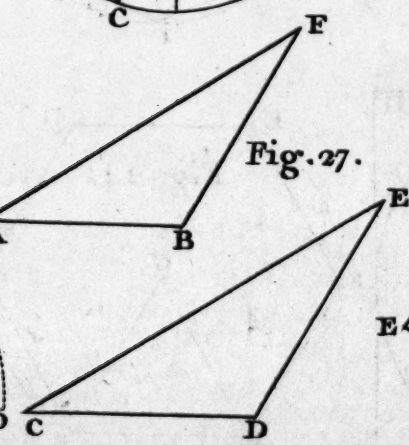
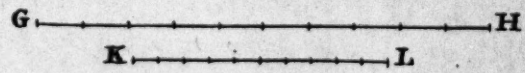
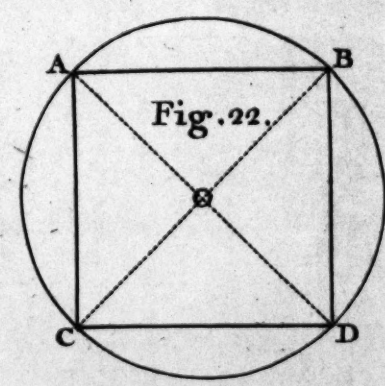
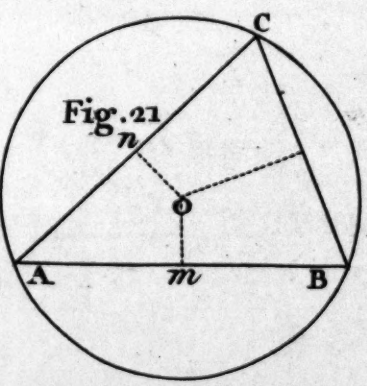
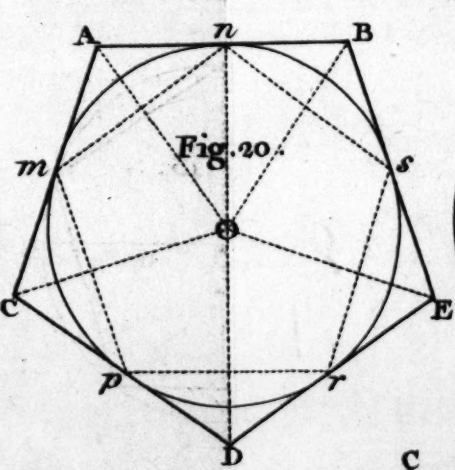
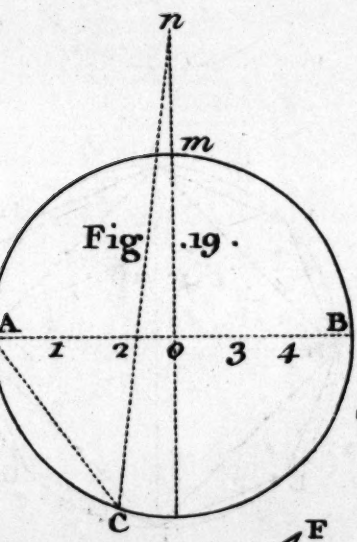
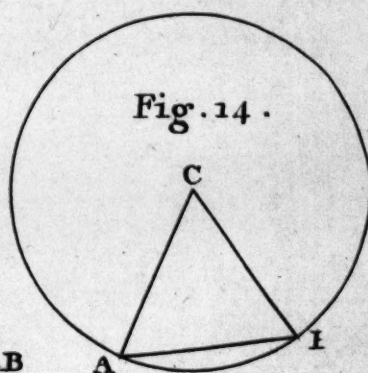
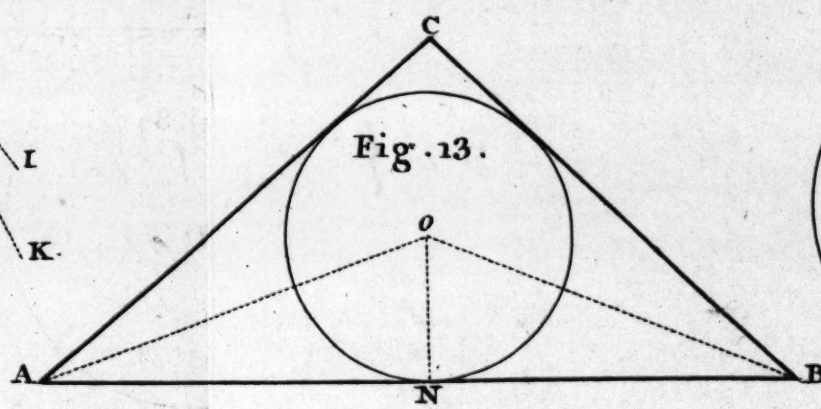
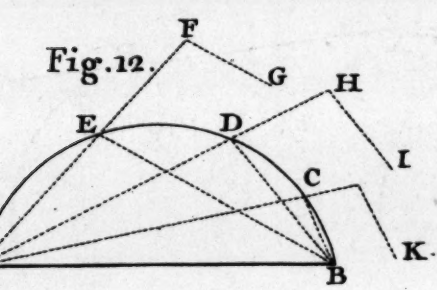
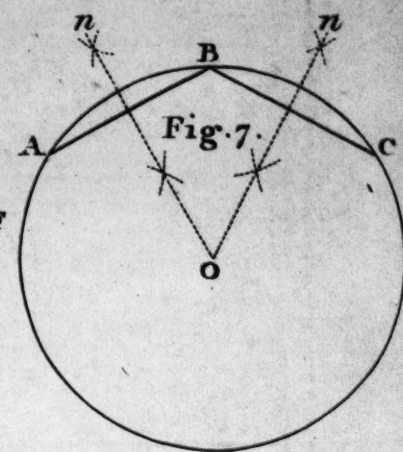
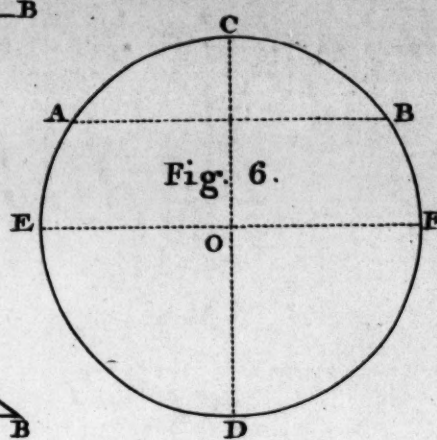
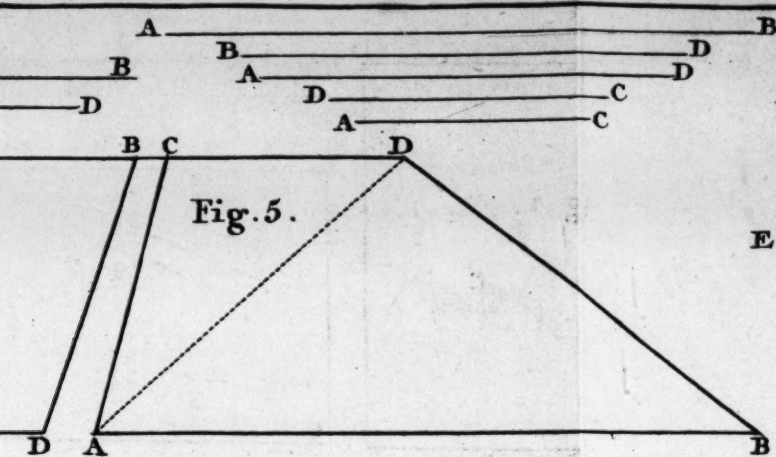
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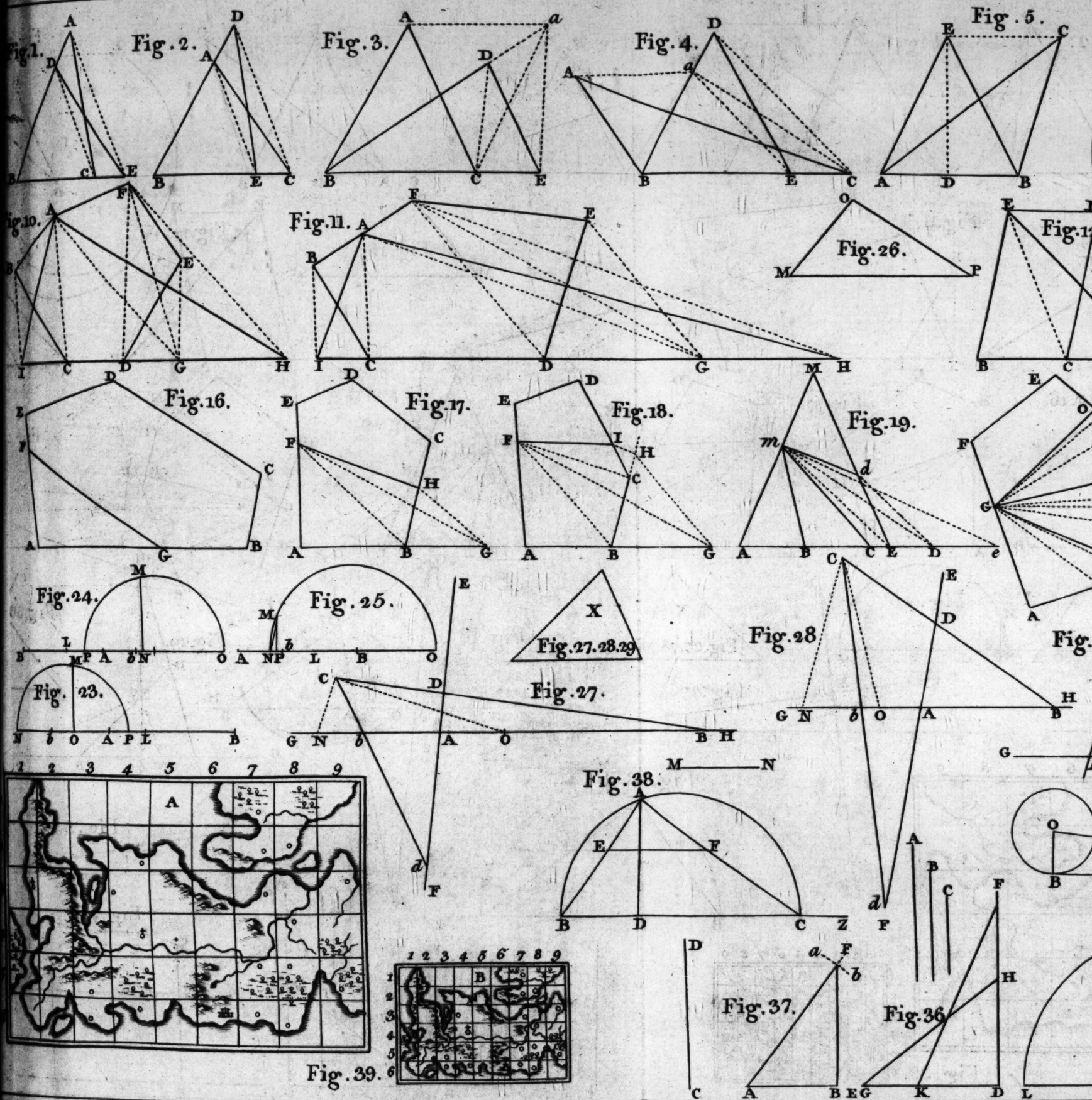
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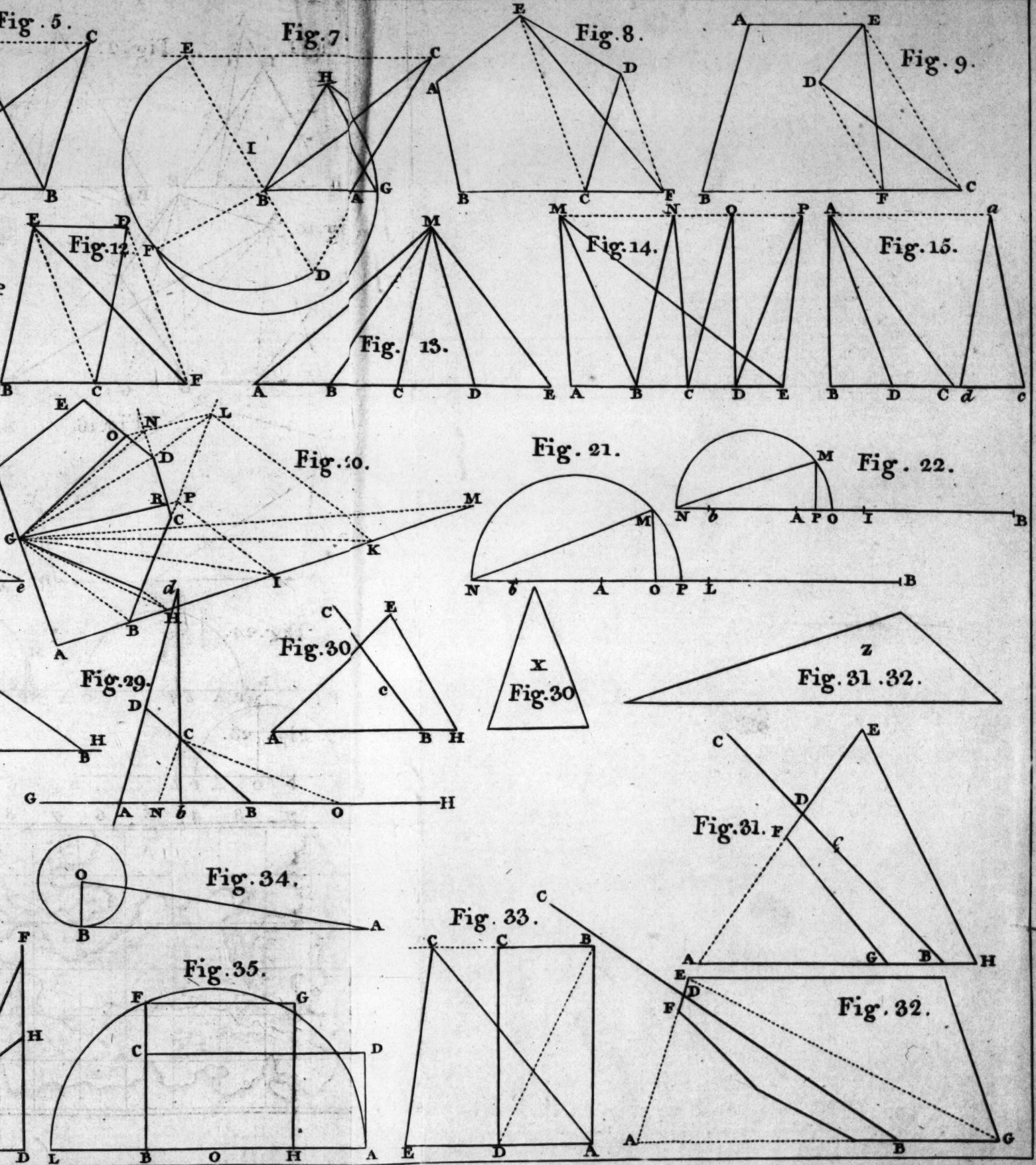
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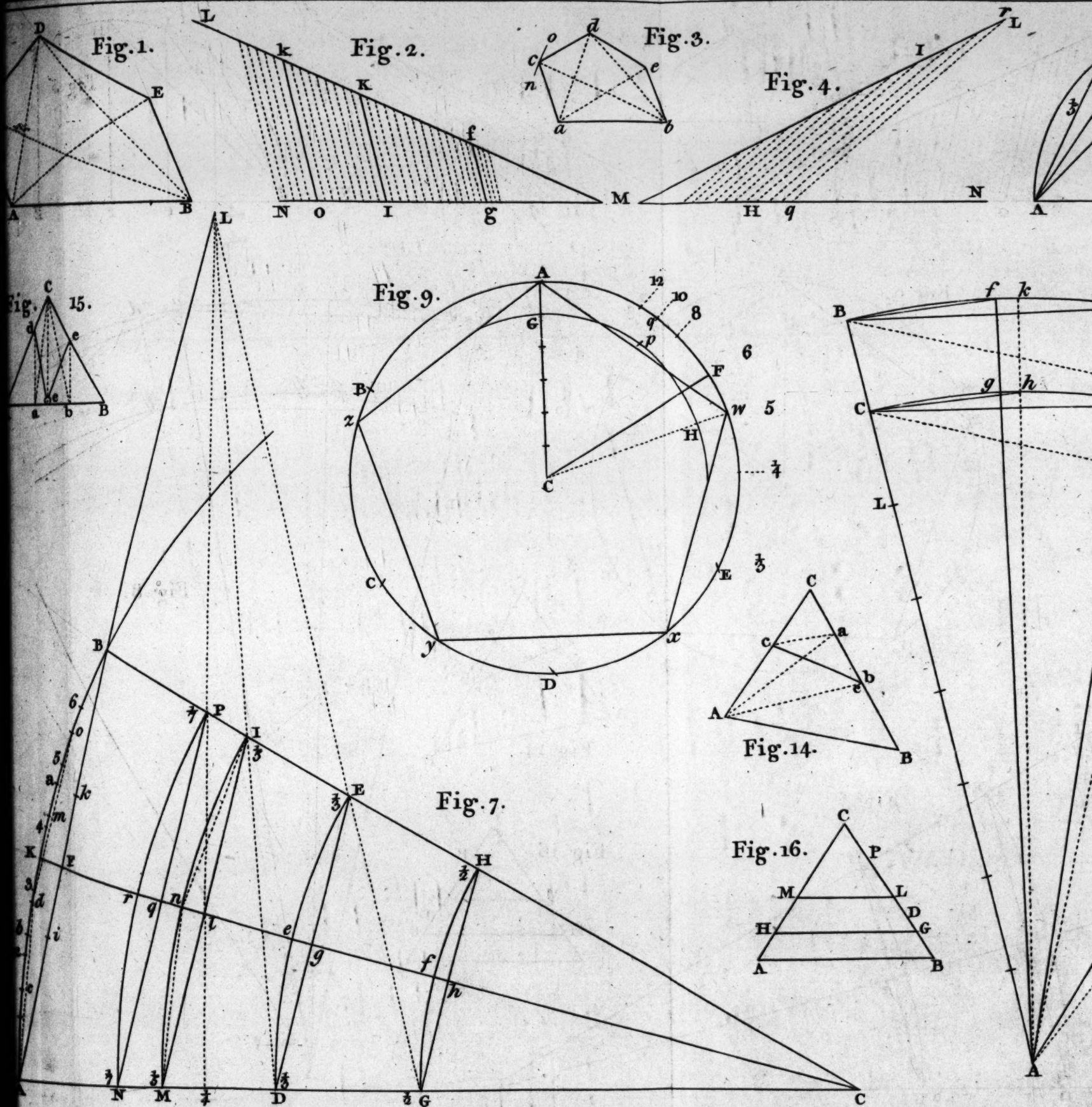


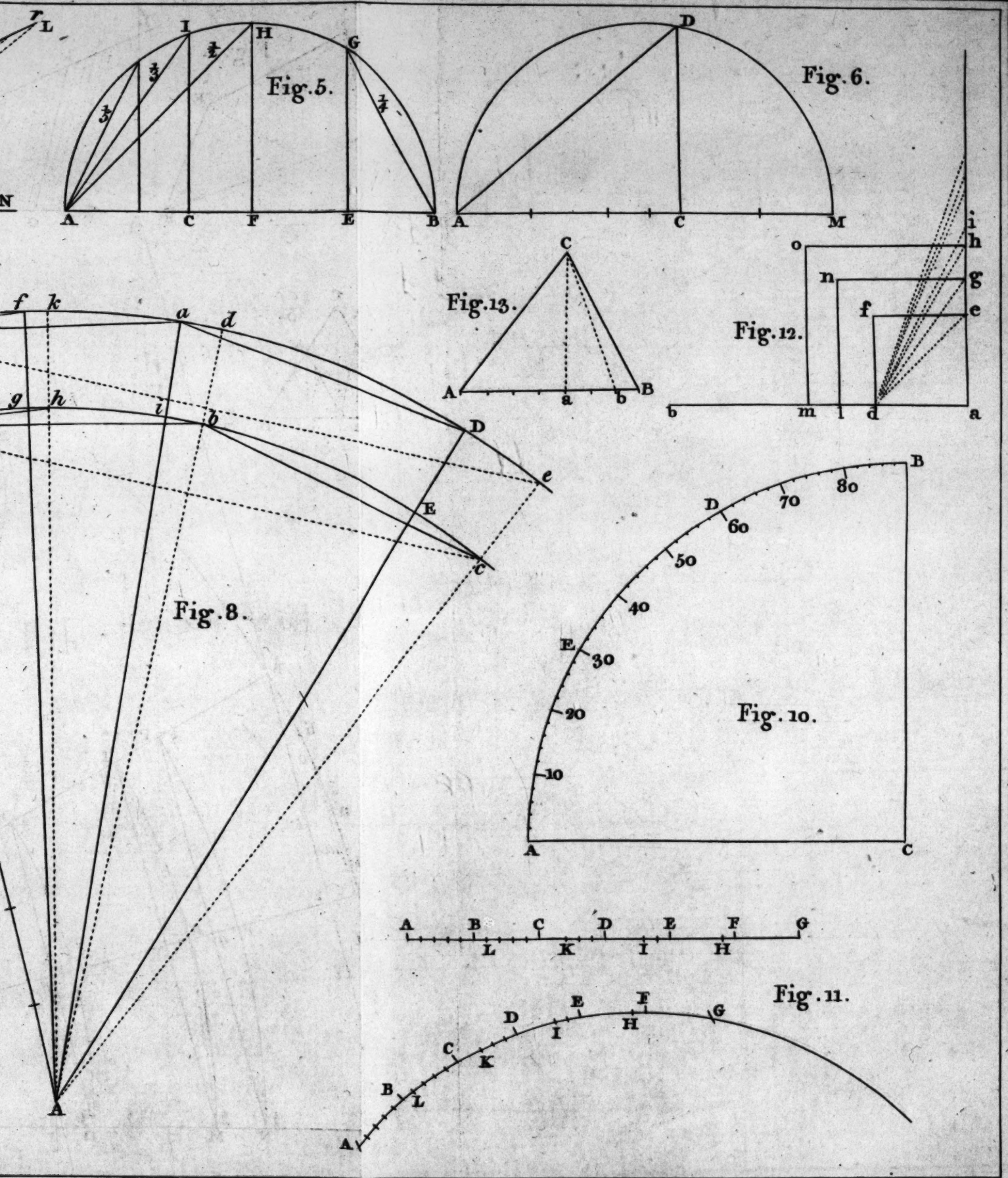


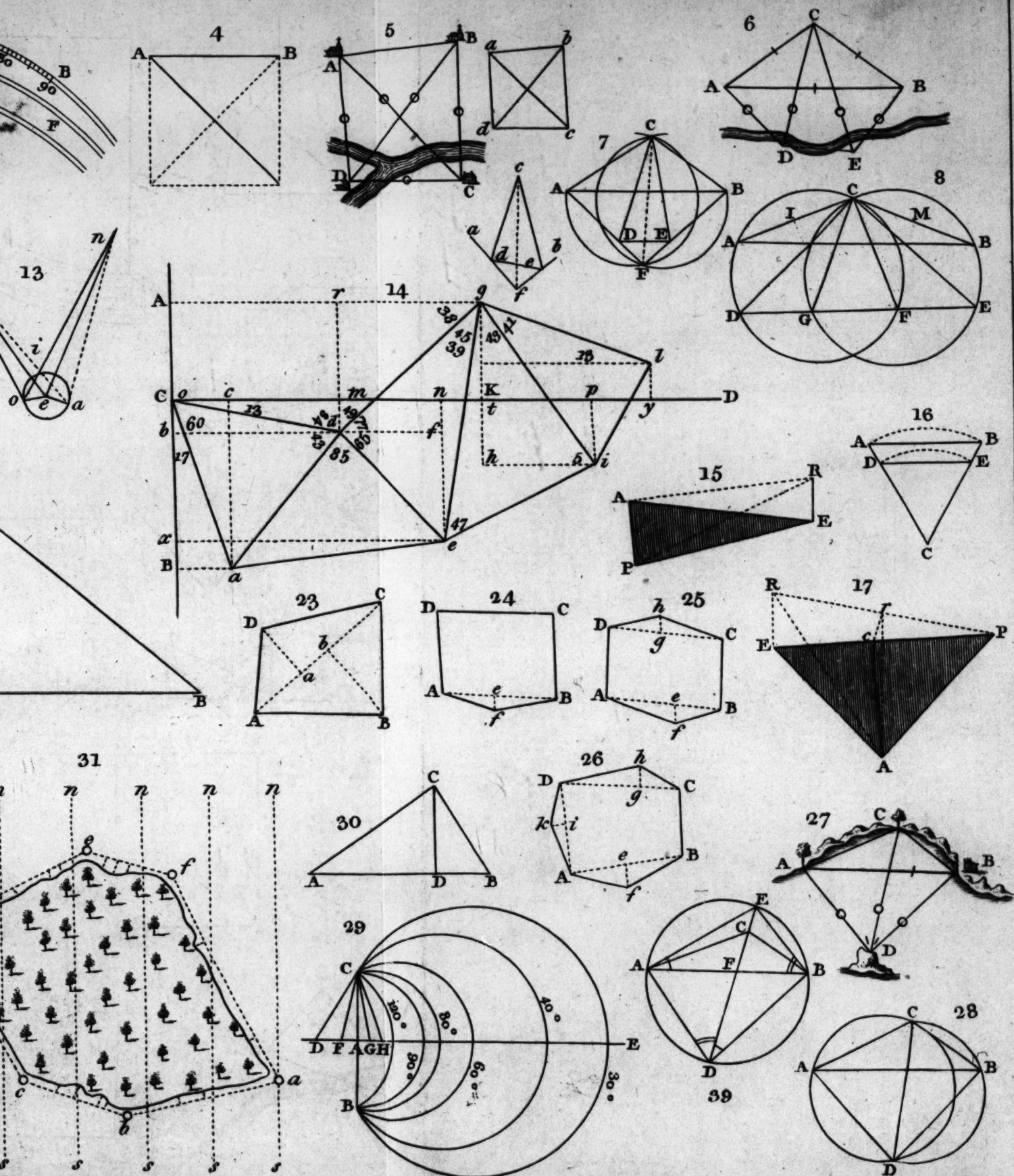












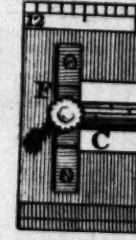
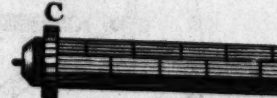
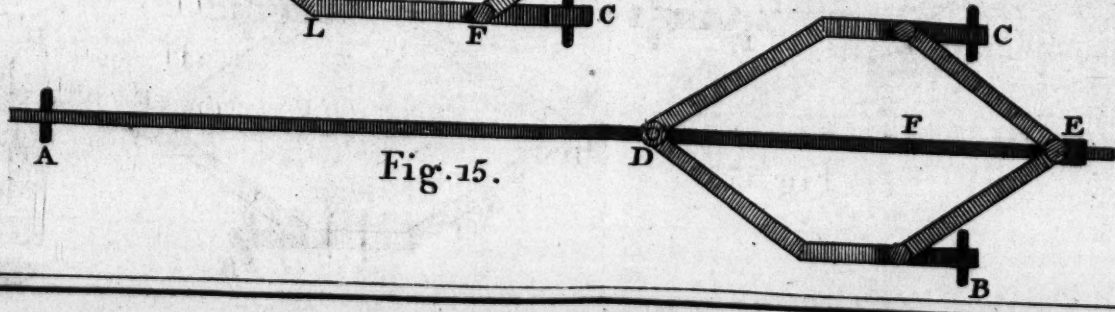
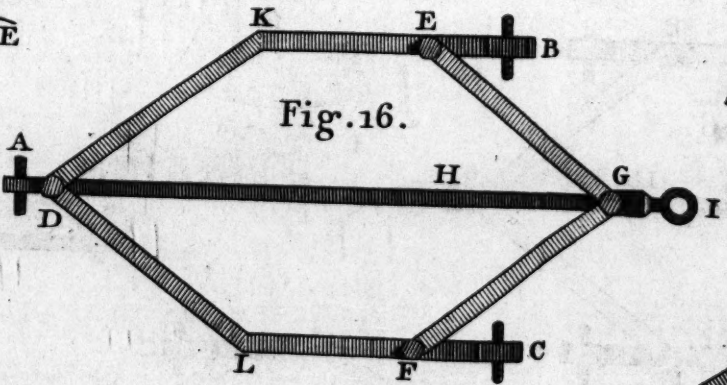
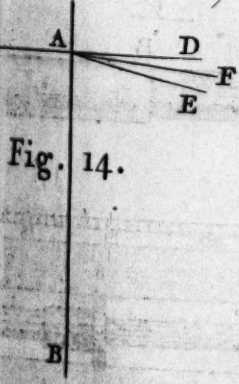
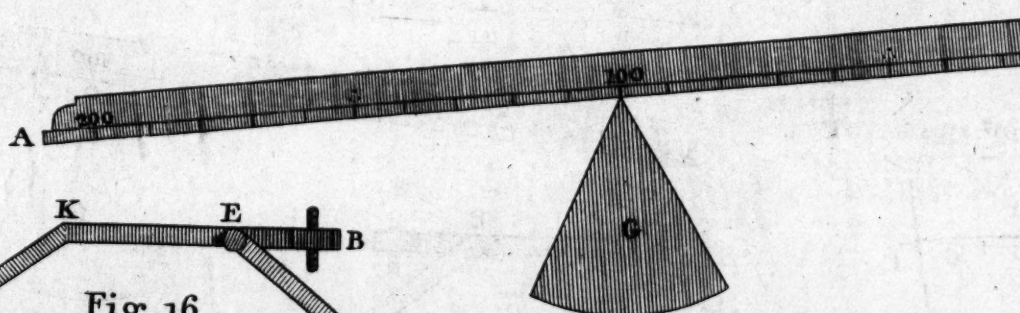
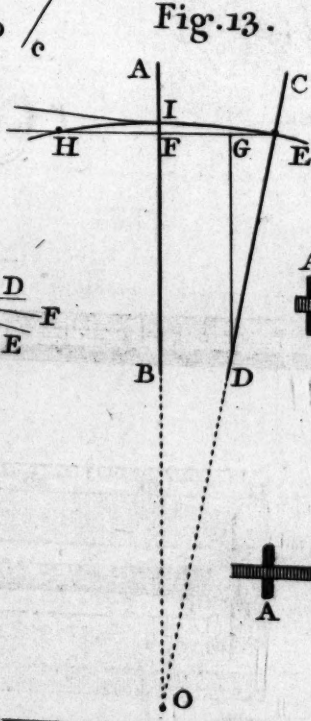
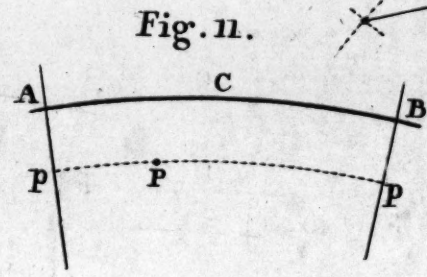
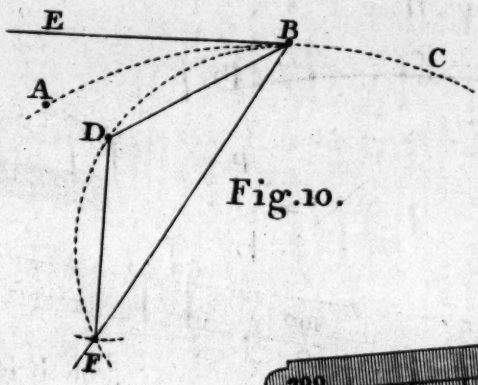
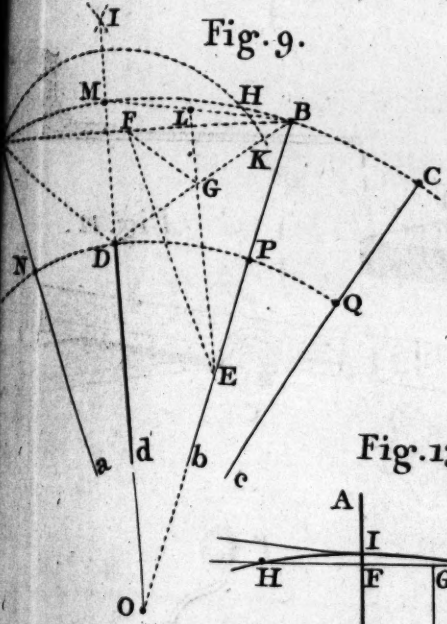
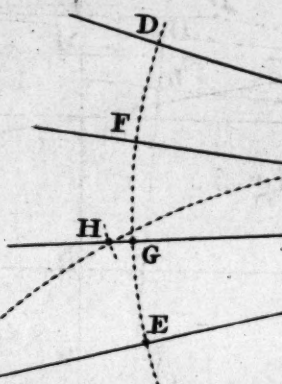
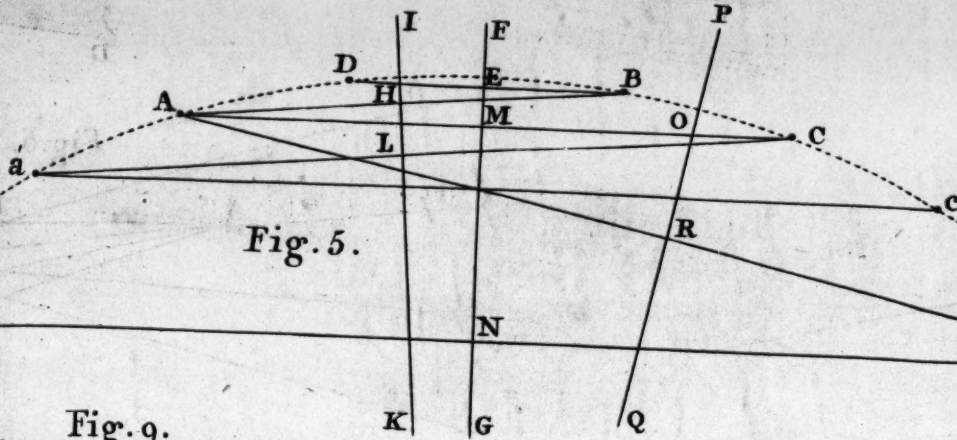
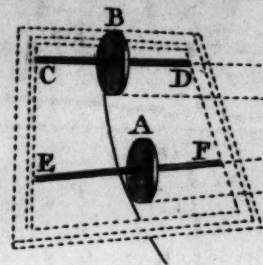
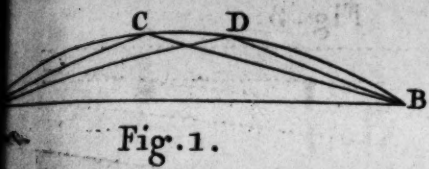


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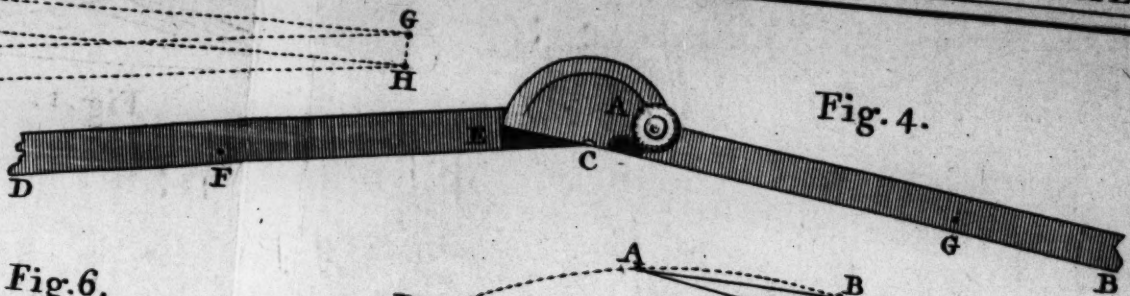


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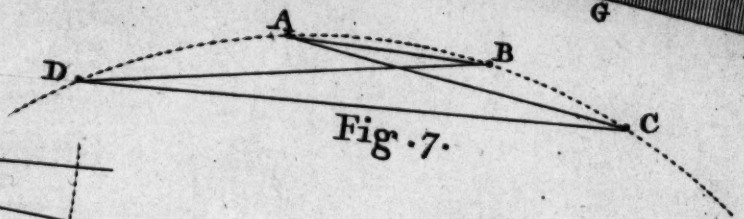


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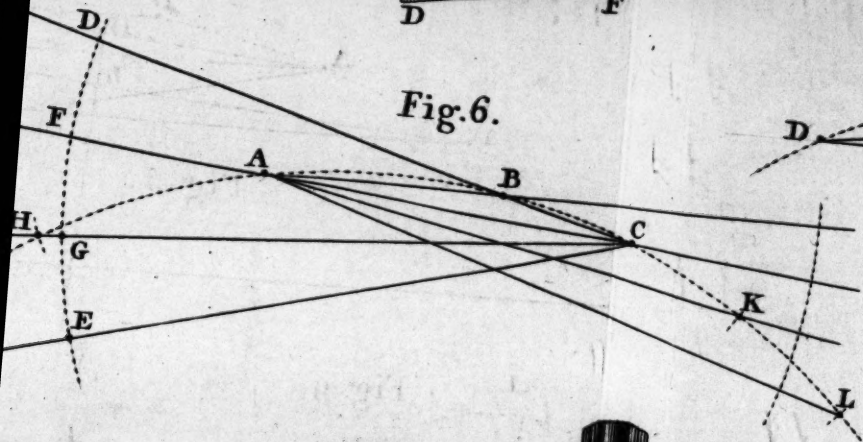


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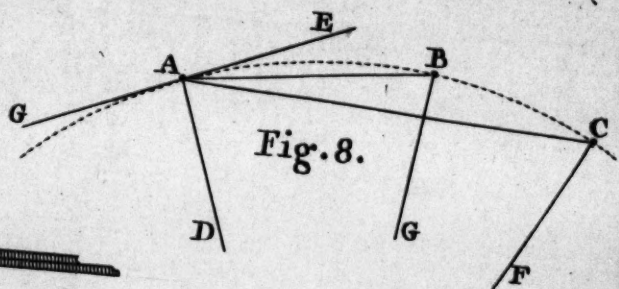


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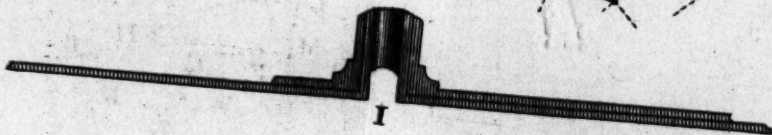


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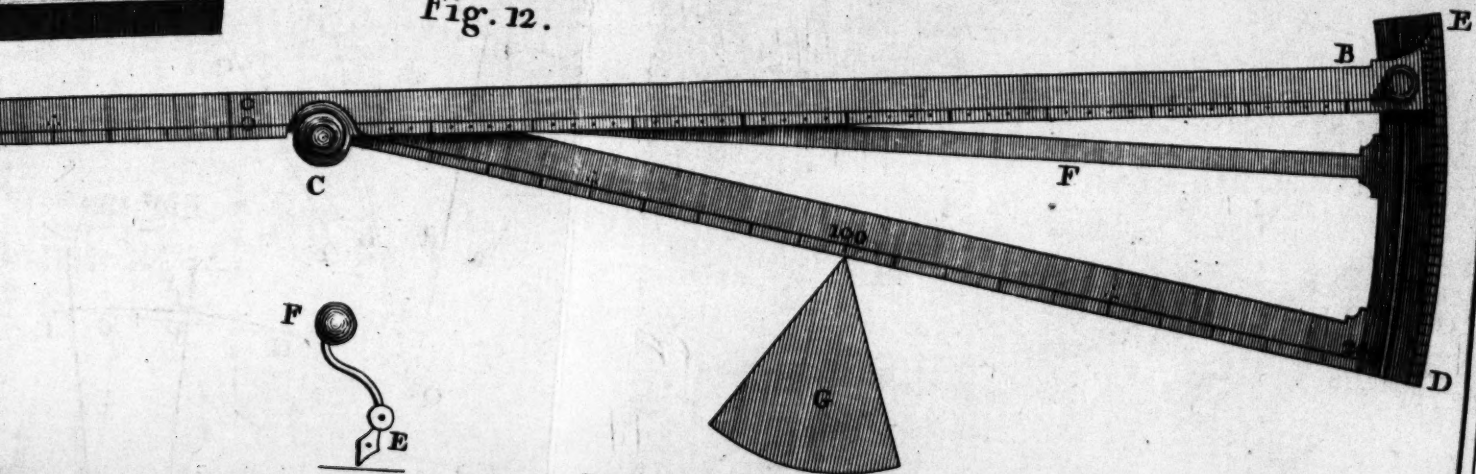
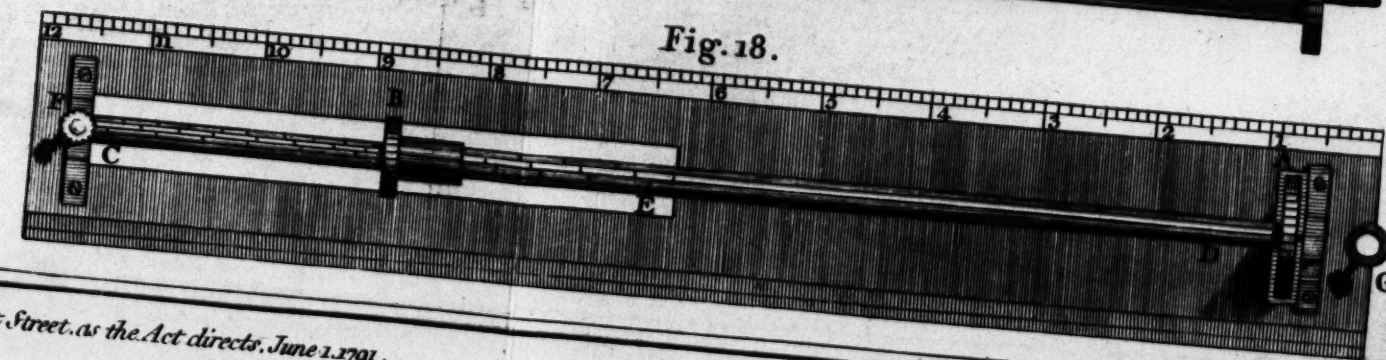


Fig. 17.



Fig. 18.



Street, as the Act directs, June 1. 1791.

In. Lodge sculp.

Fig.1.

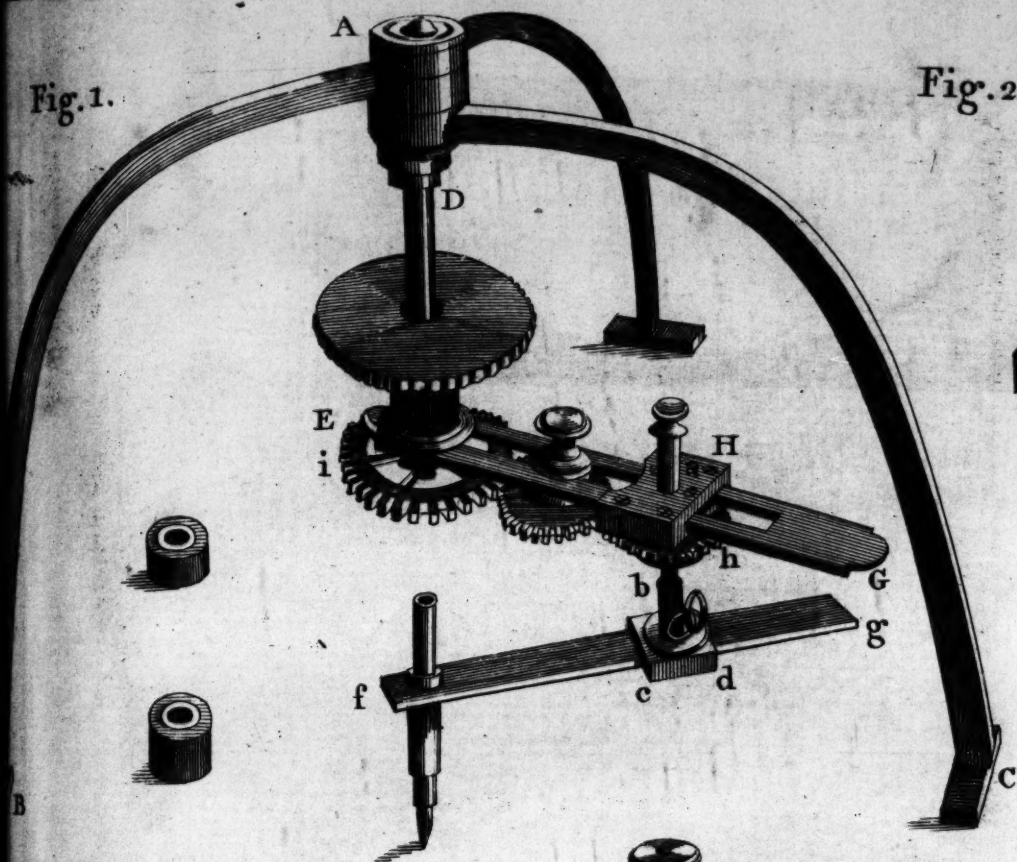


Fig.2.

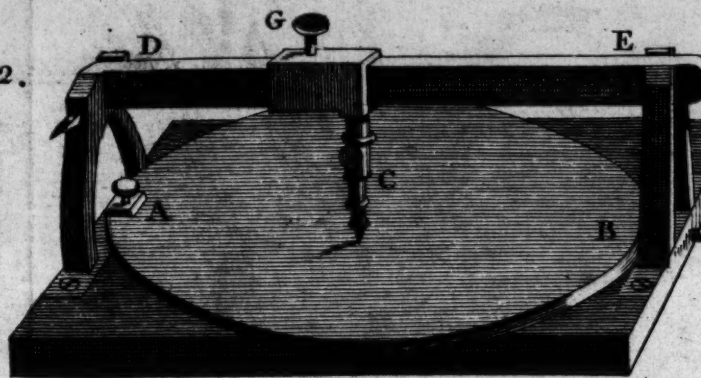


Fig.2.A

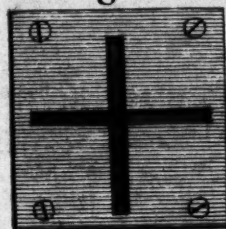


Fig.6.

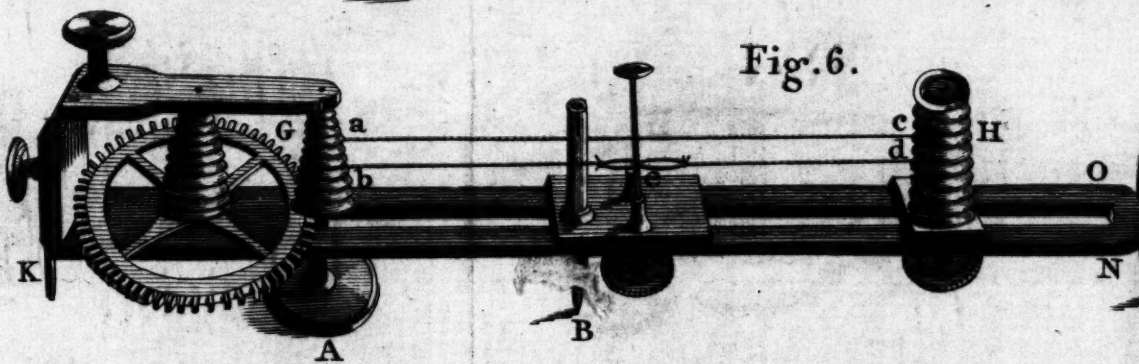


Fig.5.

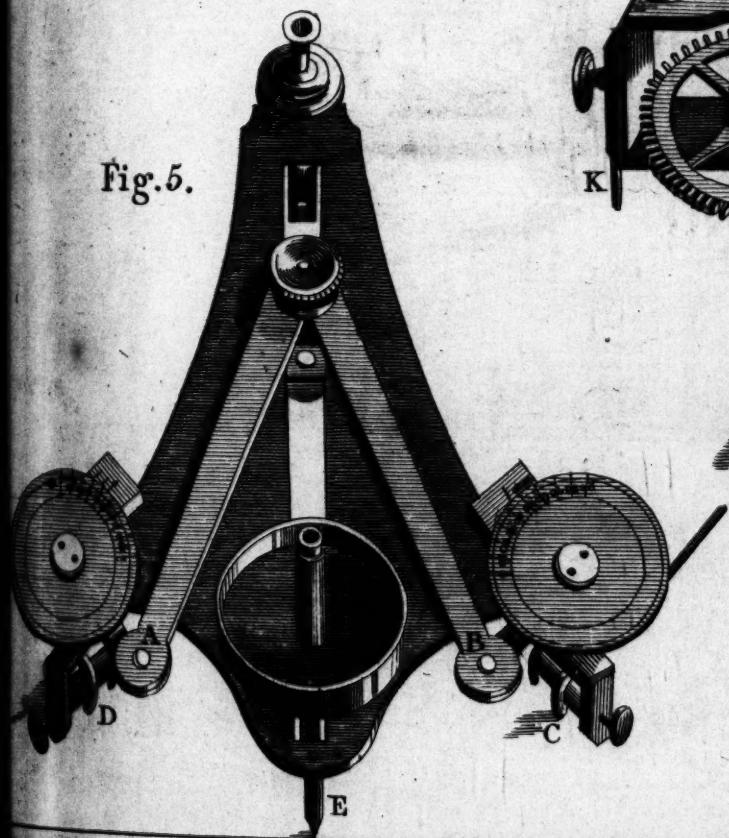
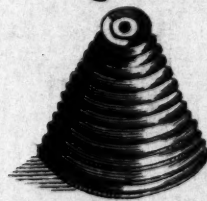
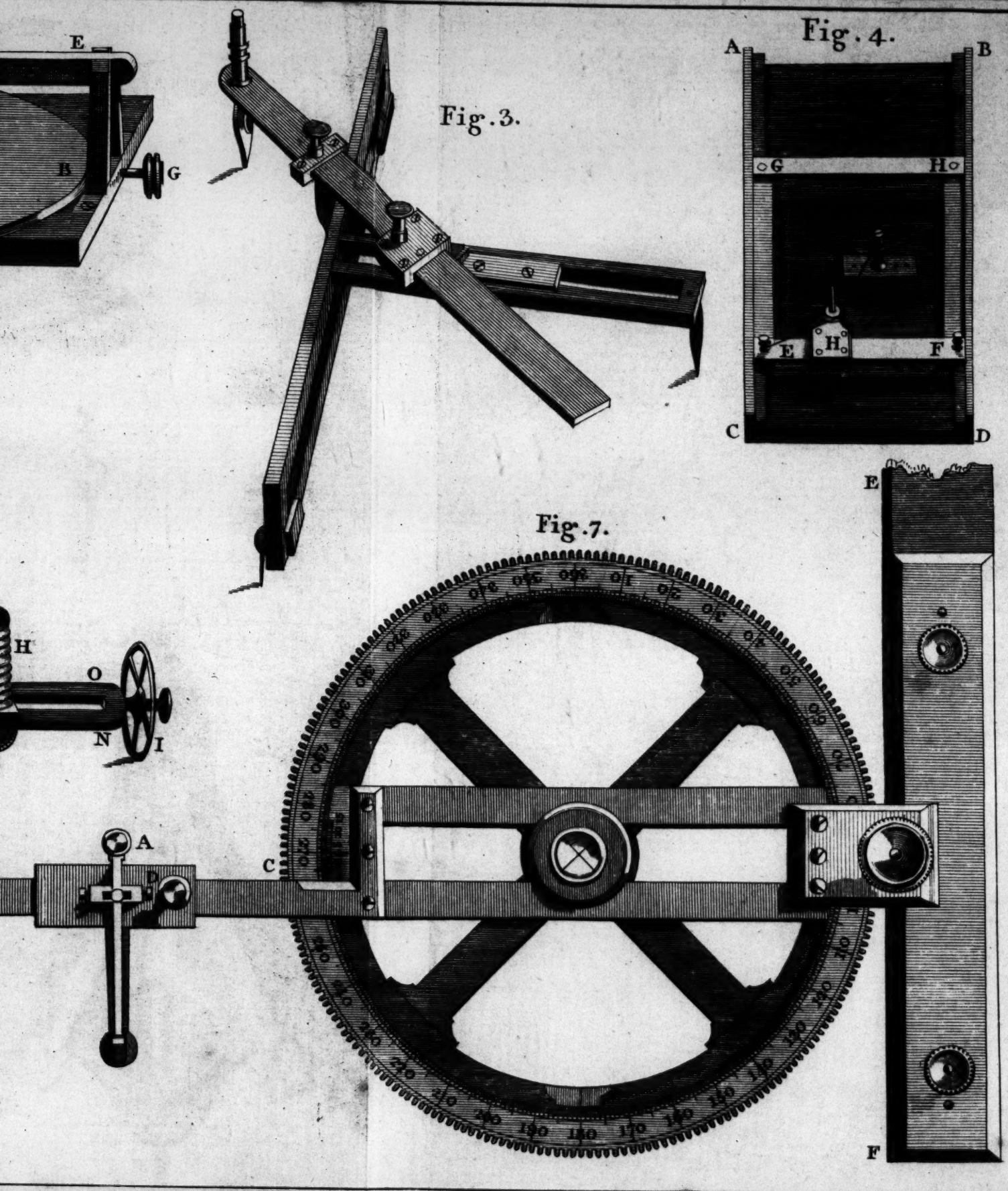


Fig.6.a.





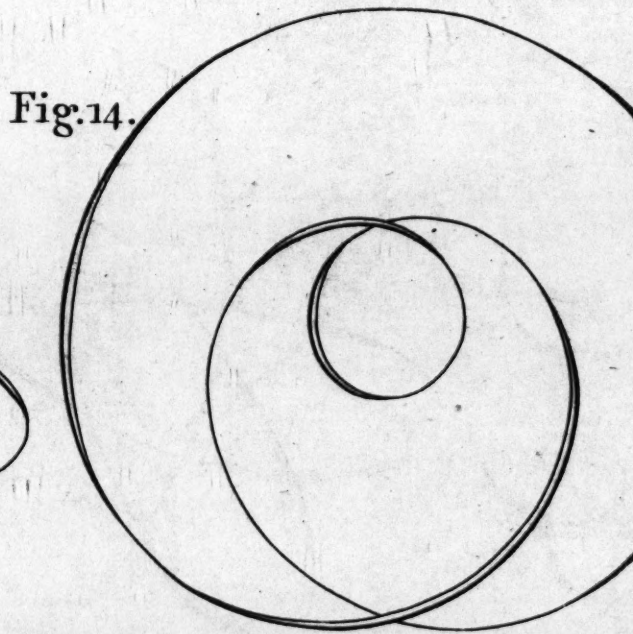
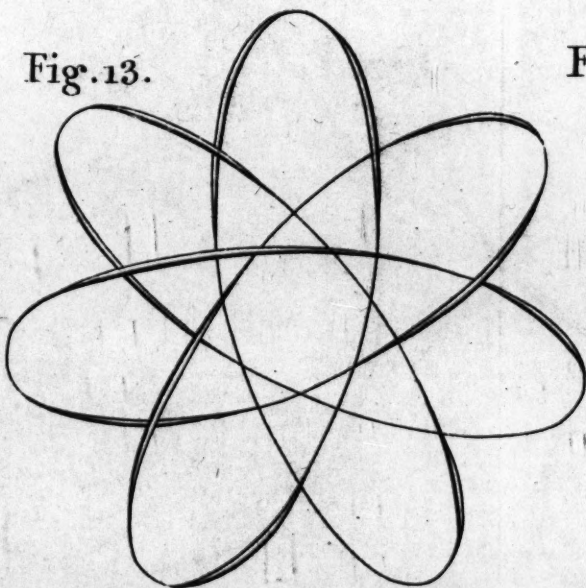
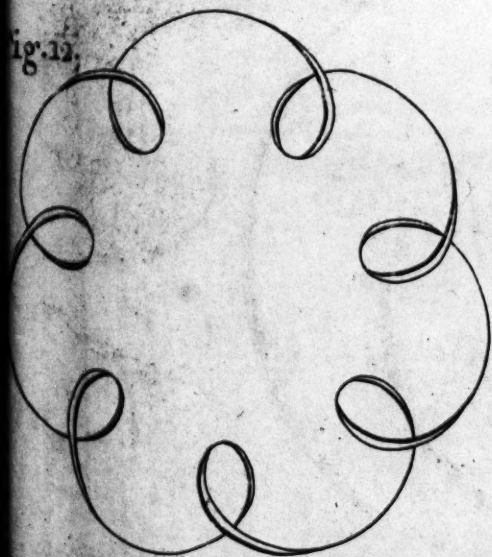
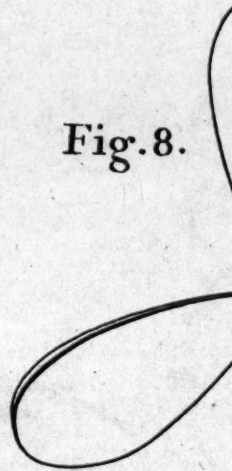
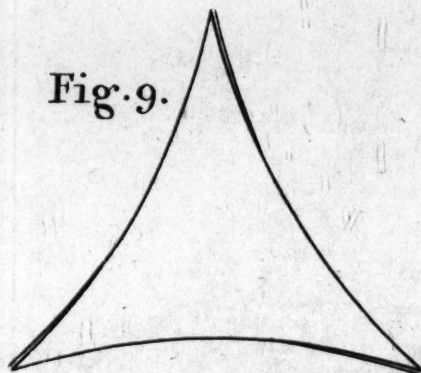
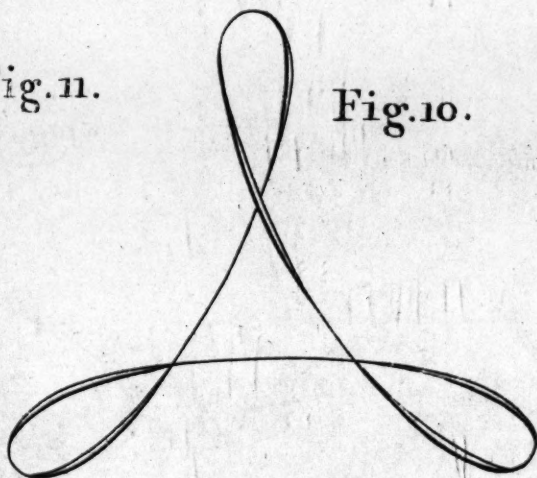
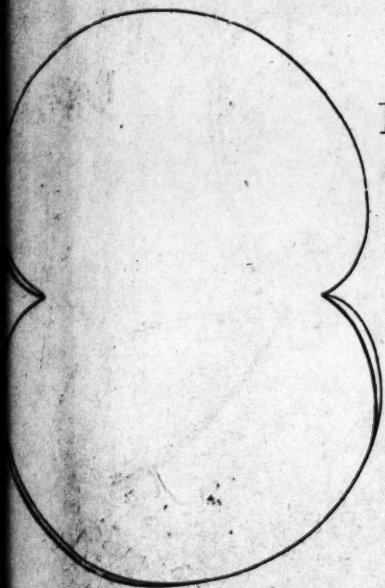
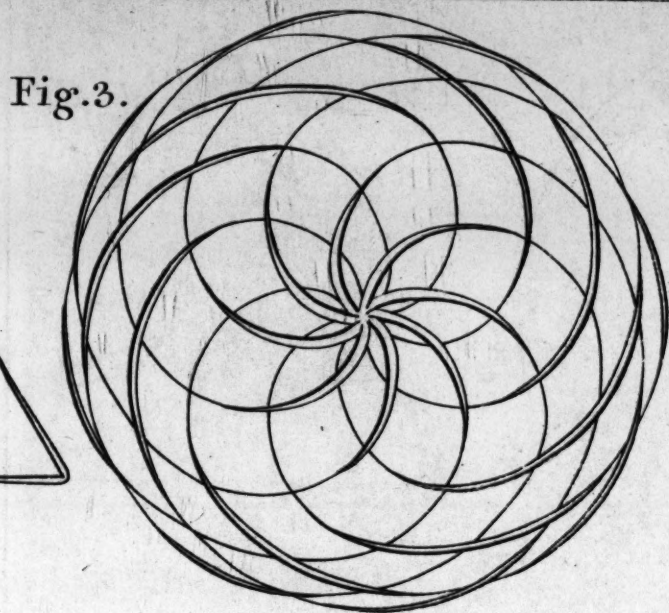
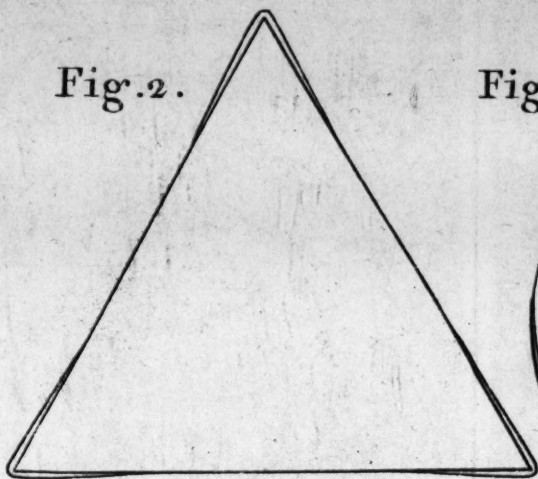
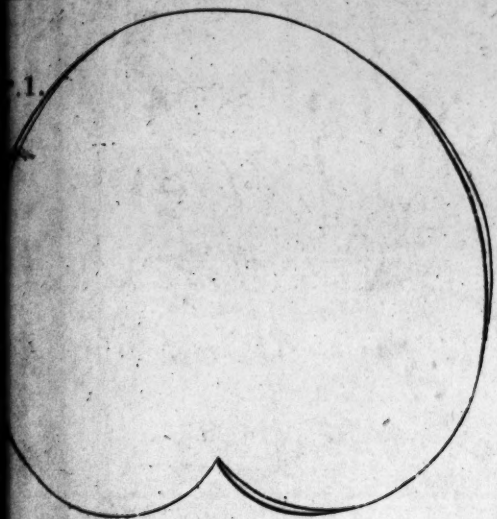


Fig. 4.

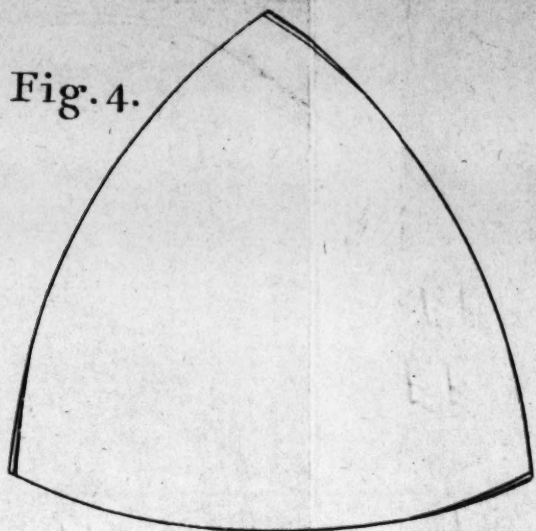


Fig. 5.

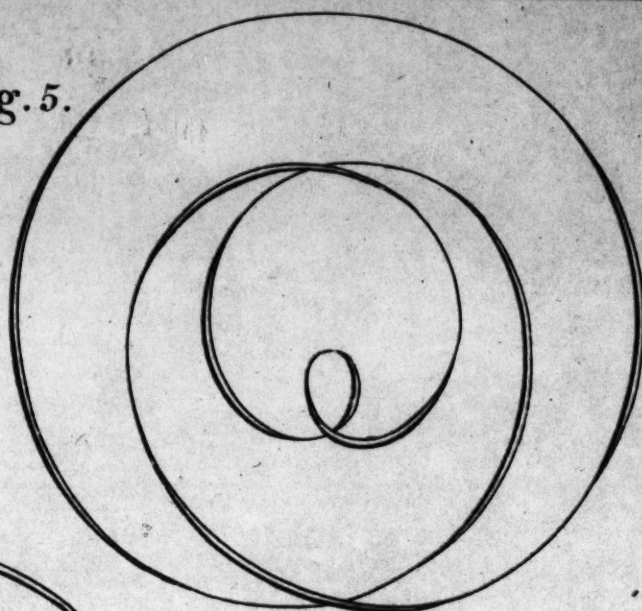


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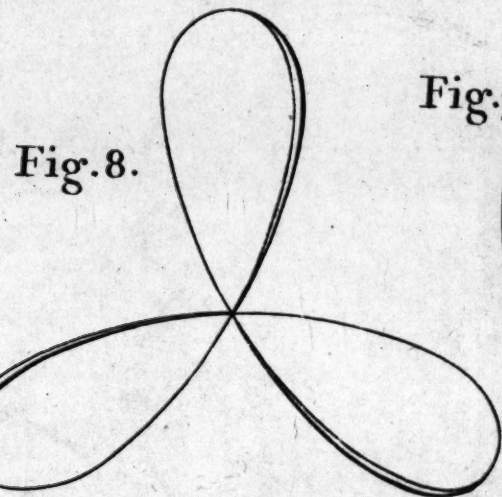


Fig. 7.

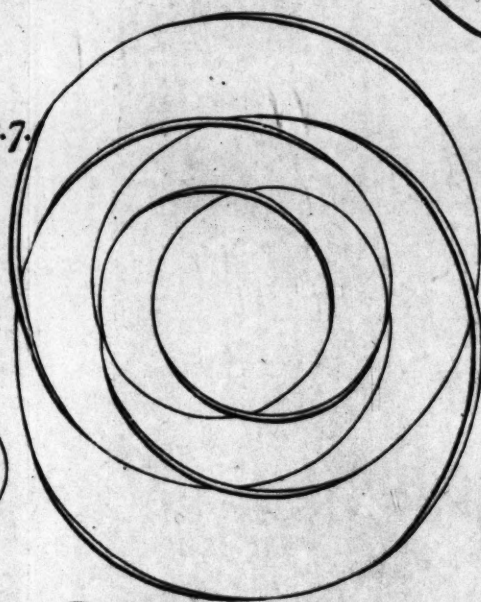


Fig. 6.

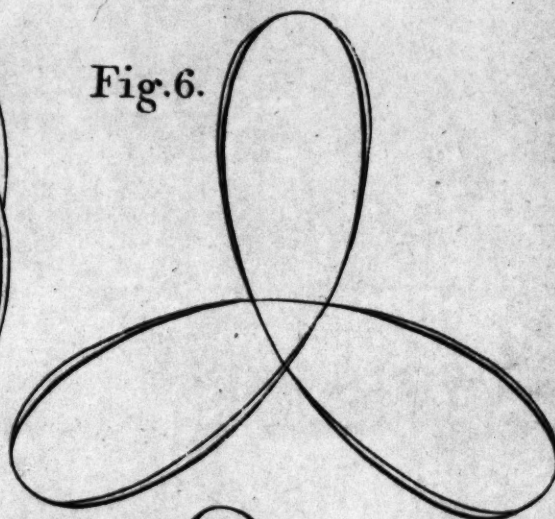


Fig. 15.

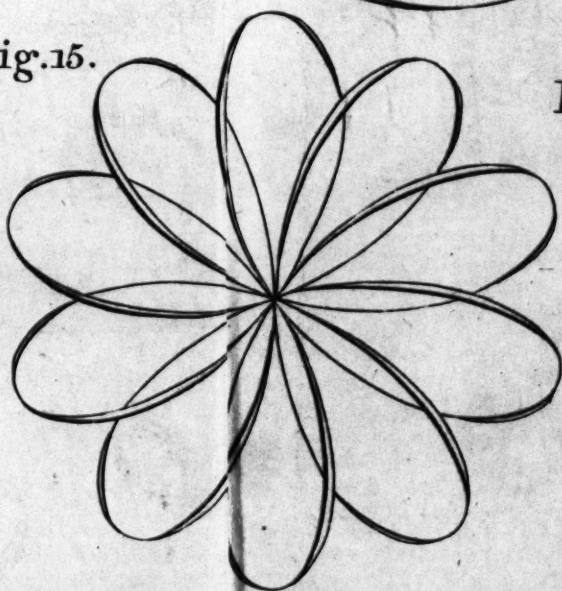
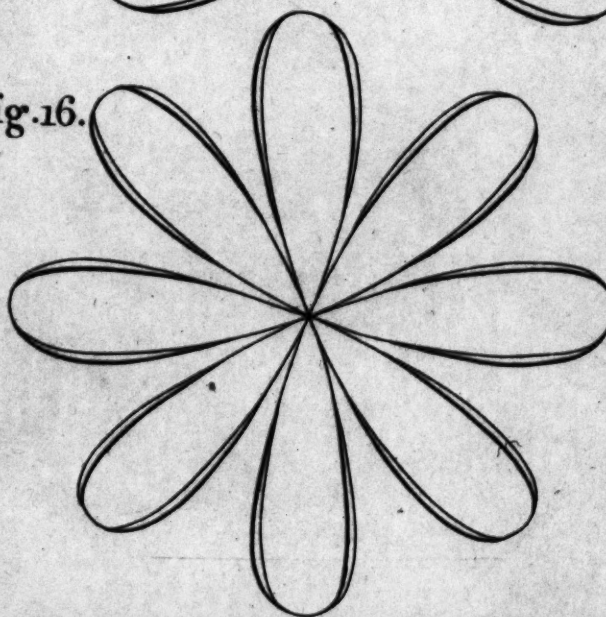
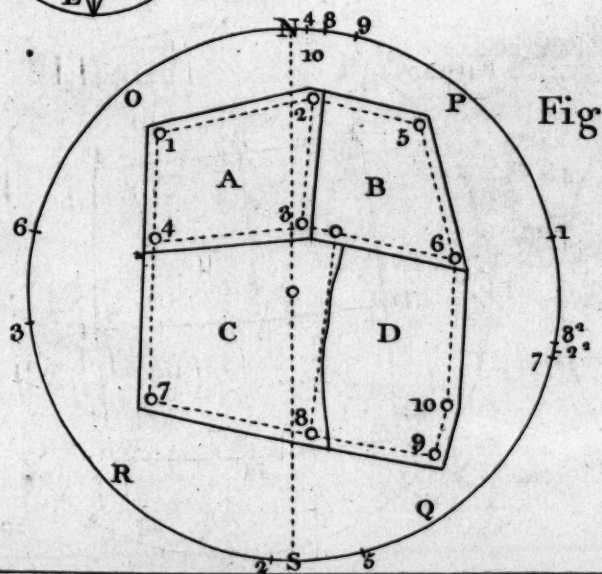
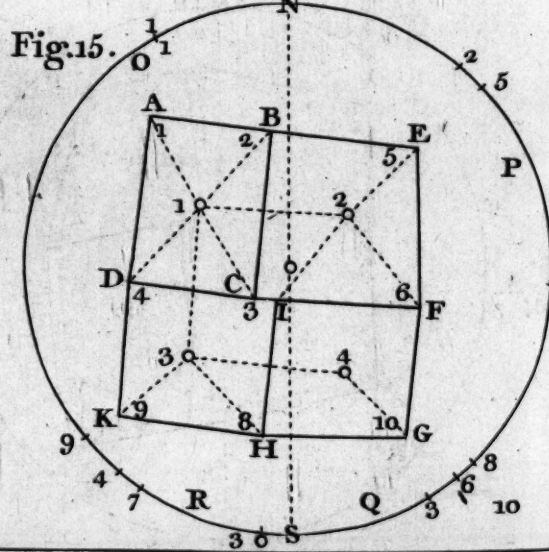
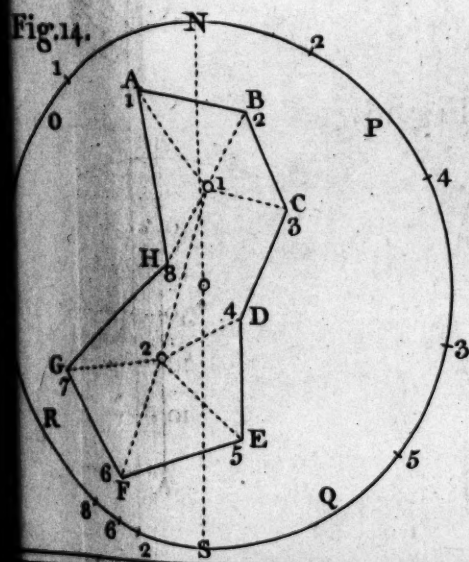
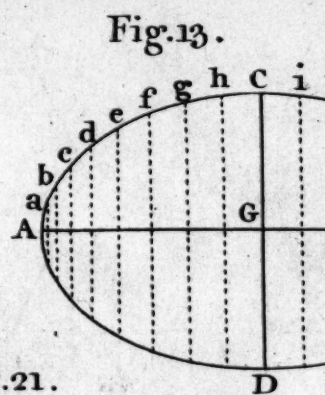
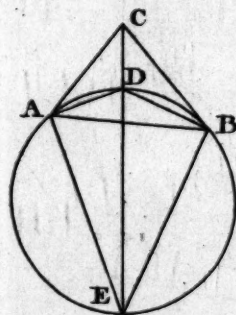
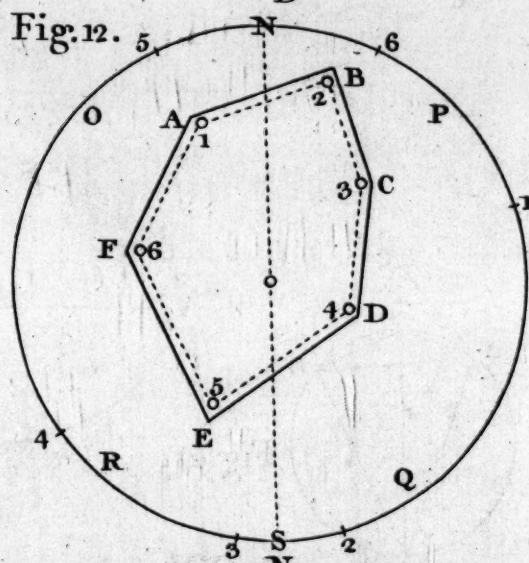
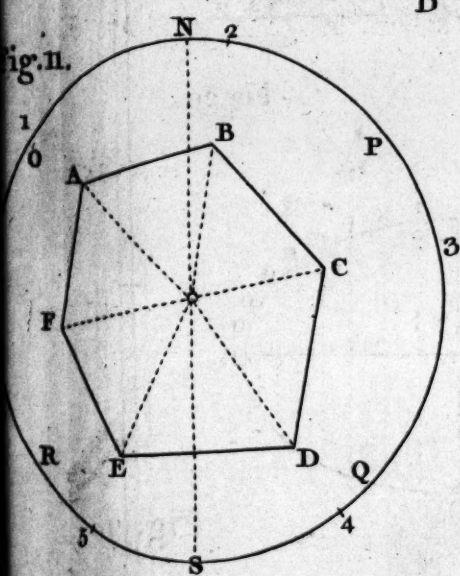
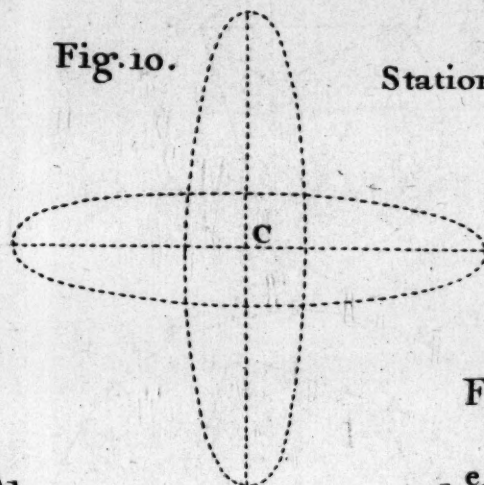
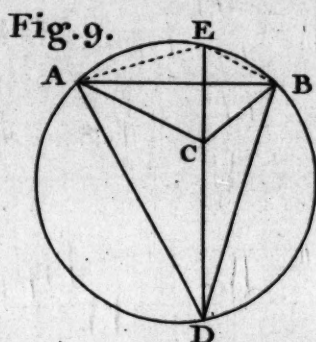
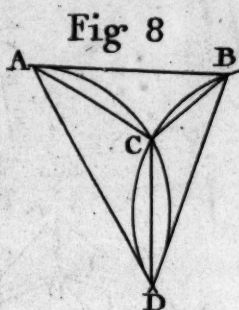
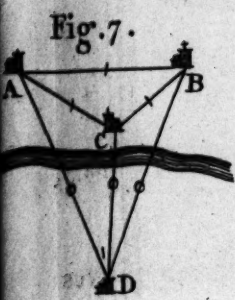
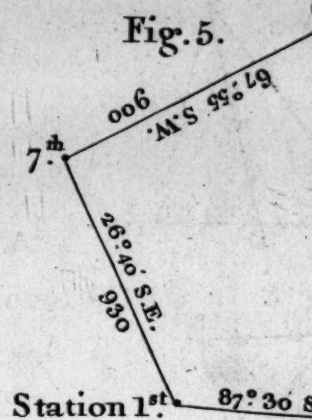
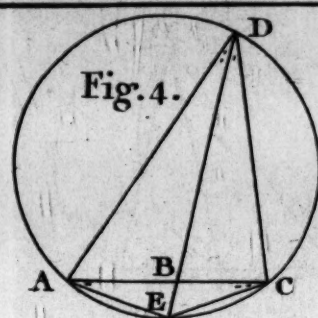
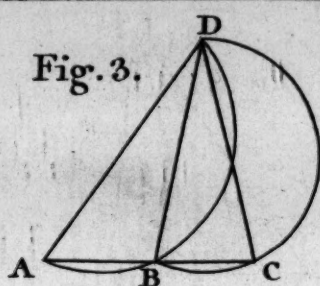
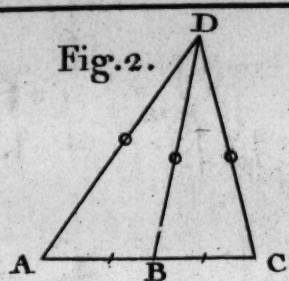
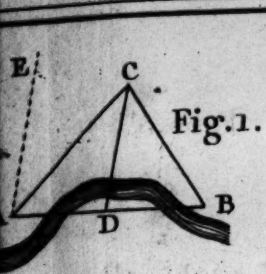


Fig. 16.





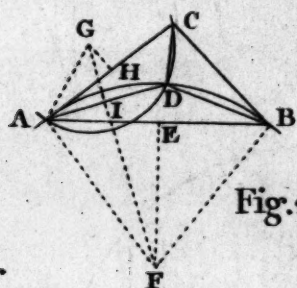
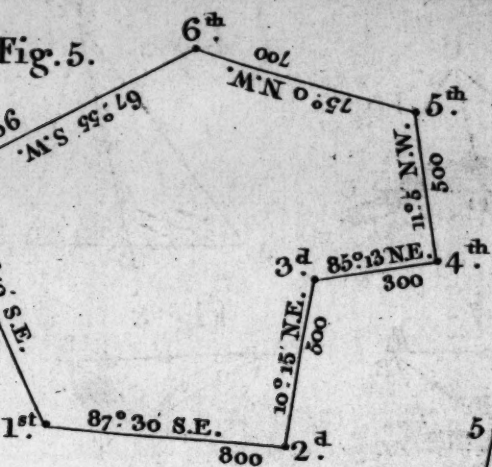


Fig.20.

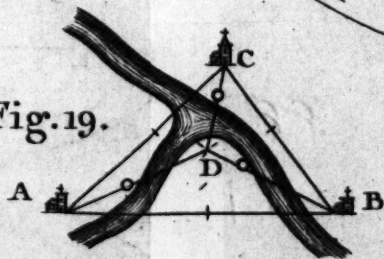
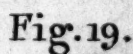
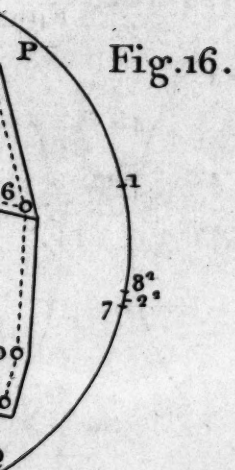
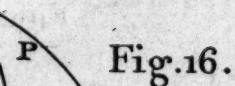
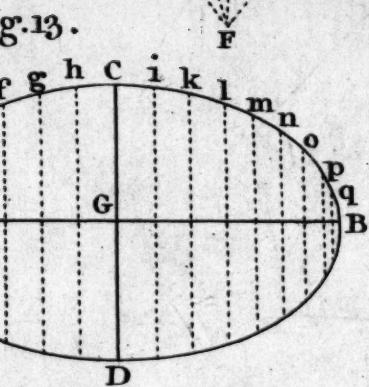


Fig.17.

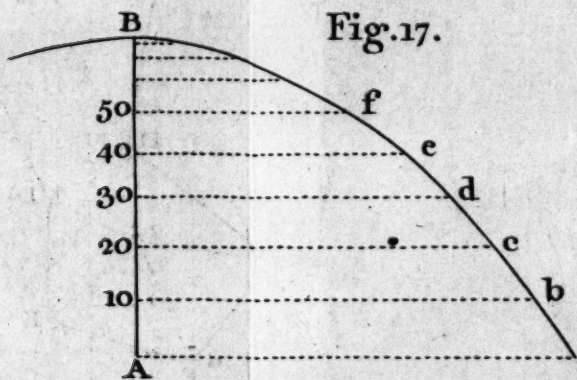


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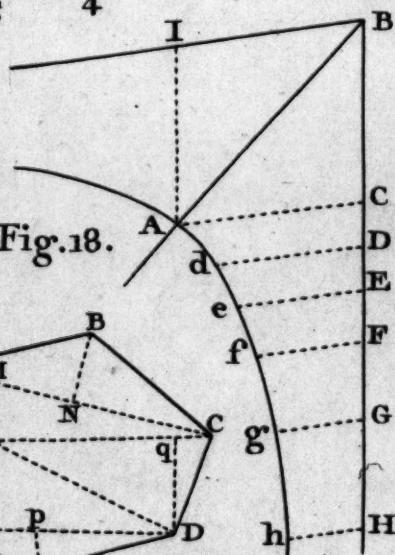


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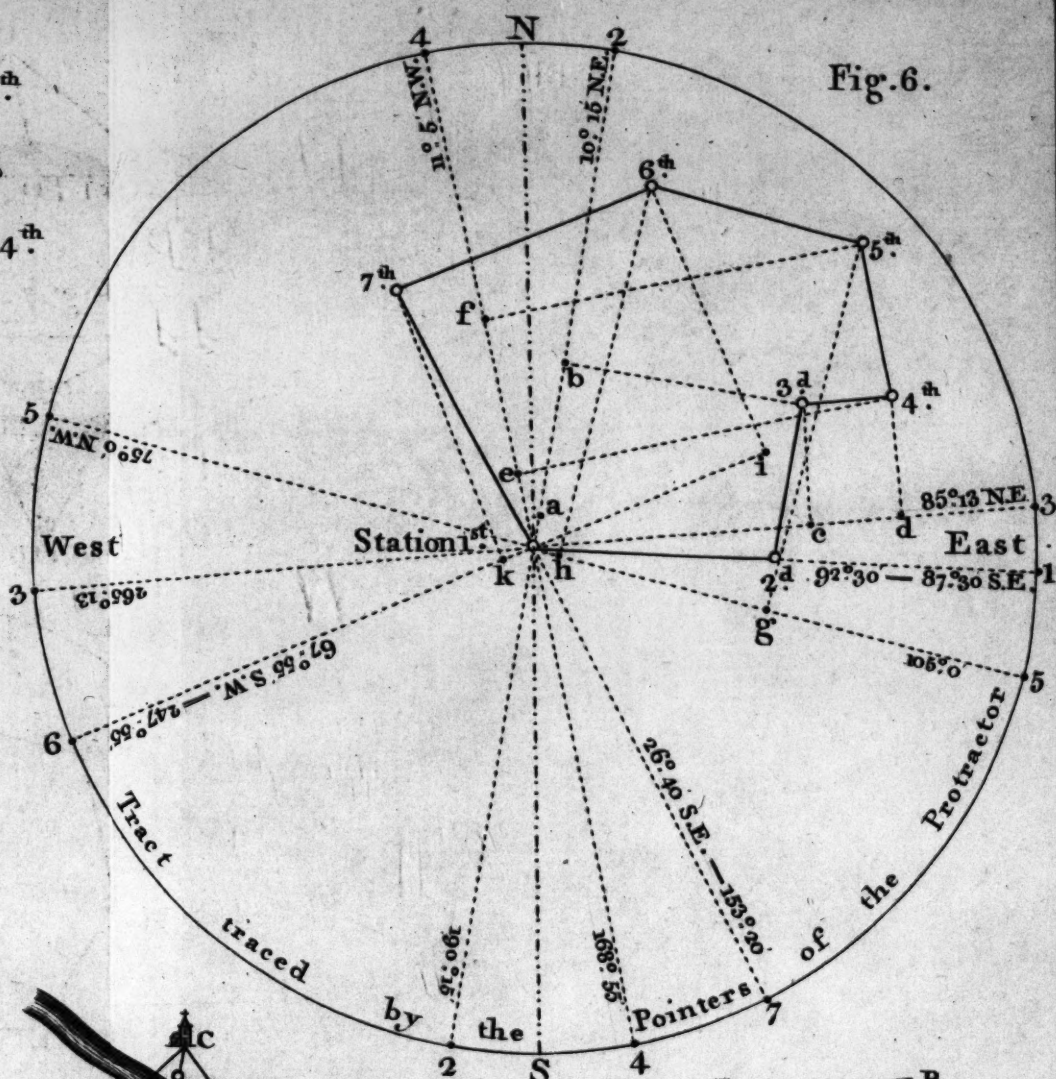
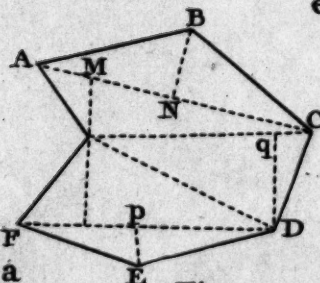


Fig.6.

Fig. 2.

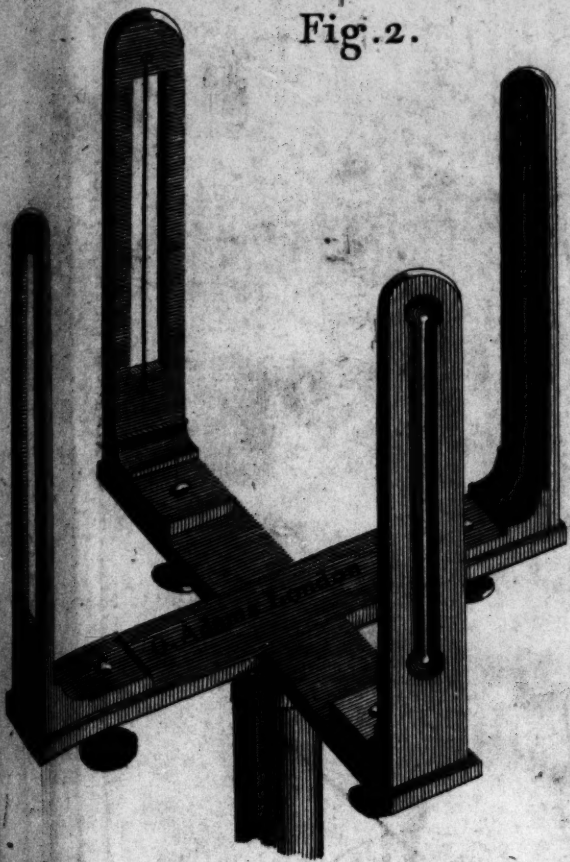


Fig. 5.

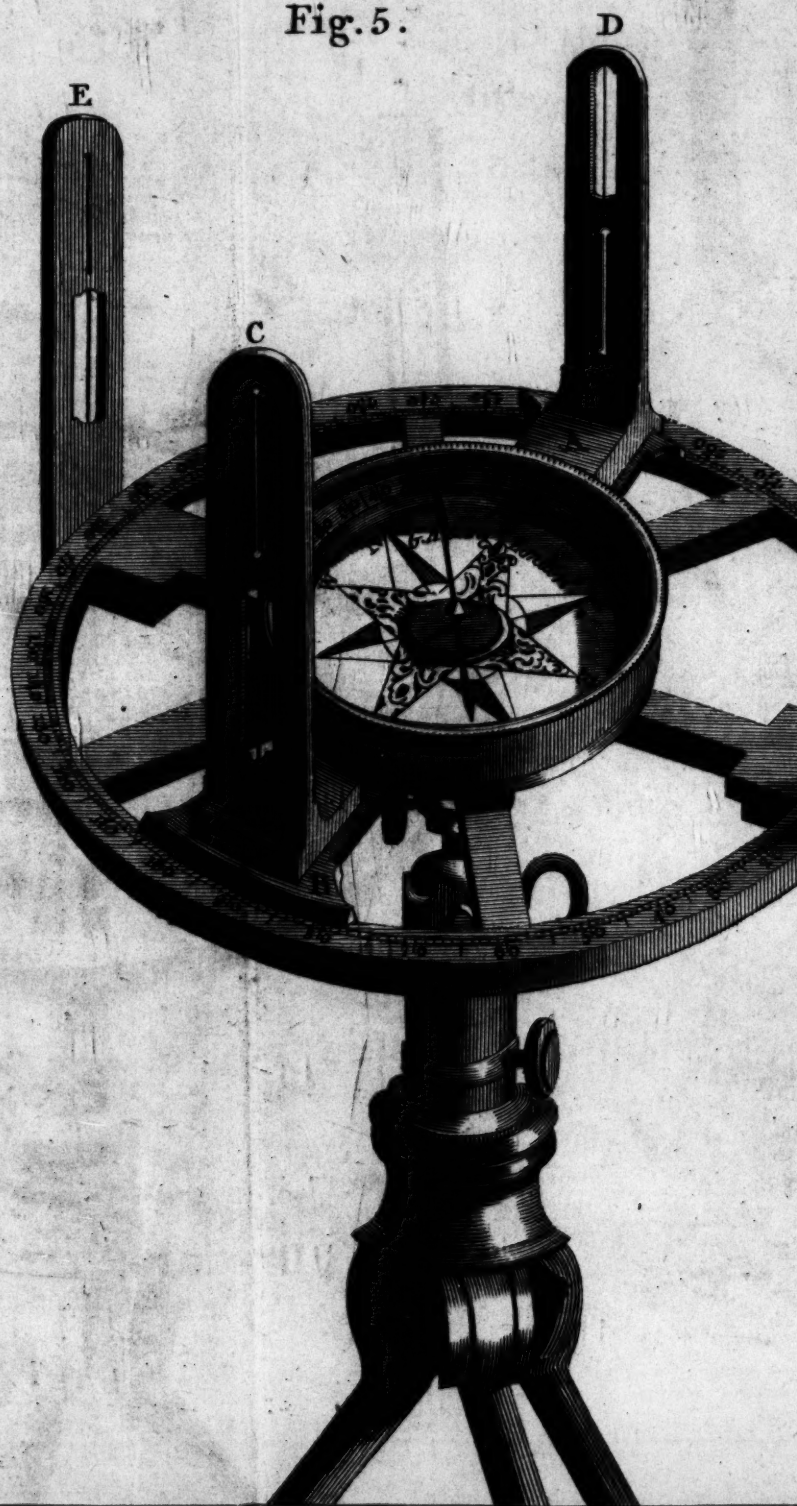


Fig. 3.

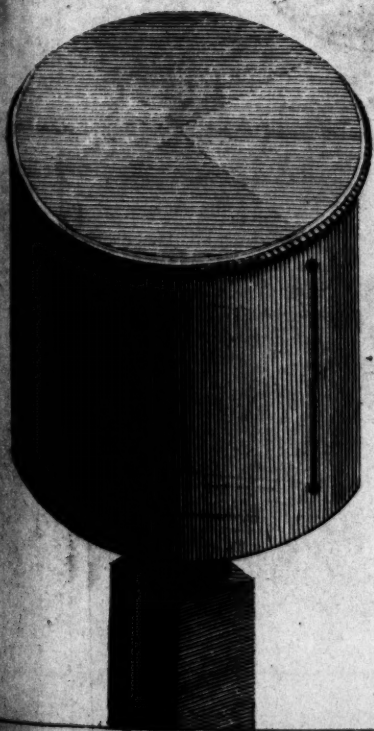


Fig. 1.

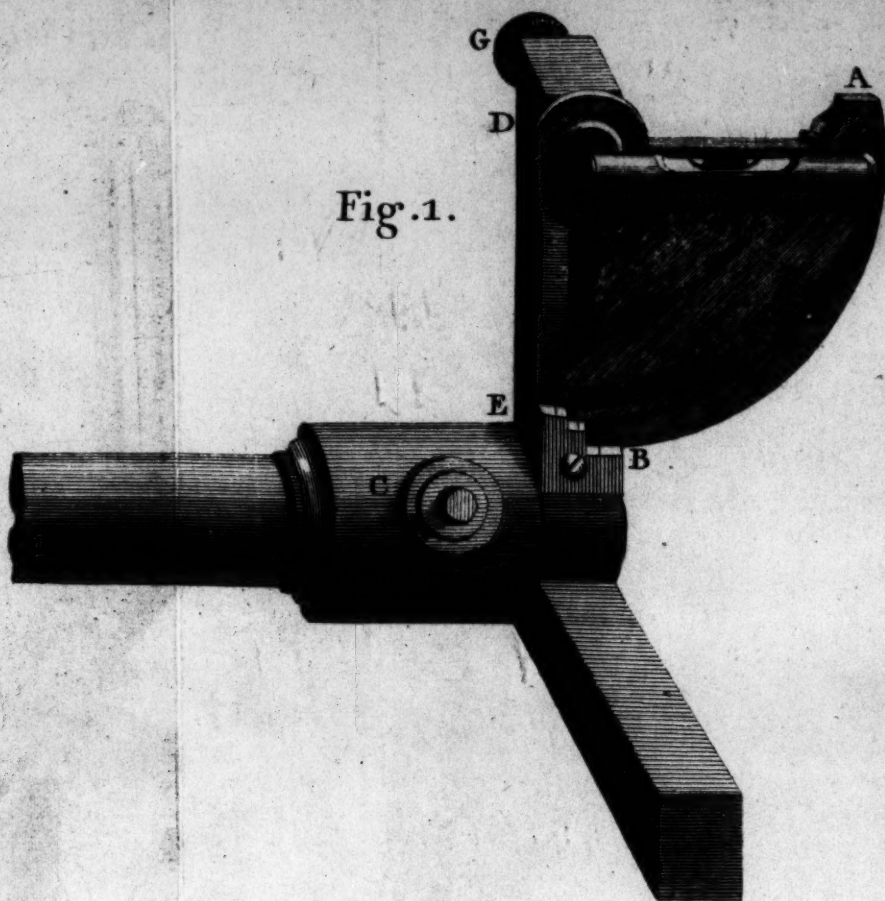


Fig. 4.

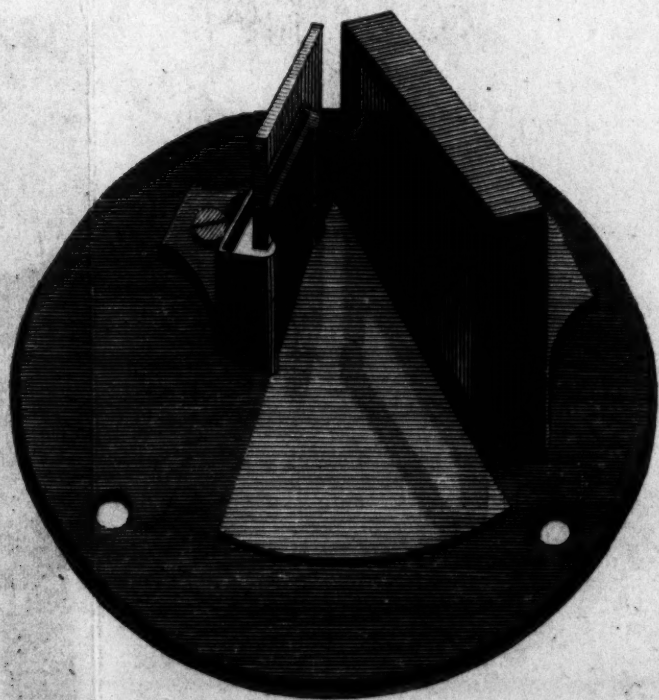


Fig. 4.

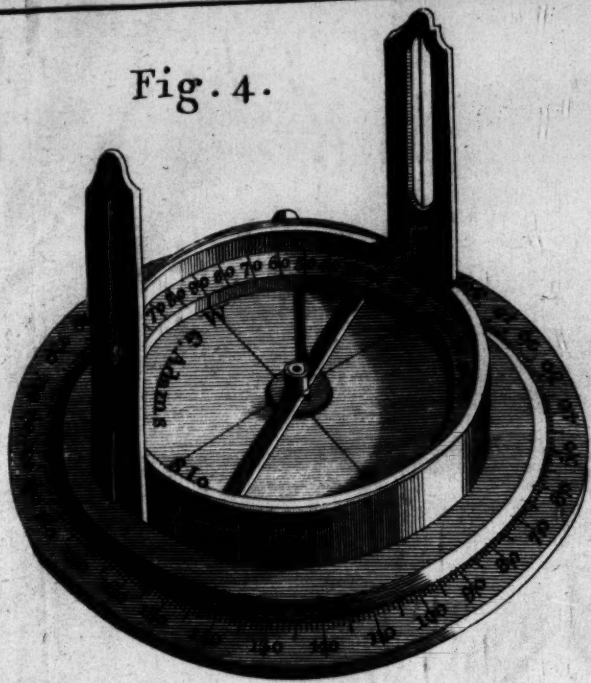


Fig. 5.

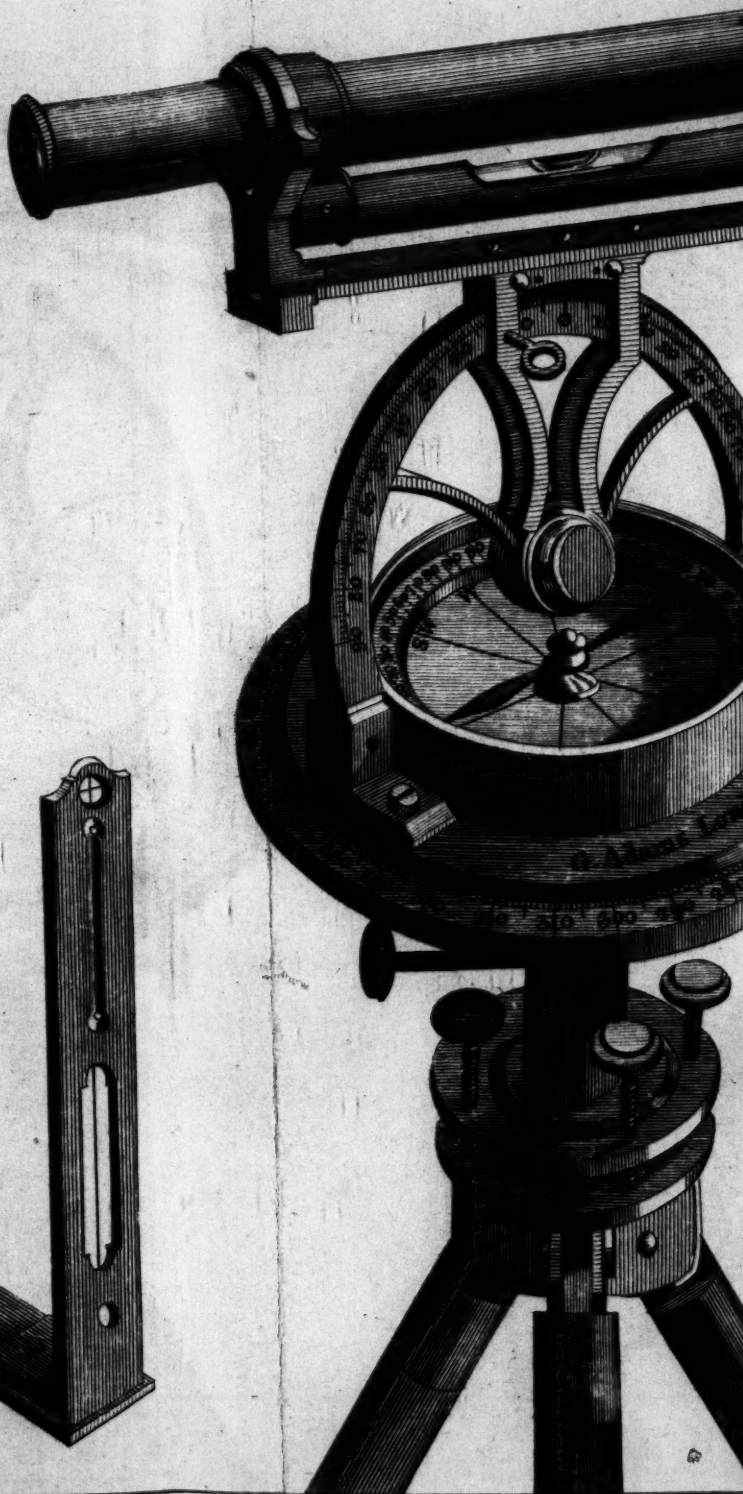


Fig. 2.

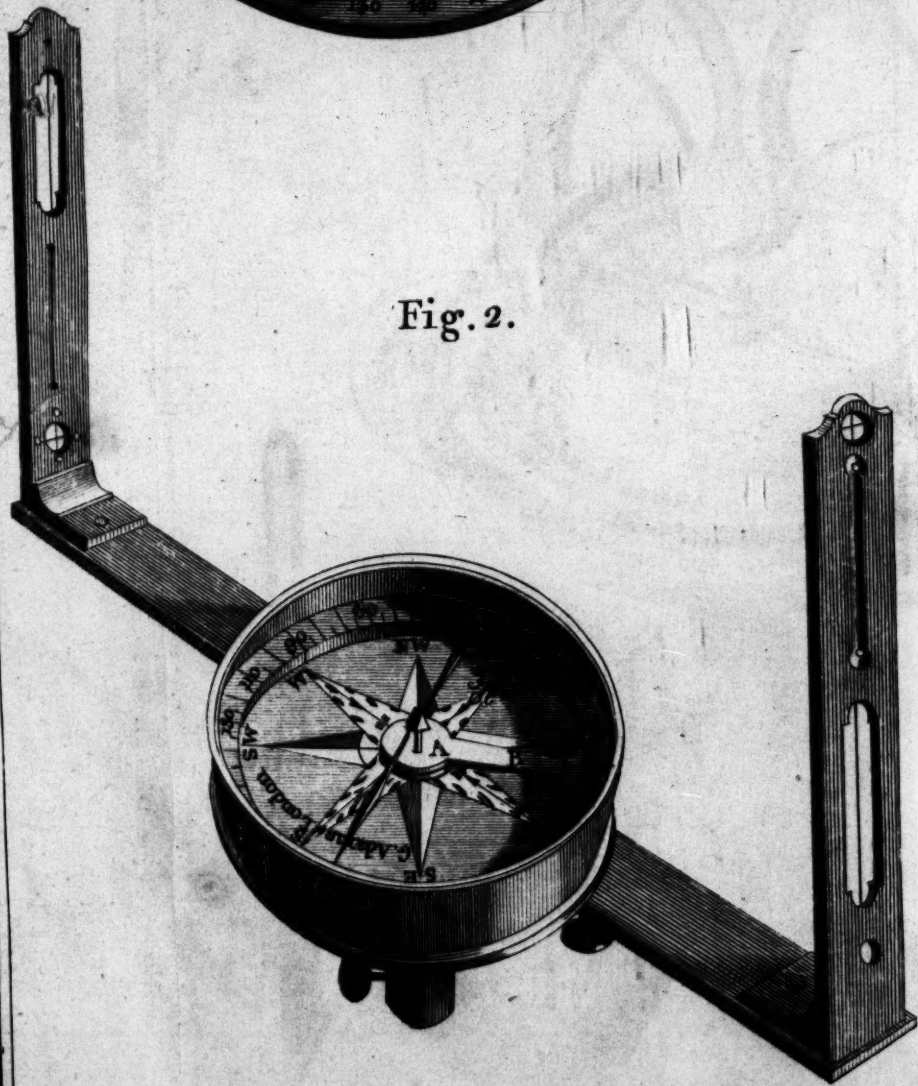


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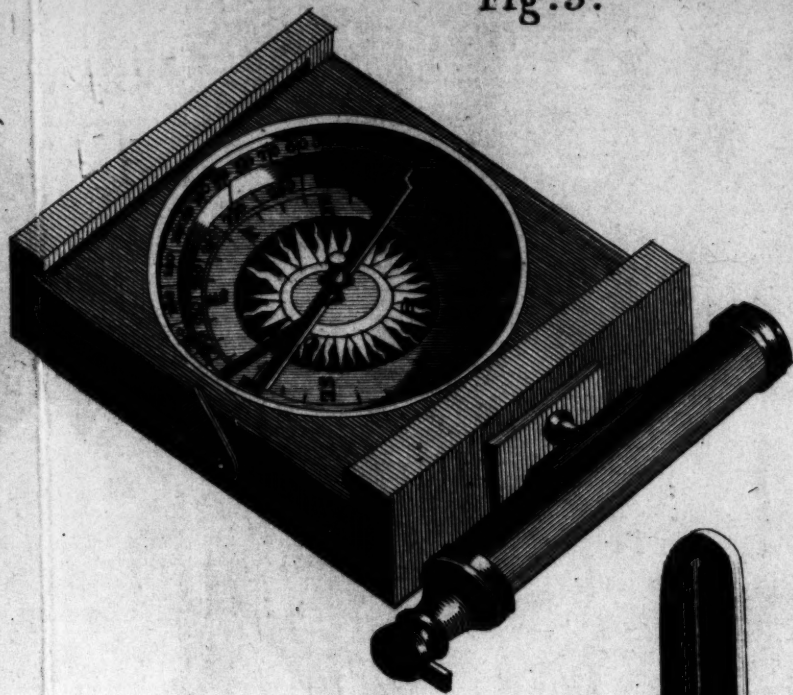
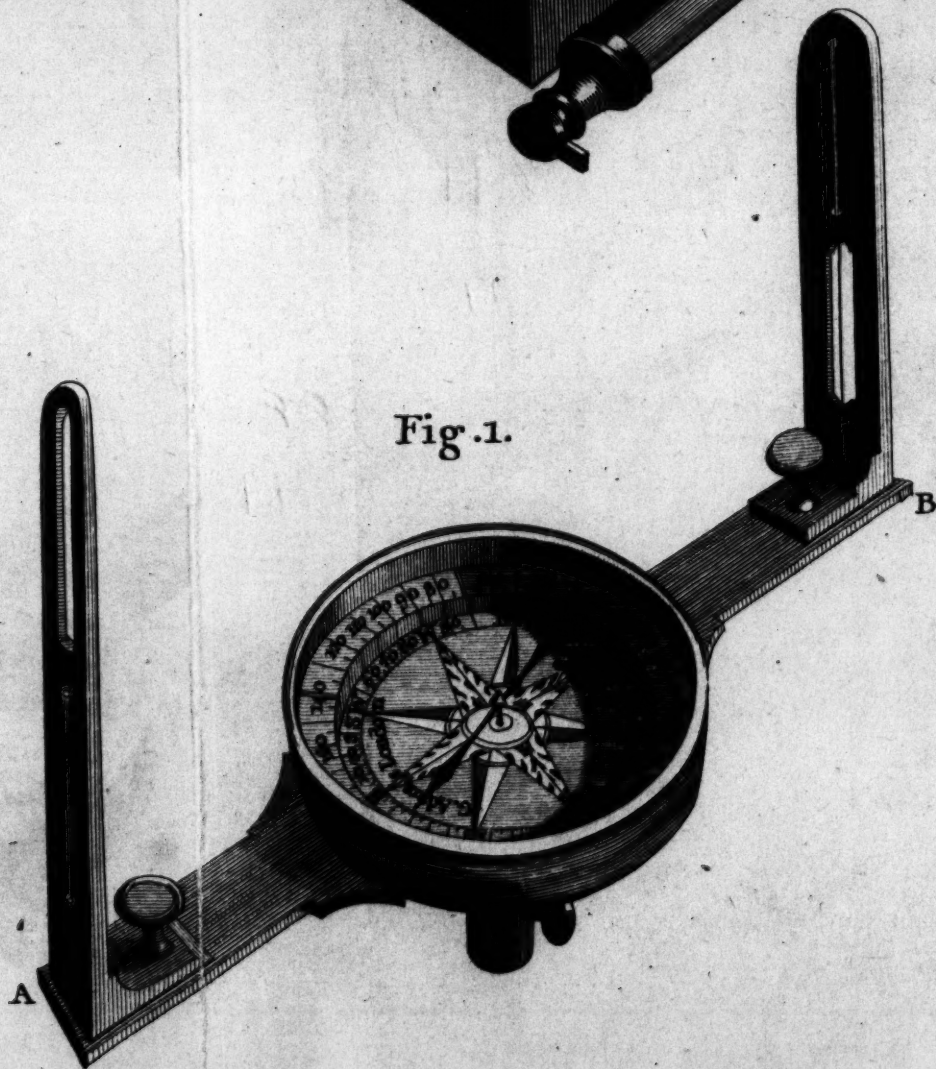


Fig. 1.



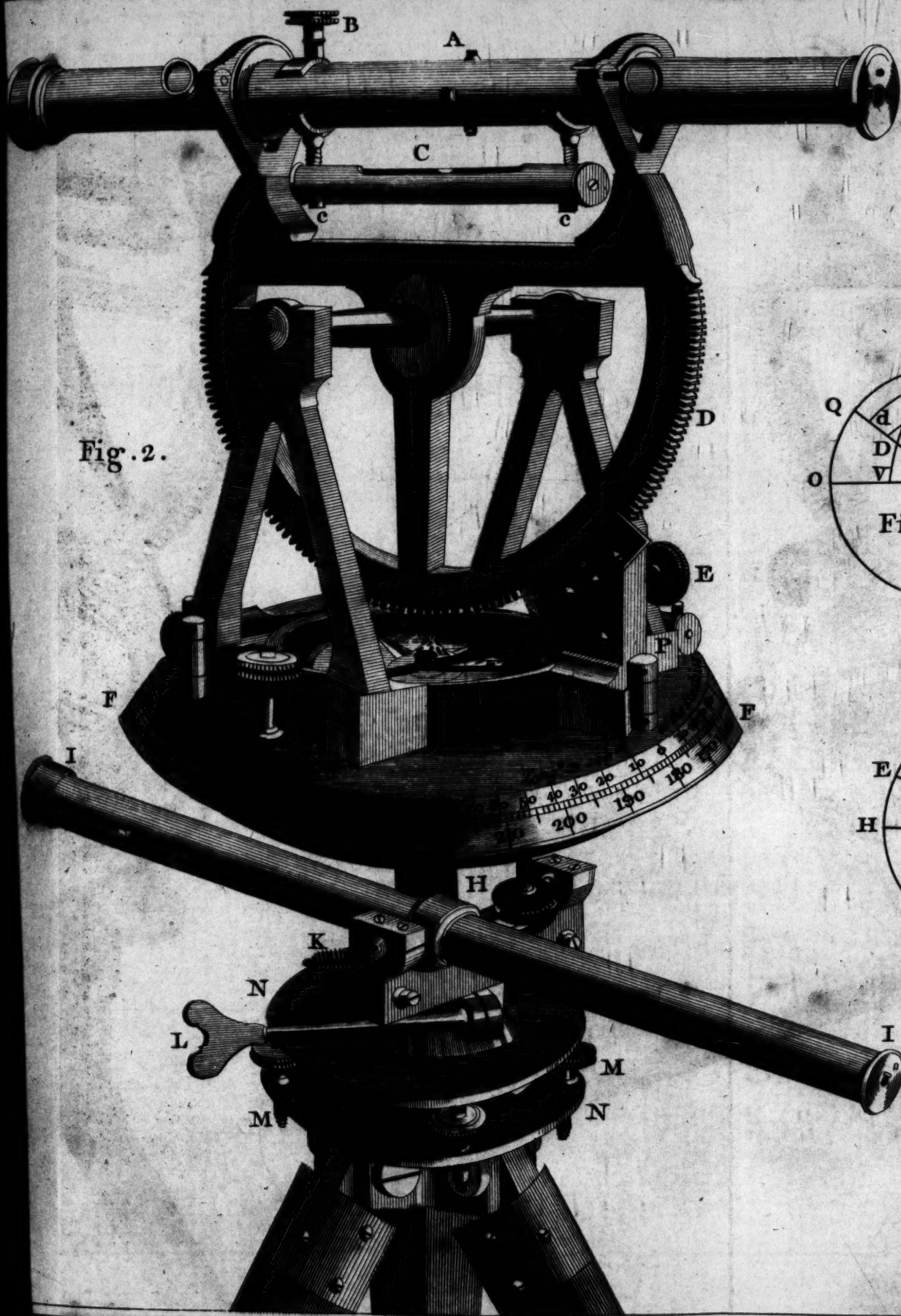


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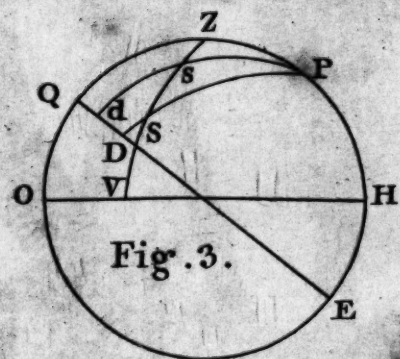


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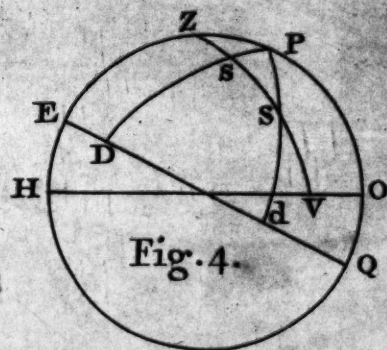


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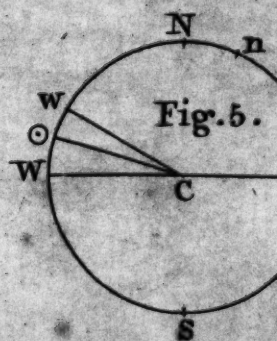
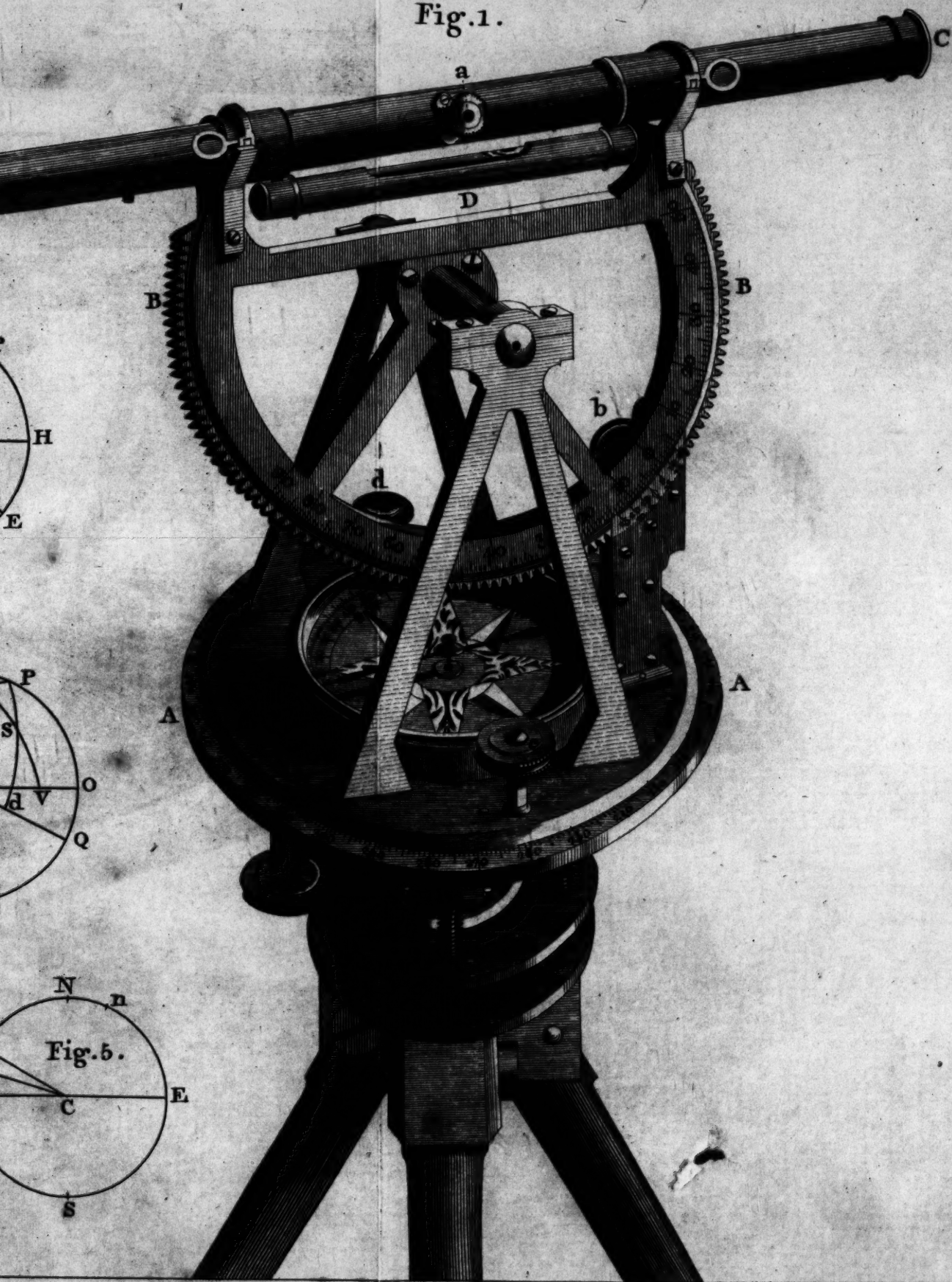


Fig. 5.

Fig. 1.



So Fleet Street. as the Act directs. June 11 1791.

Jn^r. Lodge sc.

Fig. 1.

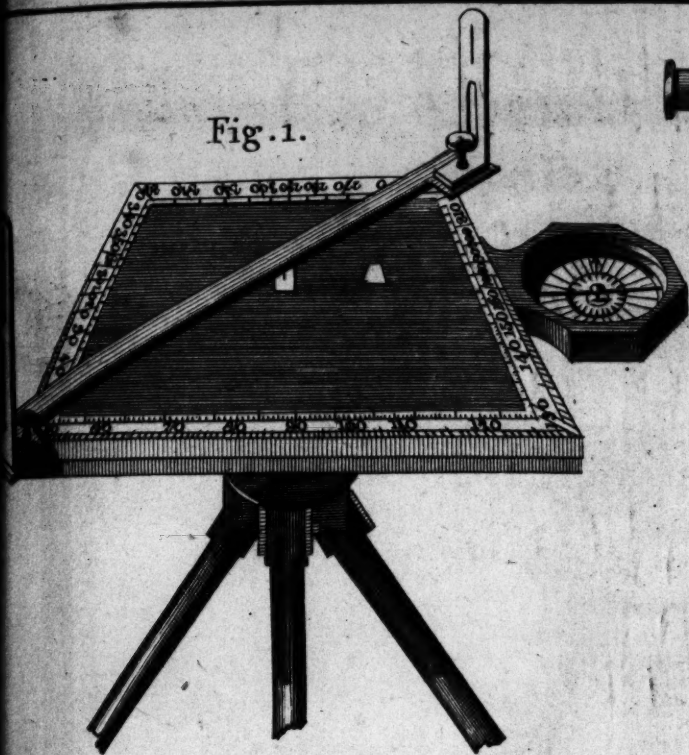


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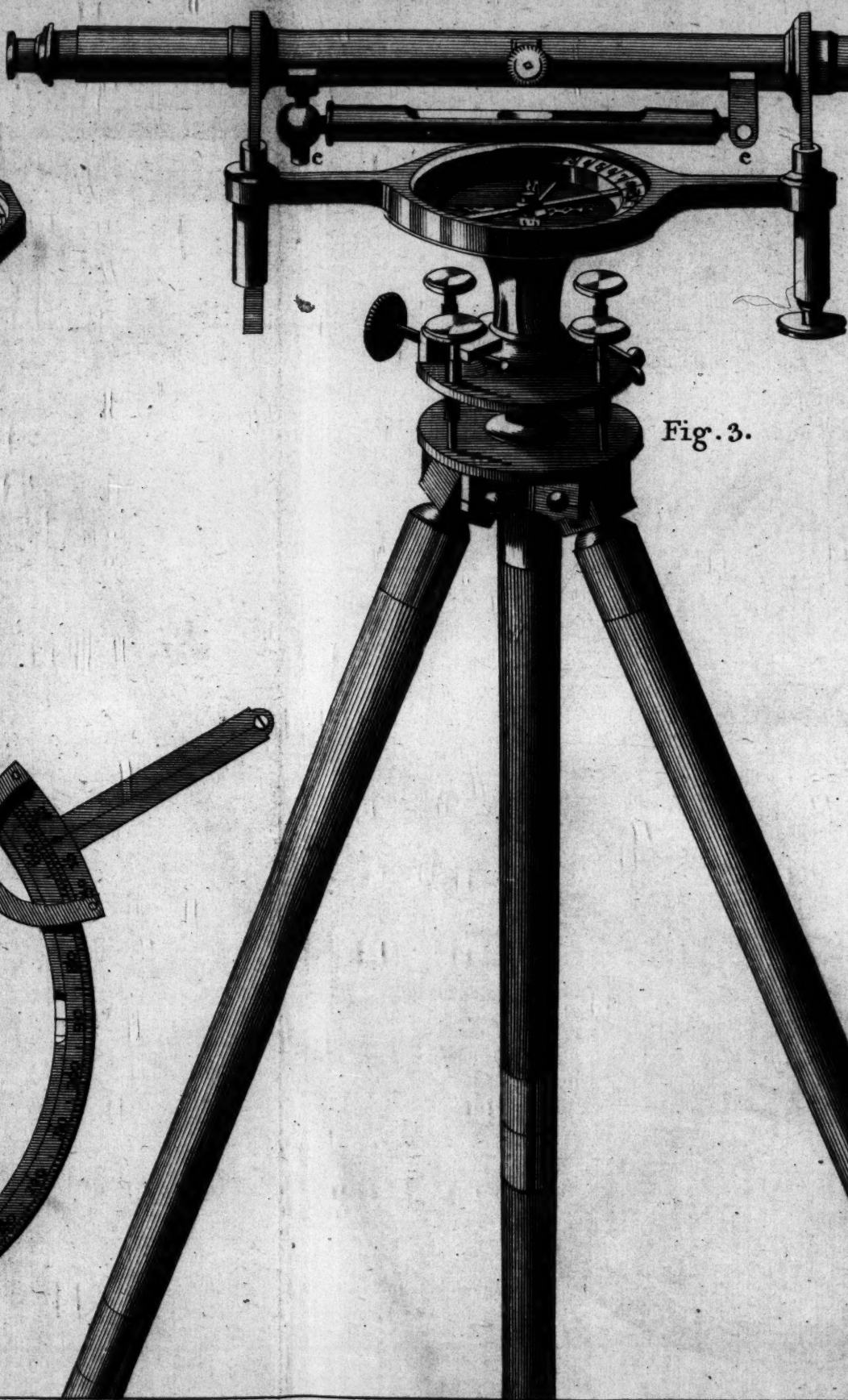
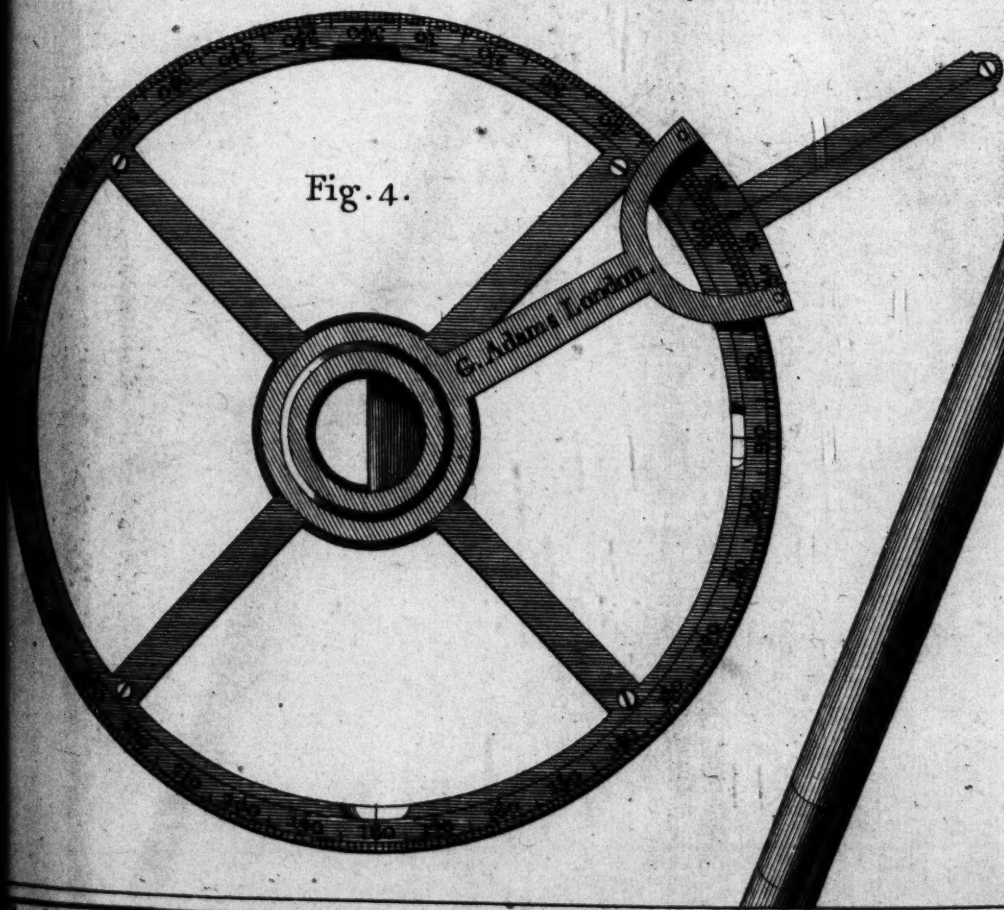


Fig. 4.



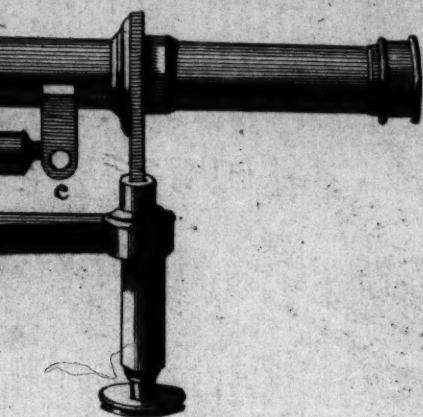


Fig. 3.

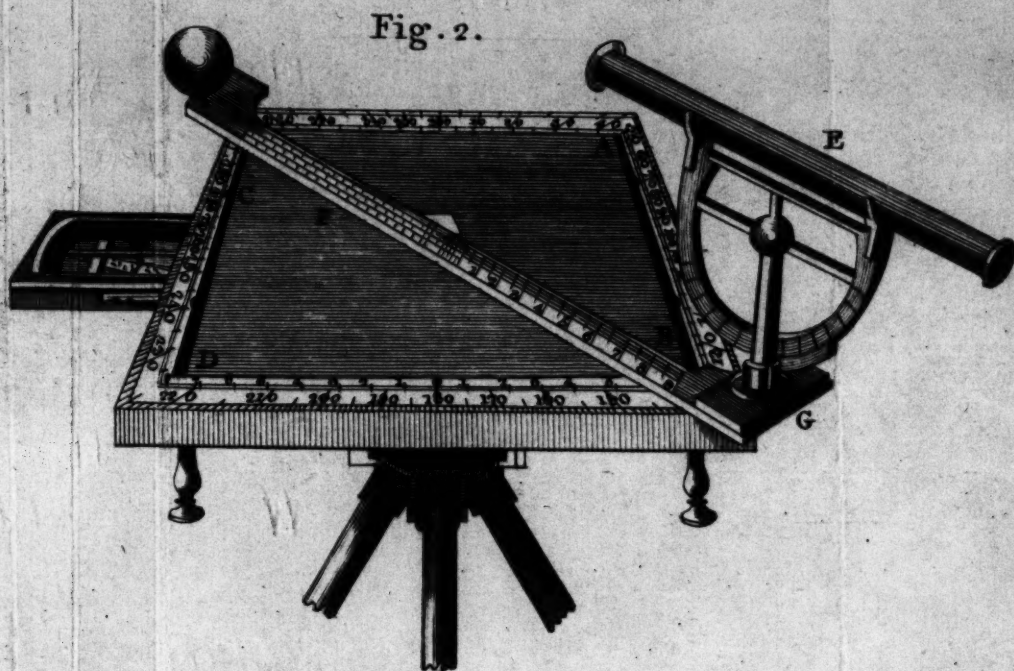


Fig. 2.

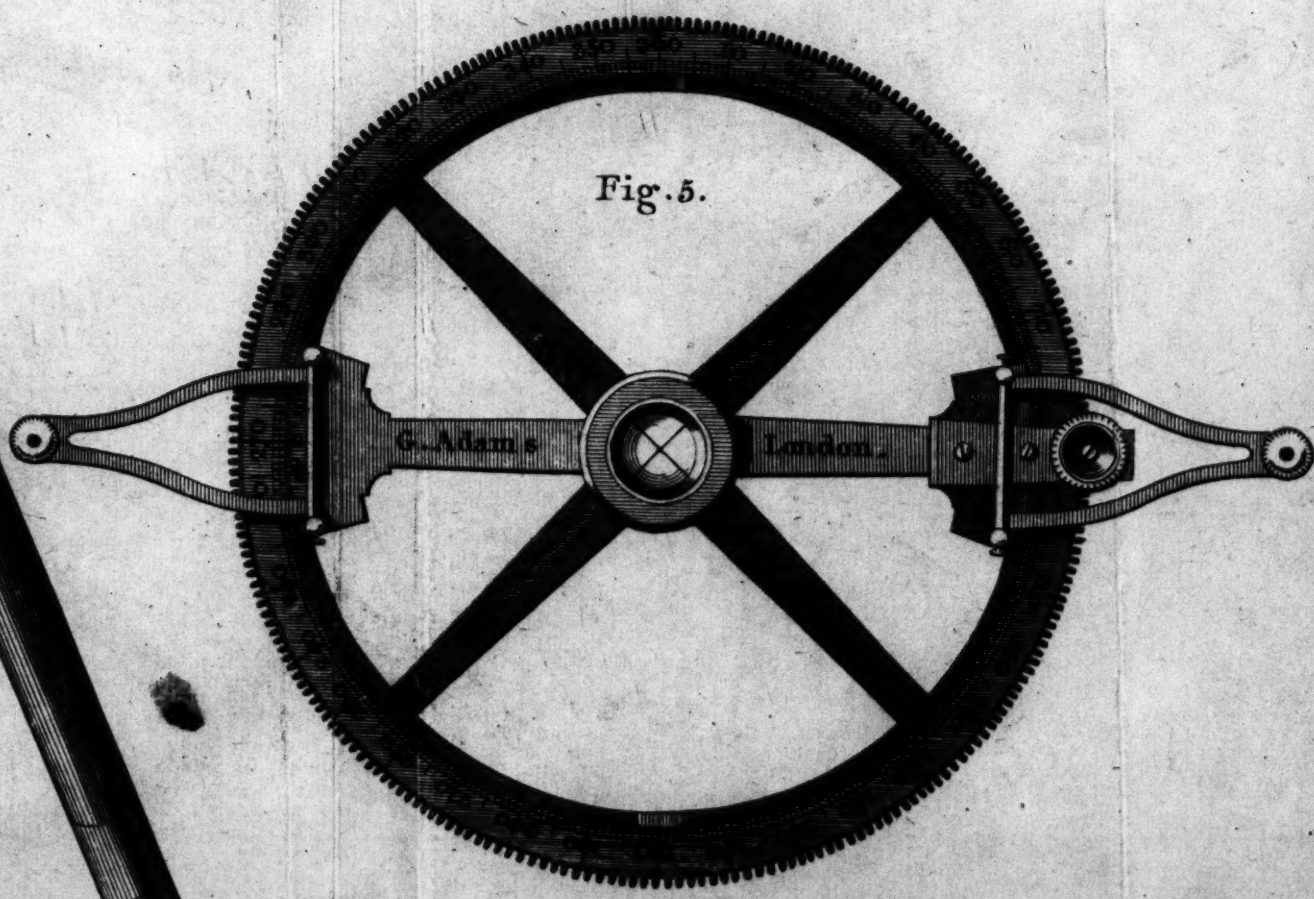


Fig. 5.

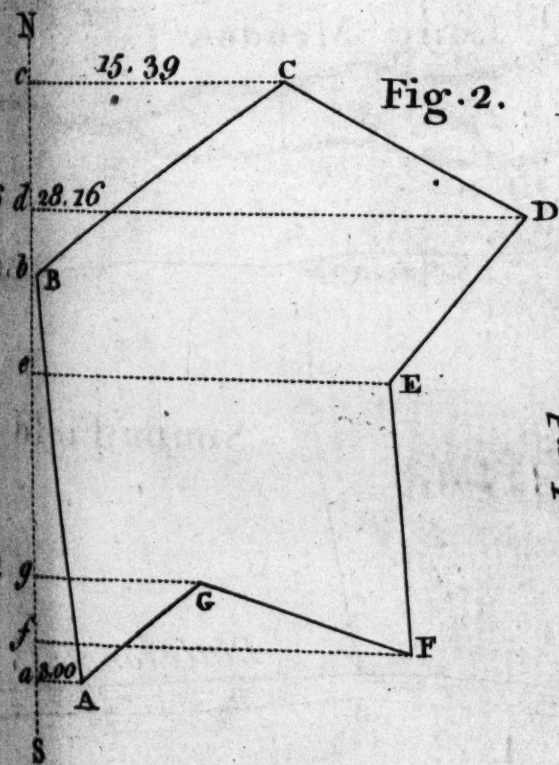


Fig. 3.

Henry Derrans Land

Wm. Higgins's Land

David Horne's Land

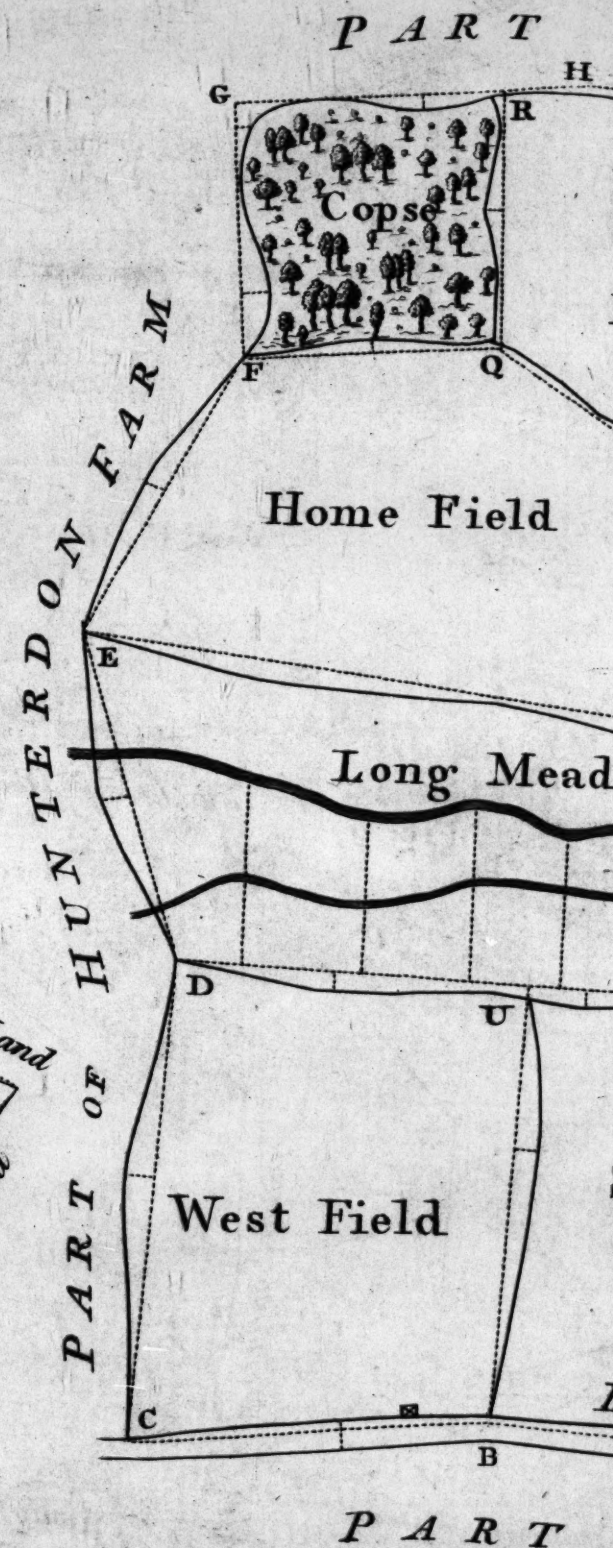
Char. & Ward's Land

Wm. Humphrey's Land

1 2 3 4 5 6 7

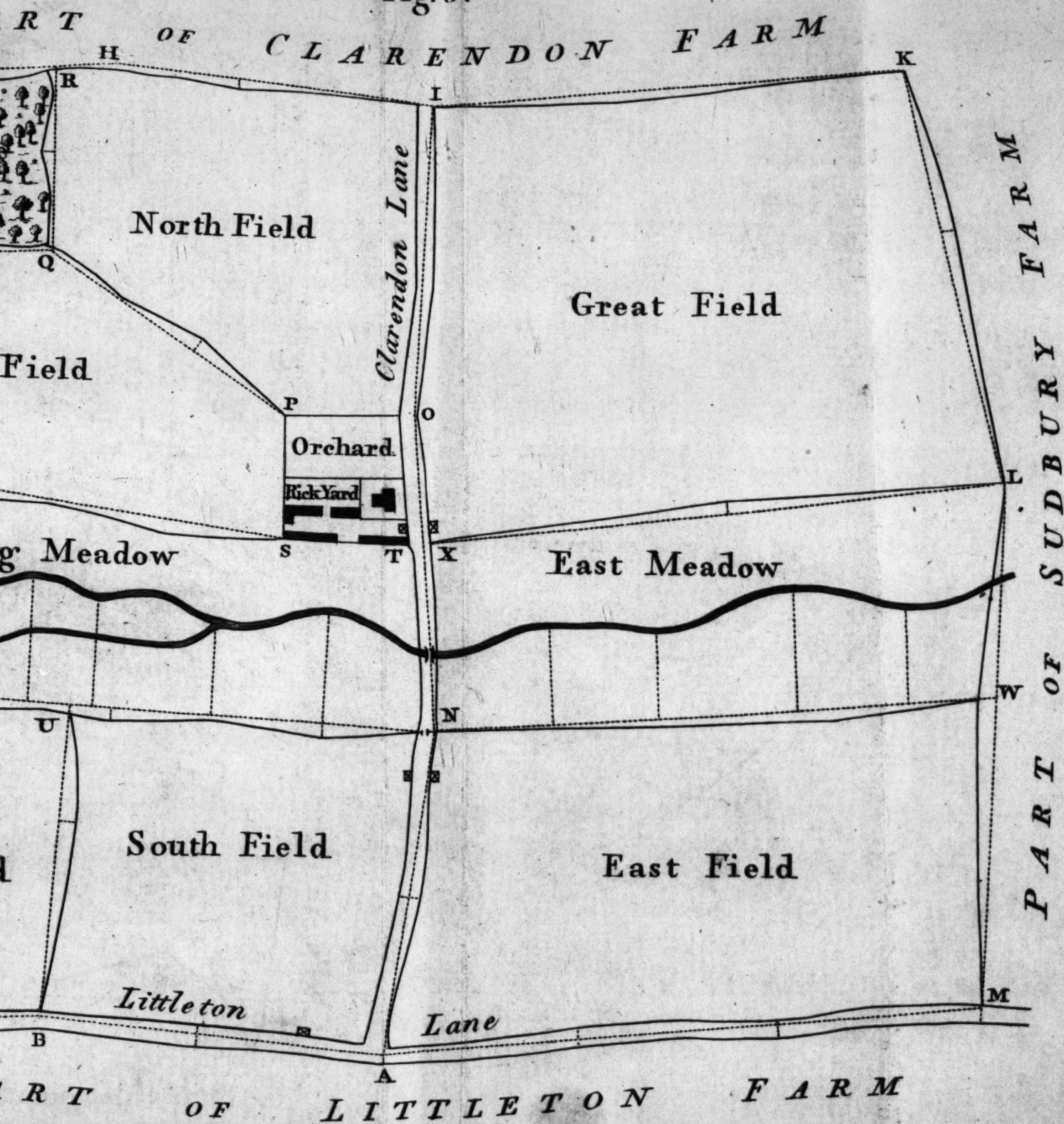
A hand-drawn map of three fields: North Field, West Field, and East Field. The map is bounded by "Lands of J. Williams" at the top, "Lands of Henry Smith" at the bottom, "Part of Hunterdon Farm" on the left, and "Part of Winterton Farm" on the right. The fields are numbered 1 through 9. North Field is at the top, West Field is on the left, and East Field is on the right. The map is drawn with solid lines for the field boundaries and dashed lines for the surrounding land boundaries.

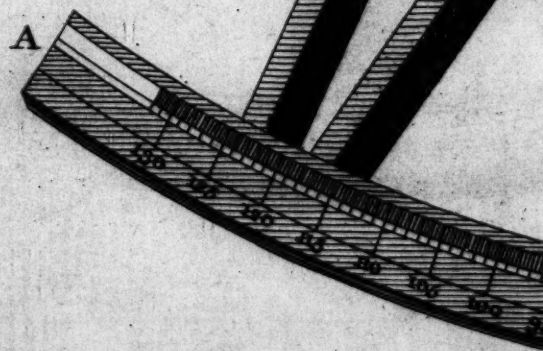
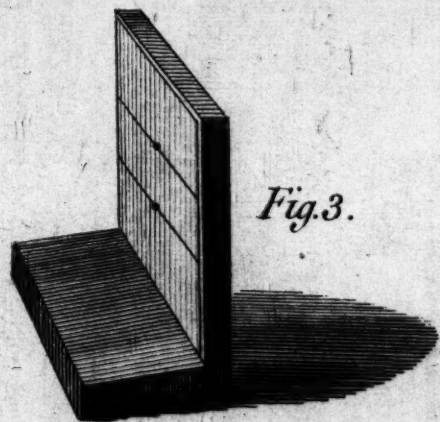
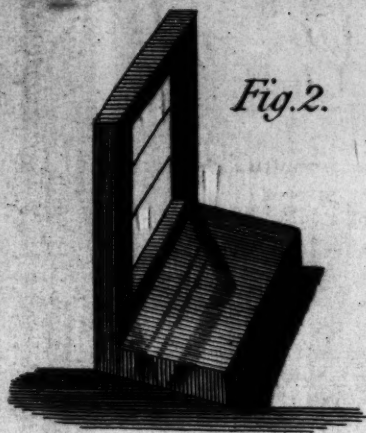
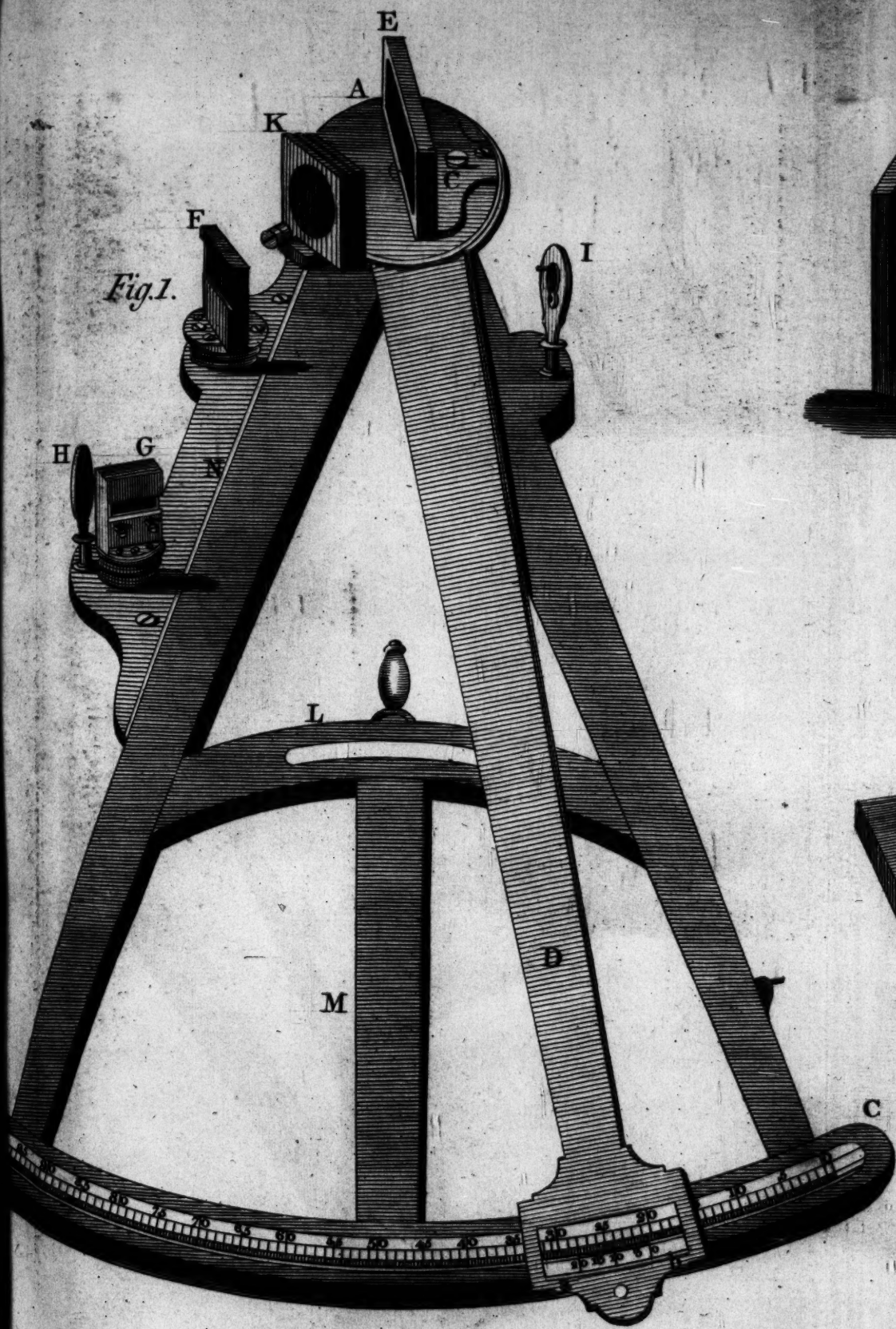
A map of a triangular tract of land. The vertices are labeled 1, 2, and 3. The land is divided into several sections by lines. The sections are labeled: Henry Derrans Land (top left), Wm Higgins's Land (top right), David Horne's Land (right), Char: Ward's Land (bottom right), Wm Humphrey's Land (bottom left), and a small section labeled 7 (bottom center). The map is oriented with a north arrow pointing towards the top right.



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Fig. 5.





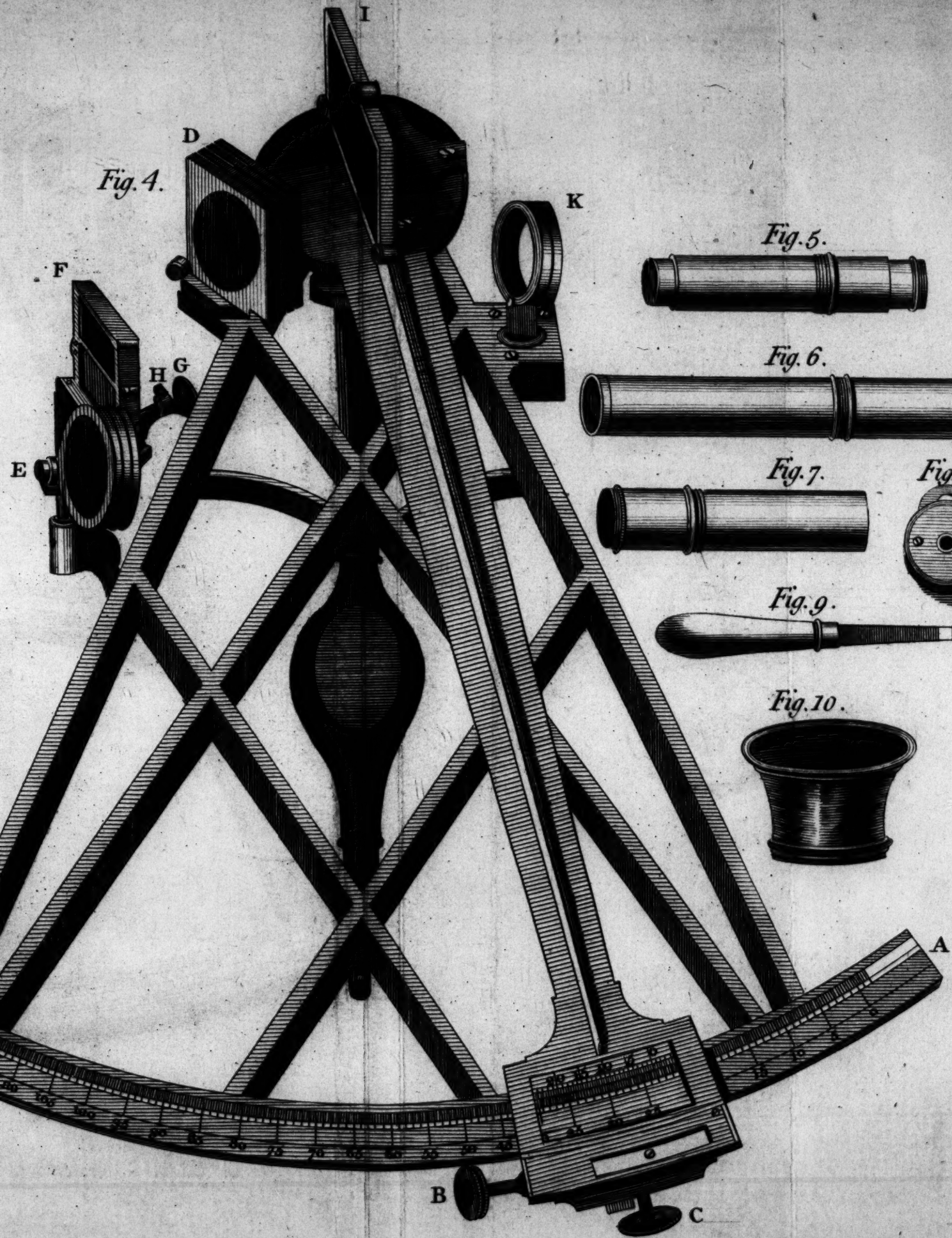


Fig. 4.

Fig. 5.

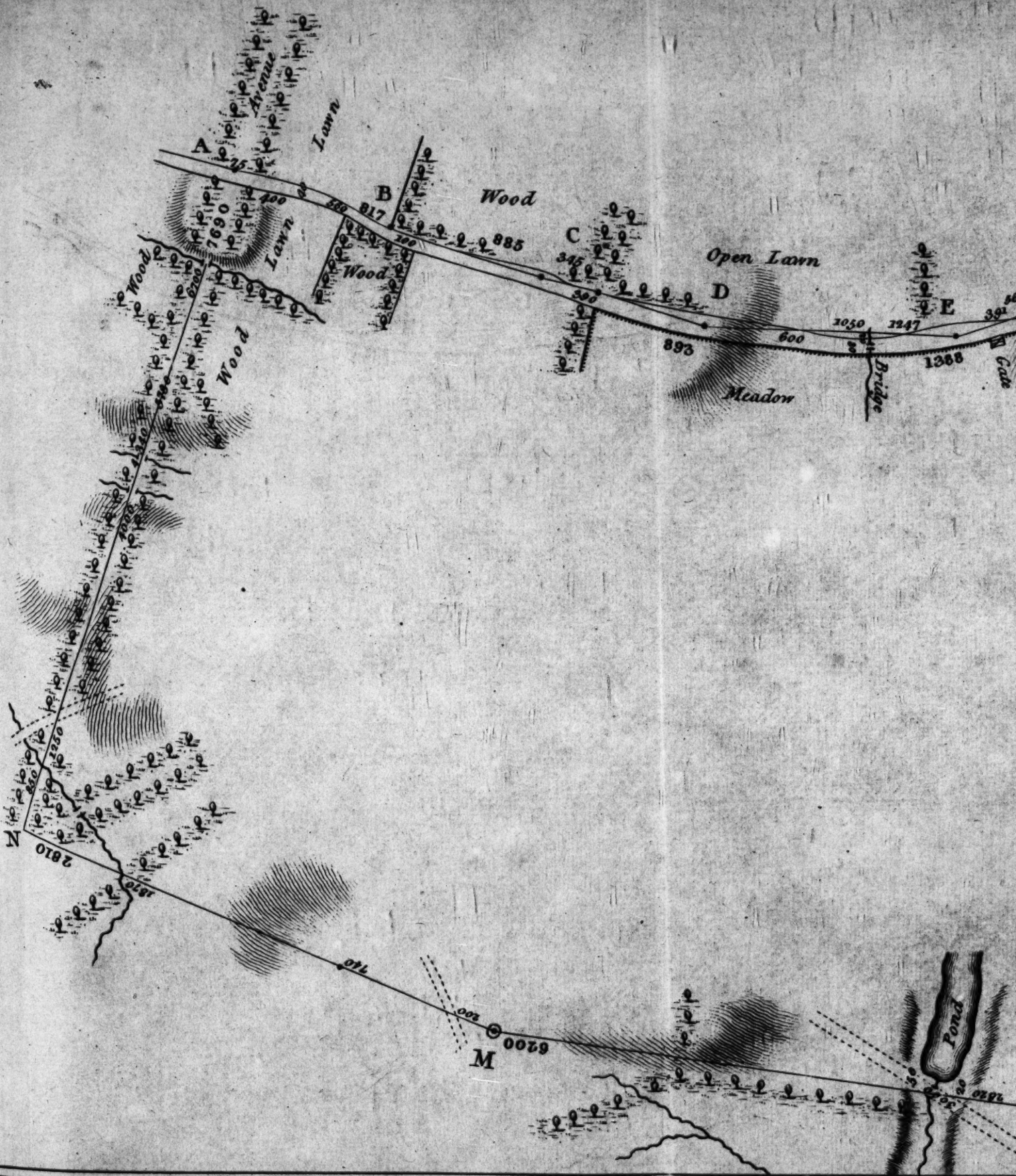
Fig. 6.

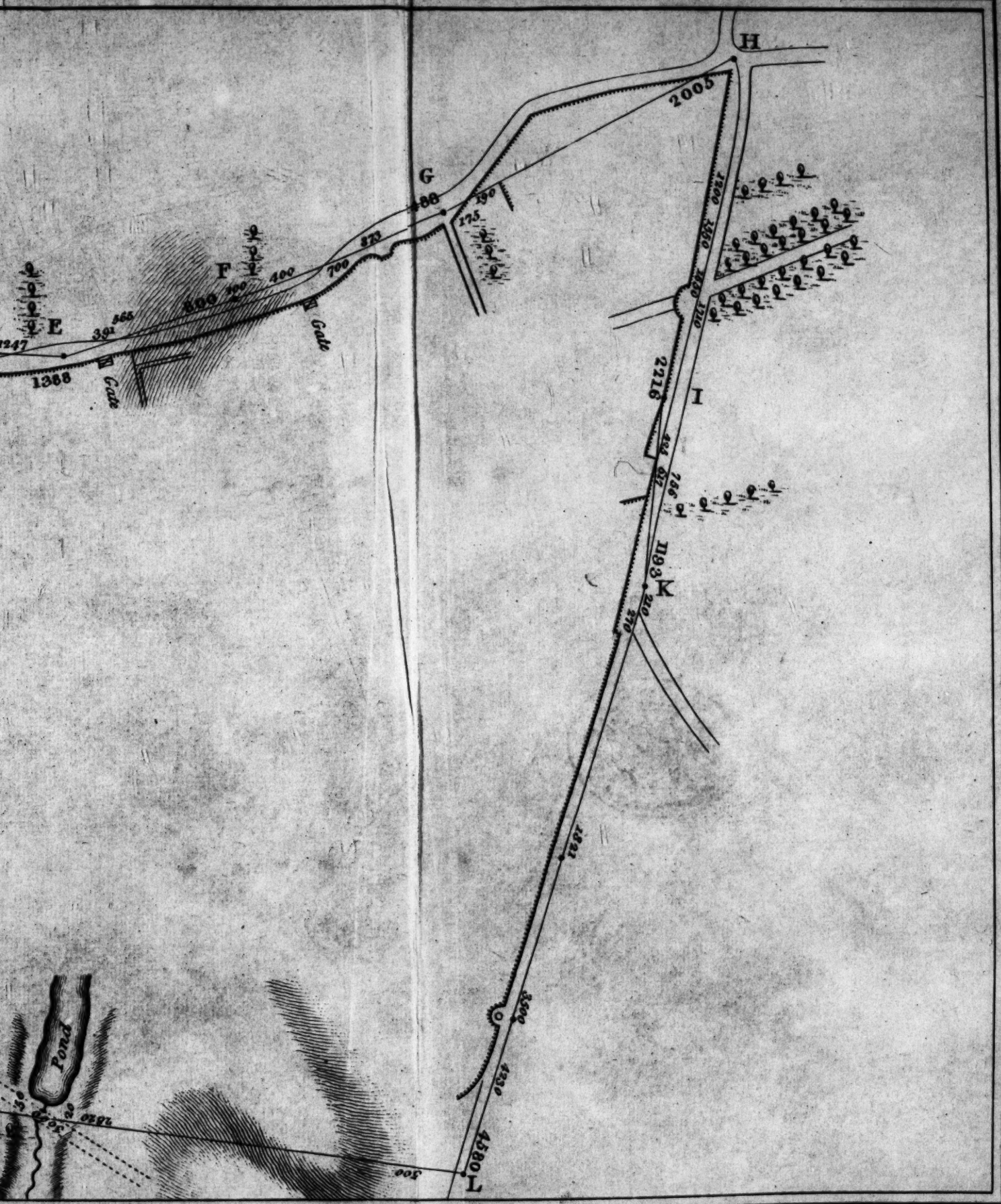
Fig. 7.

Fig. 8.

Fig. 9.

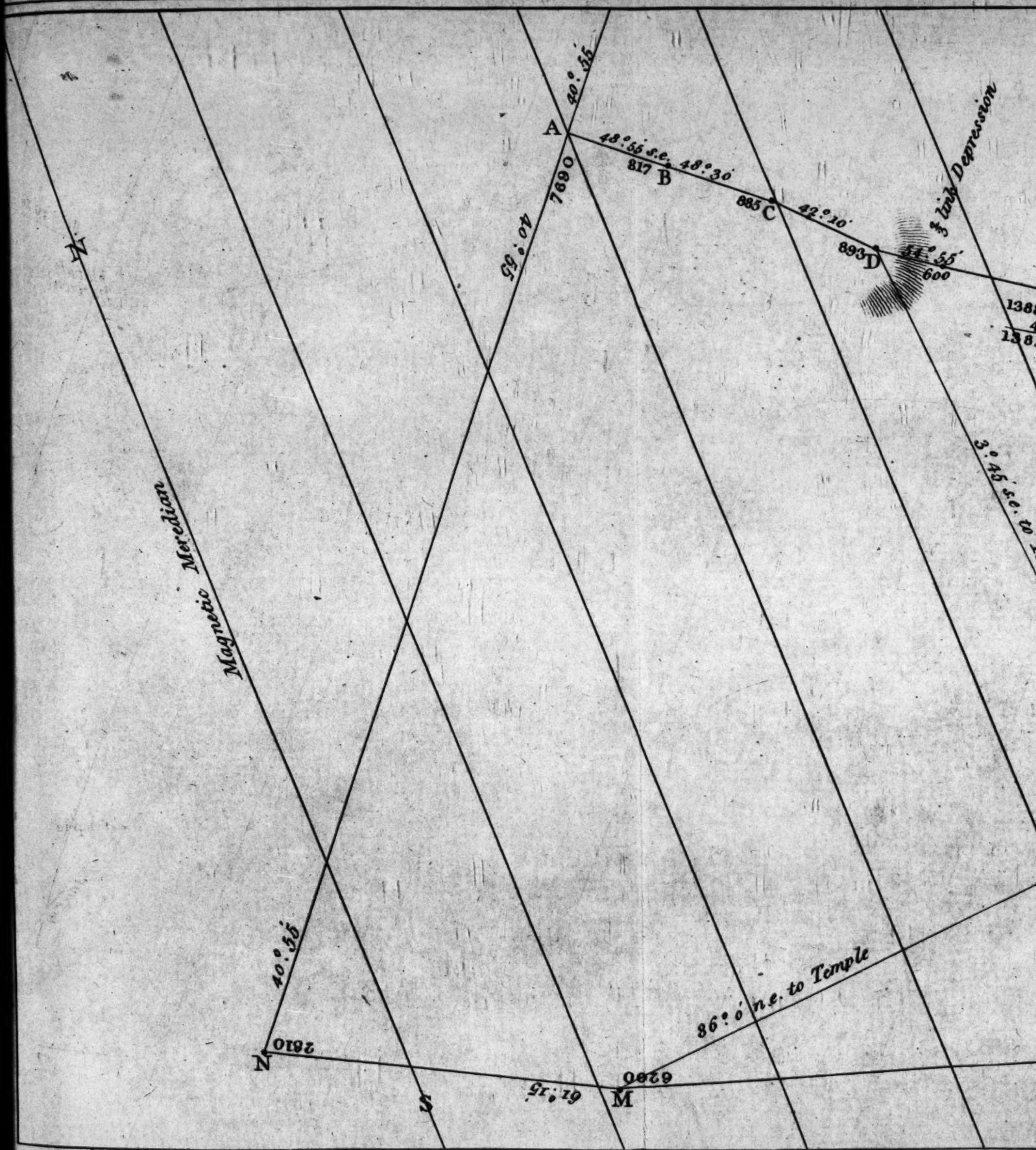
Fig. 10.

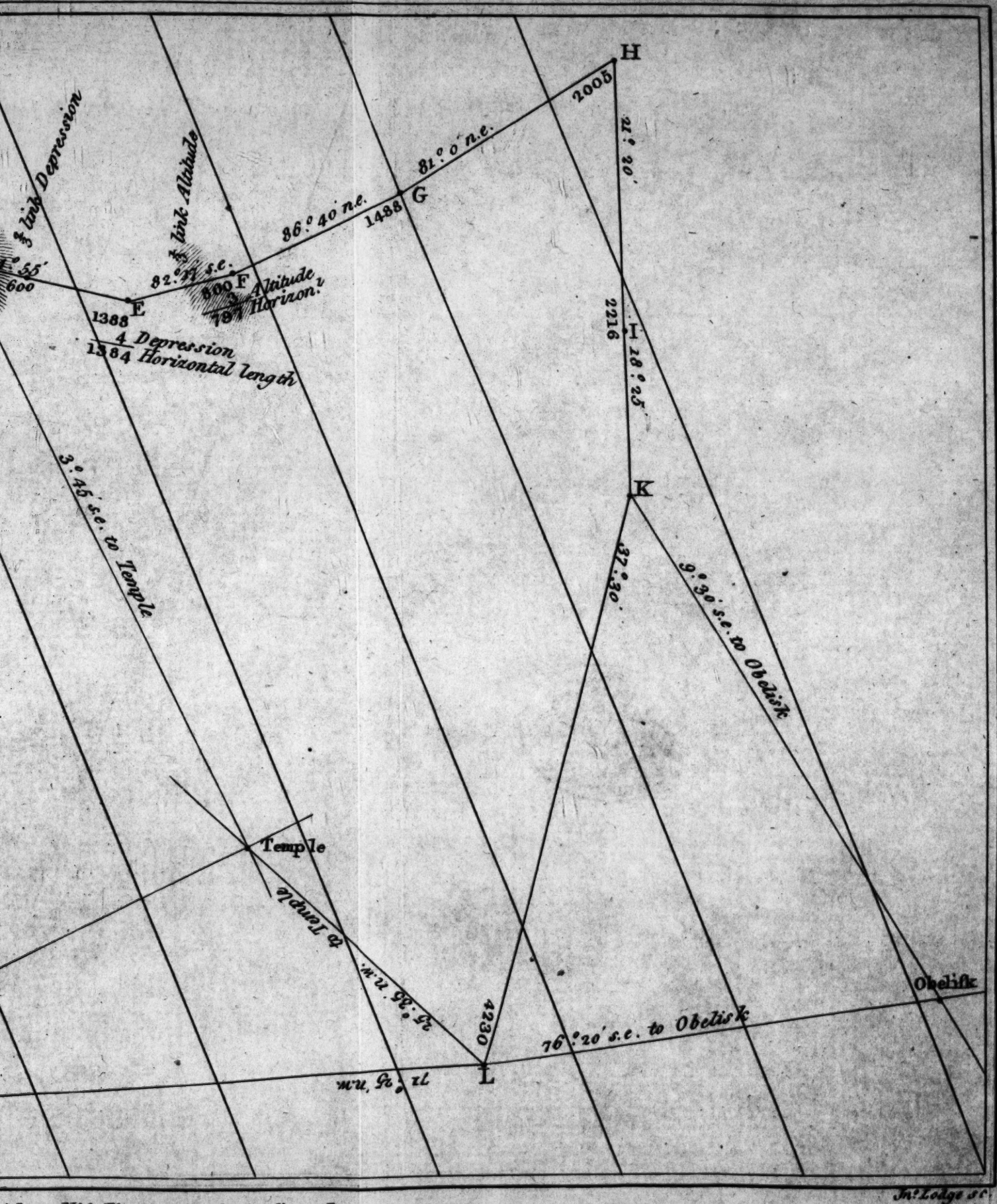


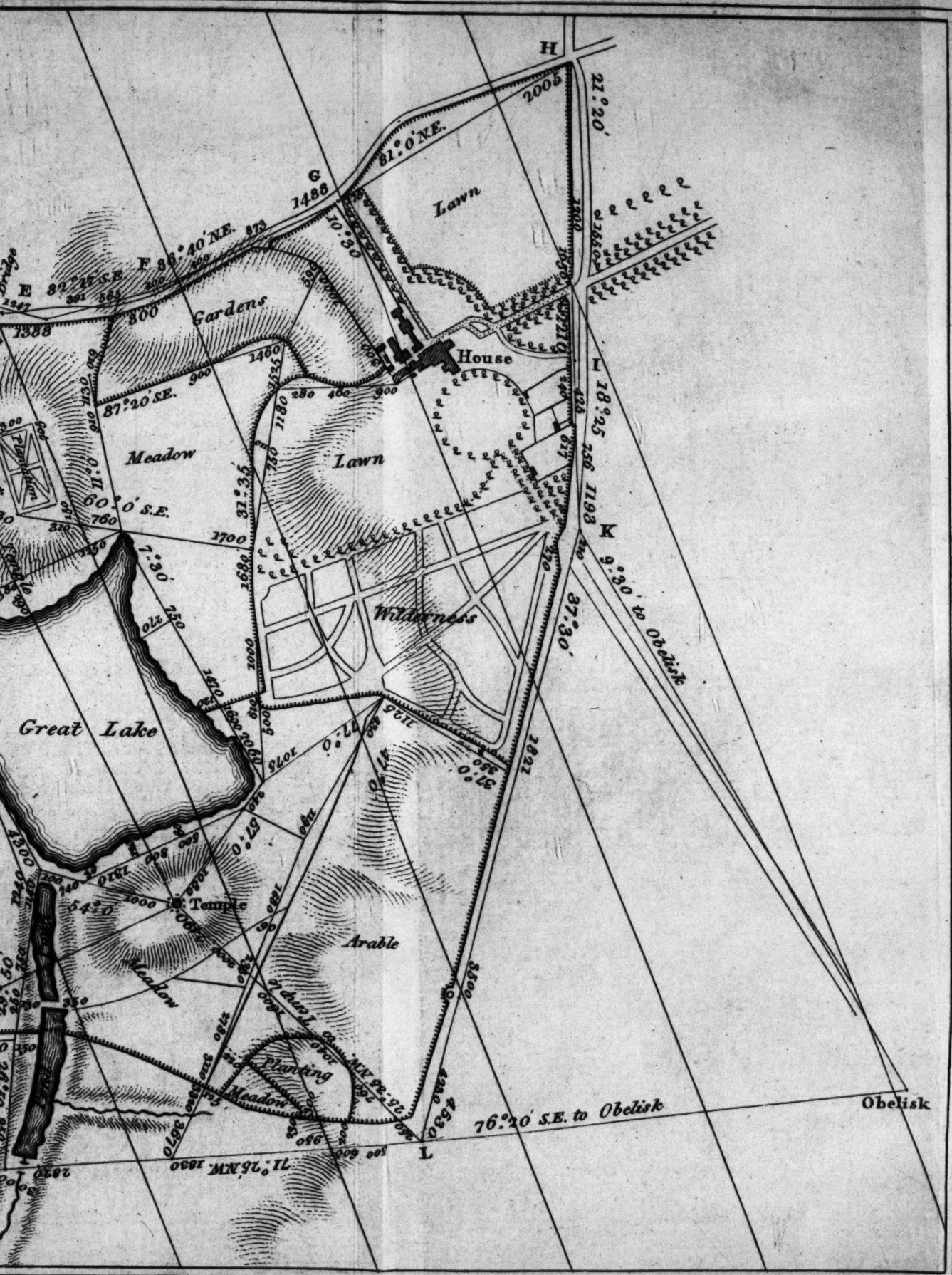


ams, N^o 60 Fleet Street as the Act directs. June 1. 1791.

Jⁿ. Lodge sc.







Fleet Street, as the Act directs. June 1. 1791.

J. Lodge sc.

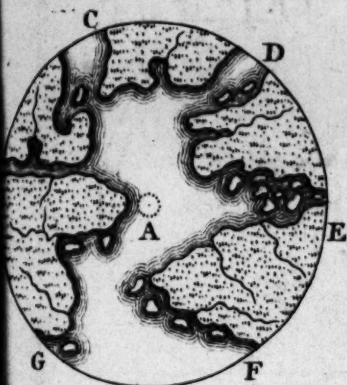


Fig. 1.

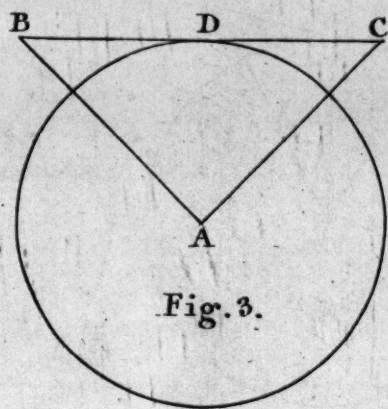


Fig. 3.

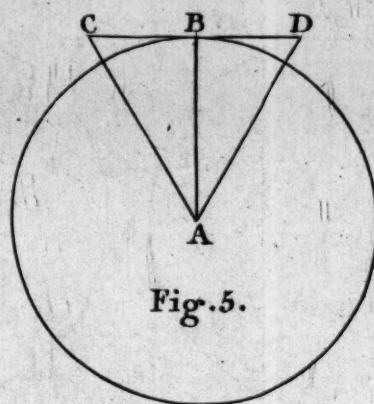


Fig. 5.

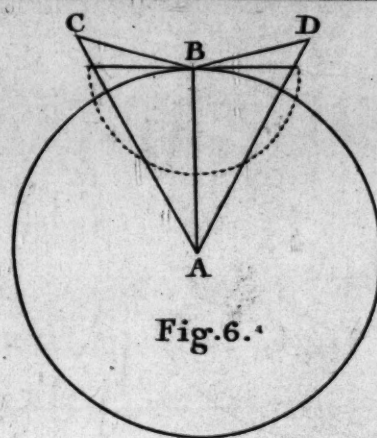


Fig. 6.

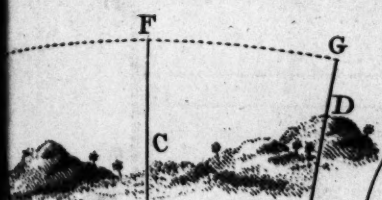


Fig. 2.

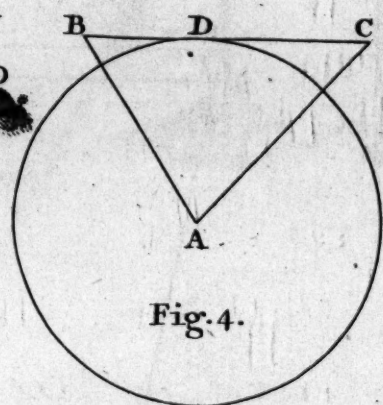


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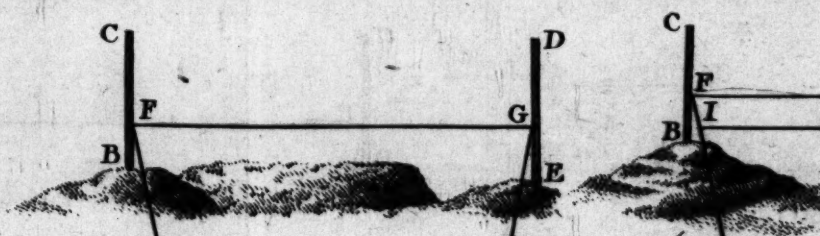


Fig. 9.

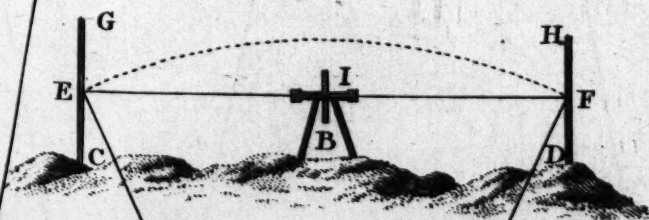


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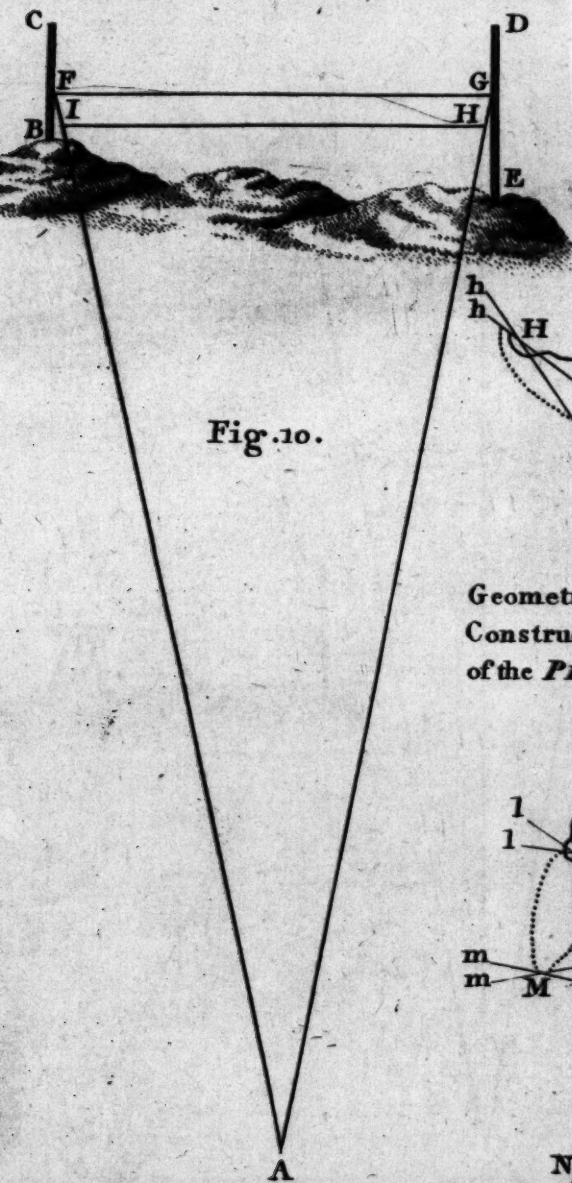
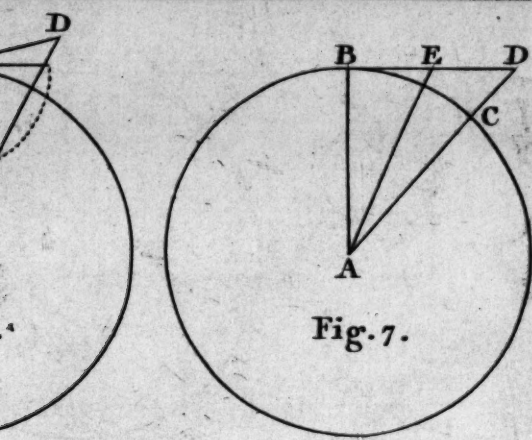
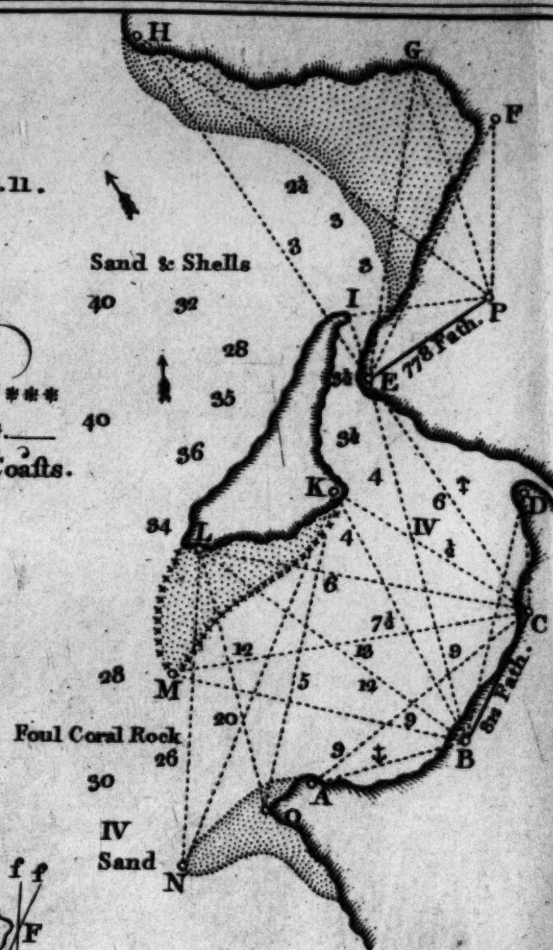


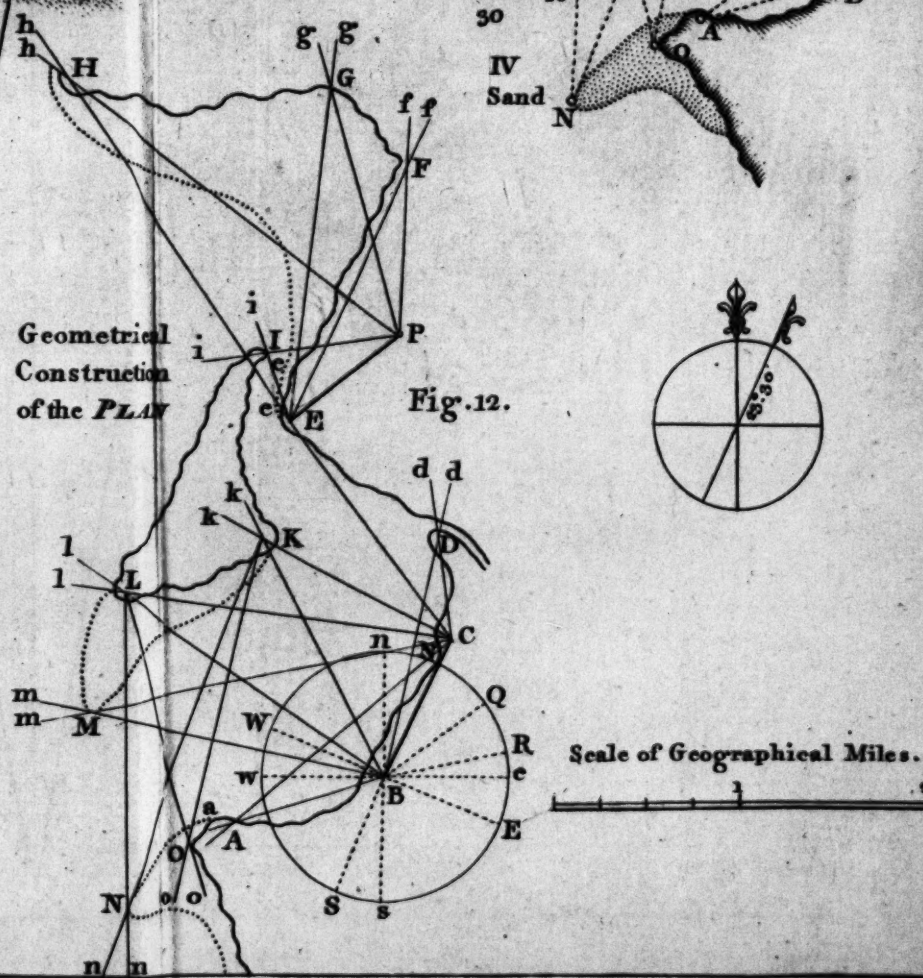
CHART
of the Harbour of ***
in LAT. LONG.
With the Adjacent Coasts.

Fig. 11.



Geometrical
Construction
of the *PLAN*

Fig. 12.



In Lodge sculp.

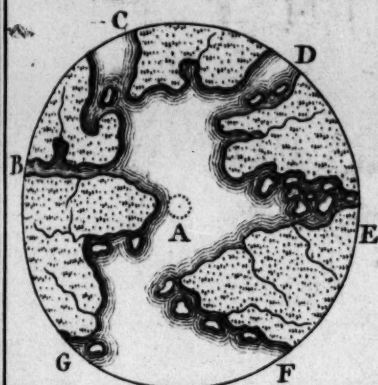


Fig. 1.

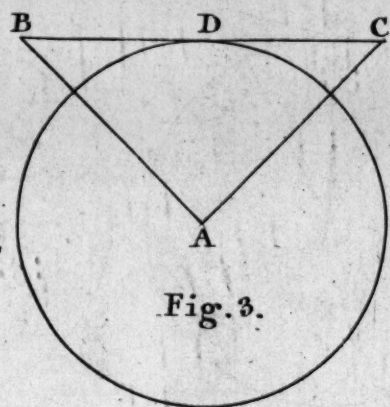


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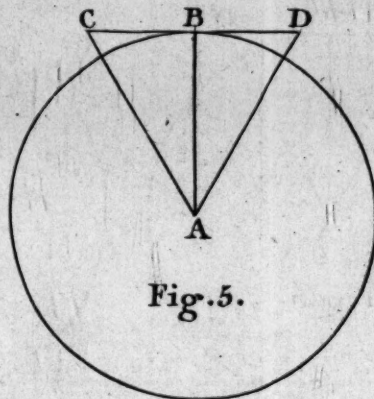


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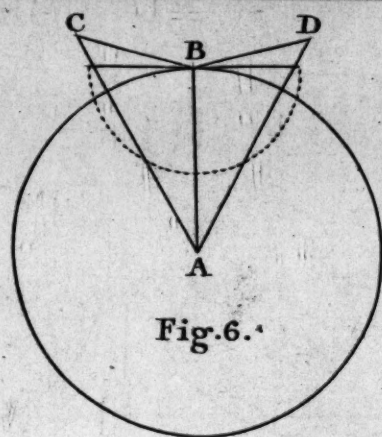


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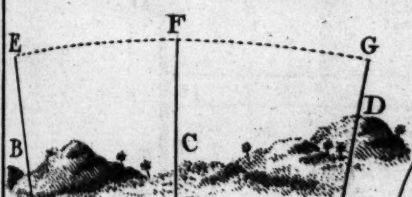


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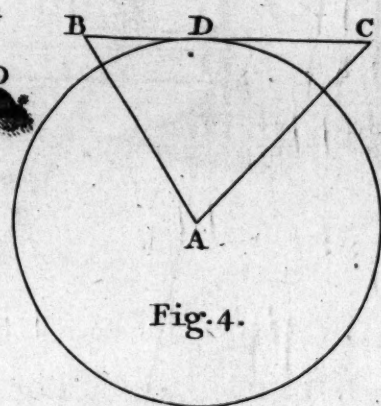


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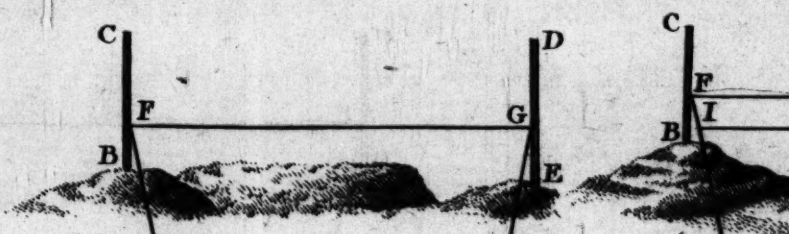


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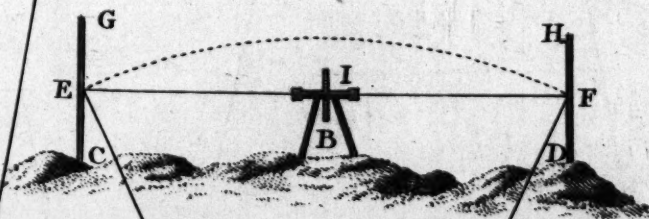


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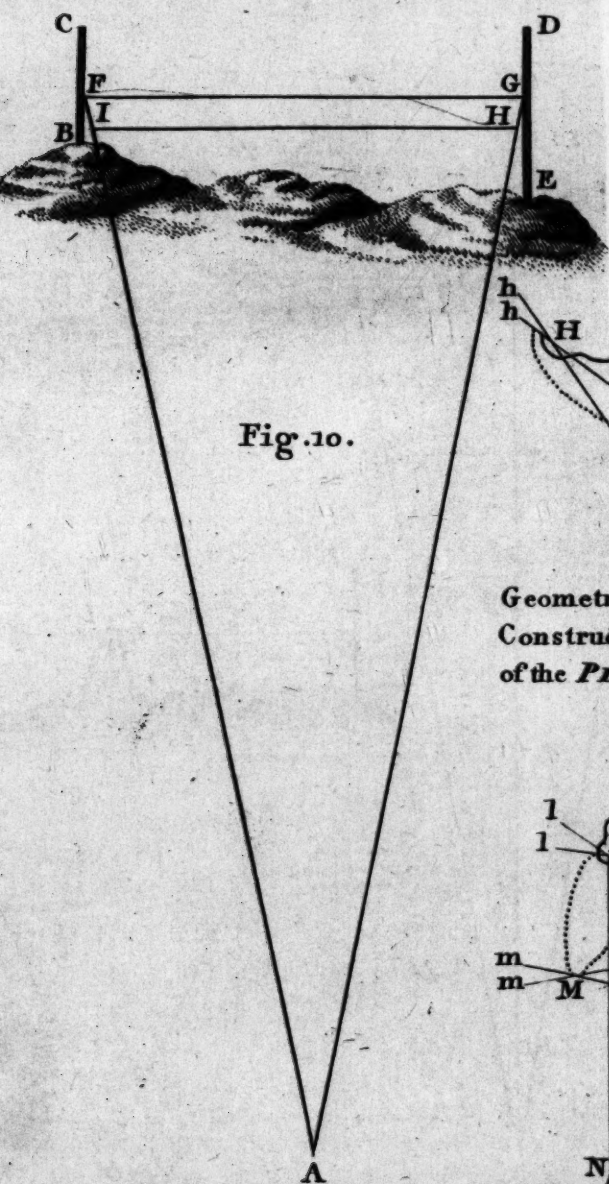
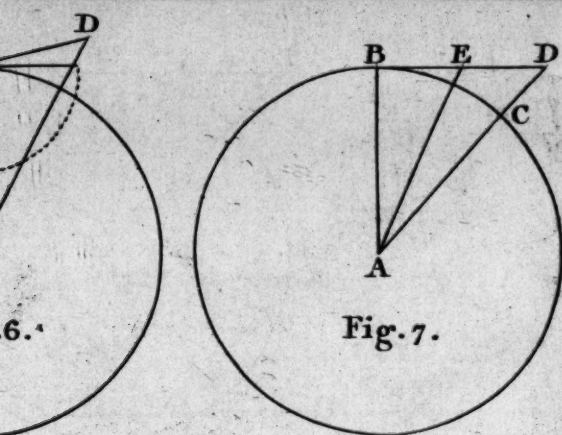
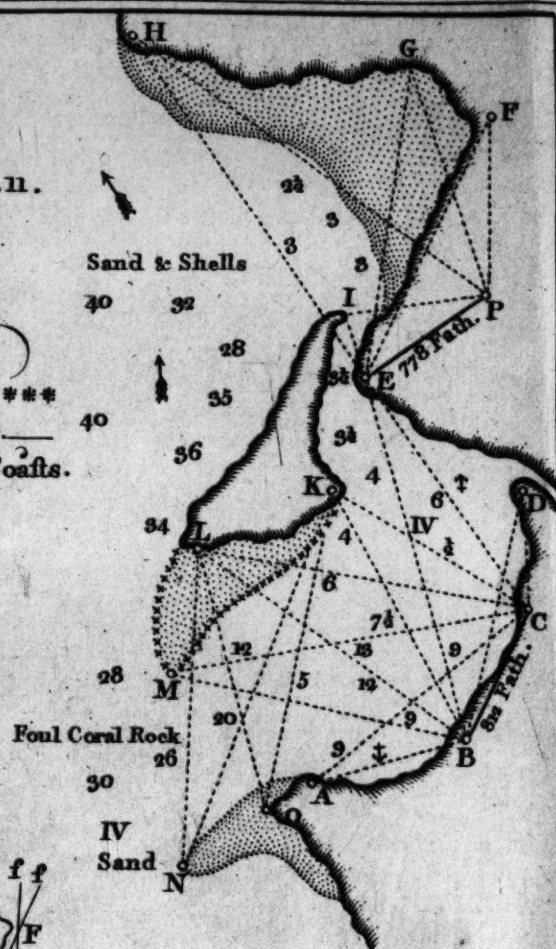


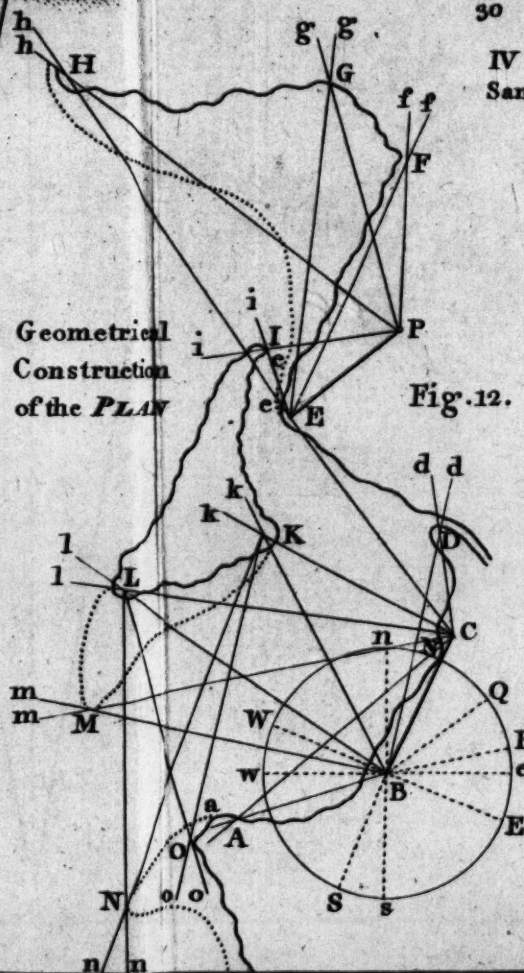
CHART
of the Harbour of ***
in LAT. LONG.
With the Adjacent Coasts.

Fig. 11.

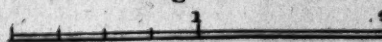


Geometrical
Construction
of the *PLAN*

Fig. 12.



Scale of Geographical Miles.



In *Lodge's* scale.

Fig. 4.

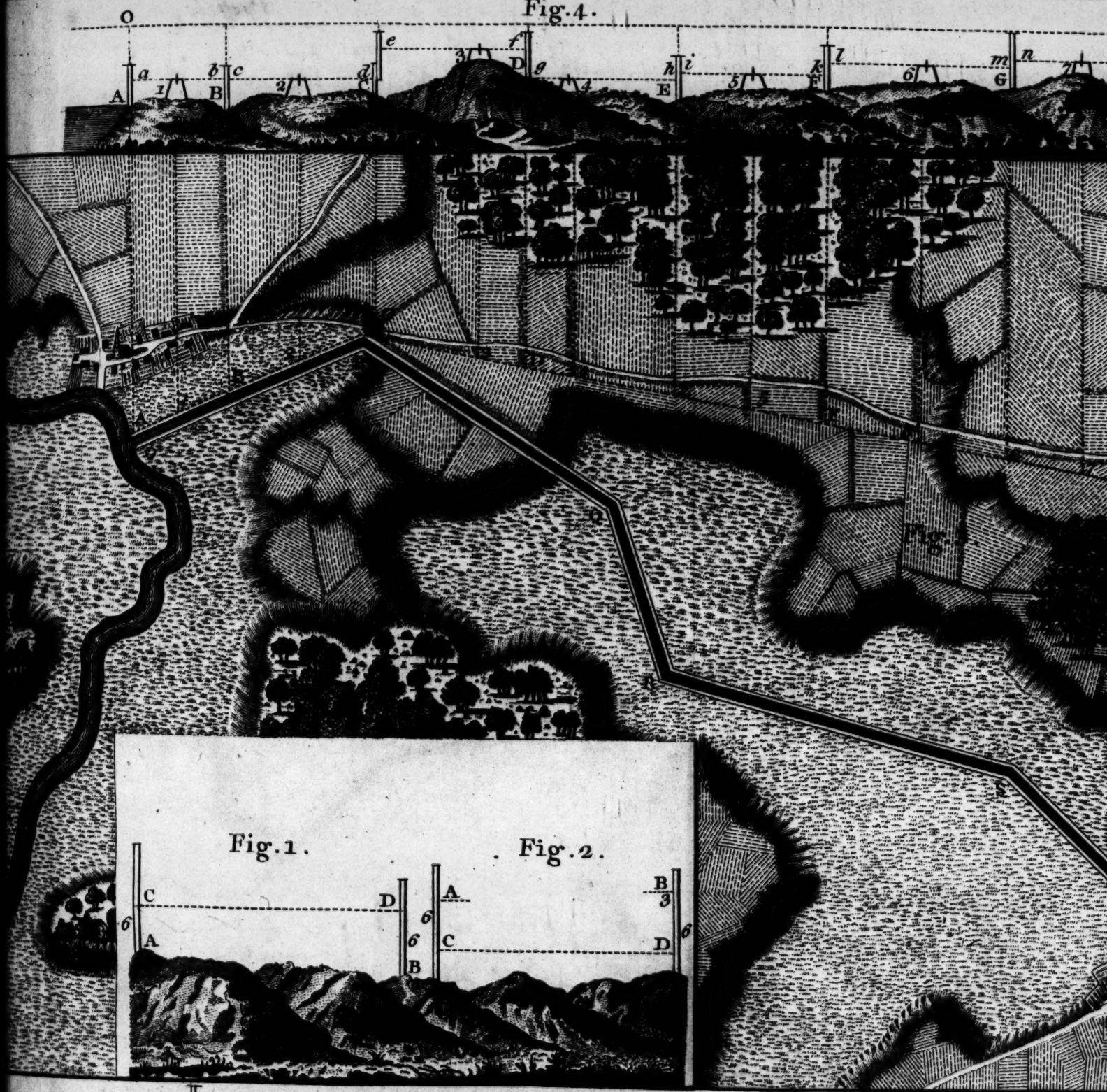


Fig. 1.

Fig. 2.

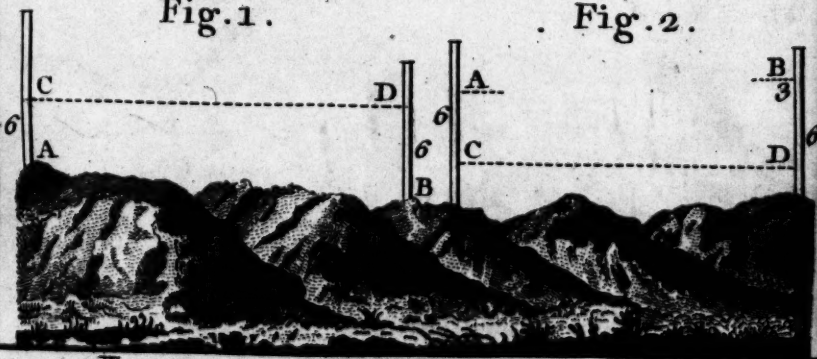
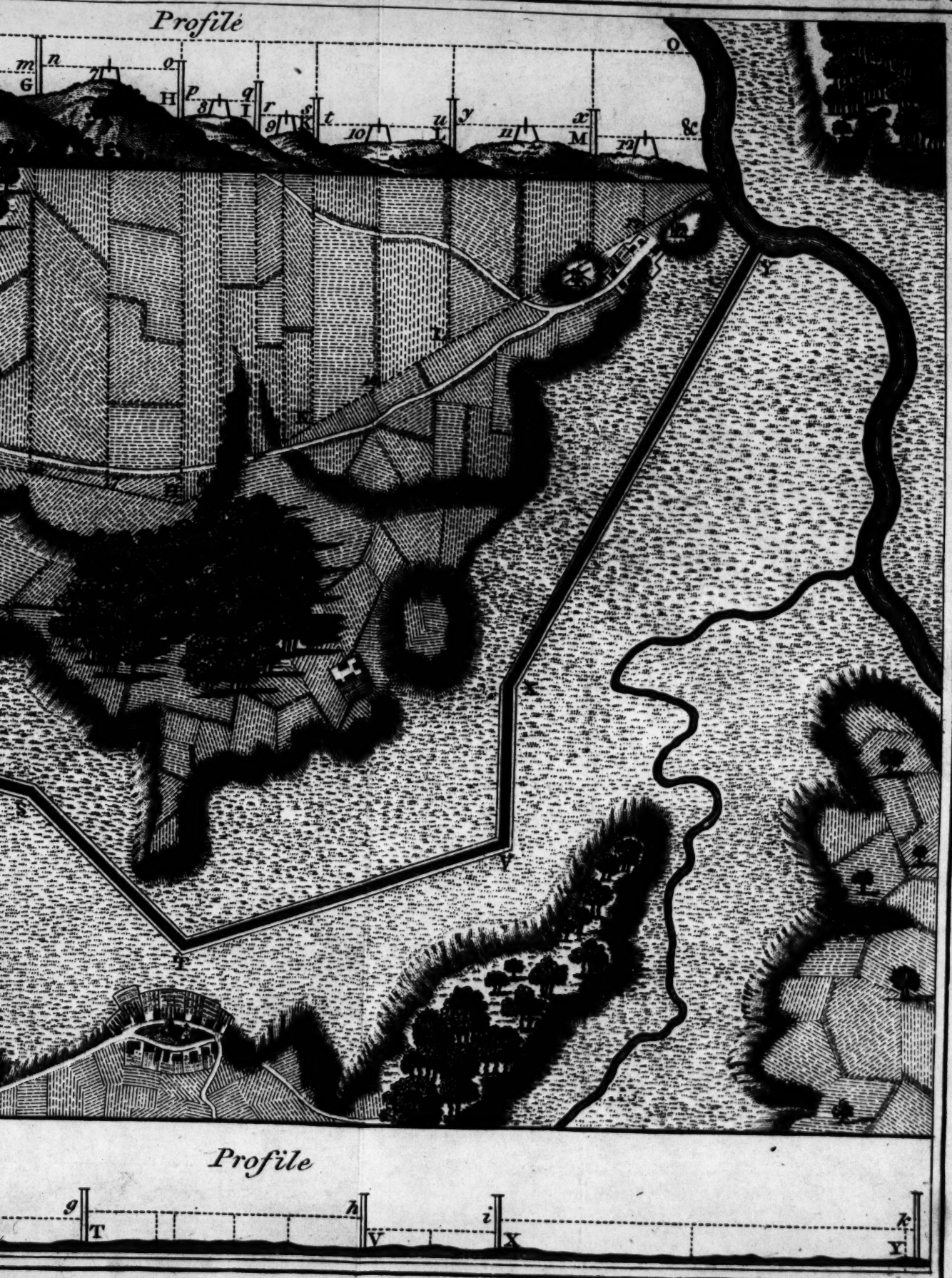


Fig. 5.



delin.



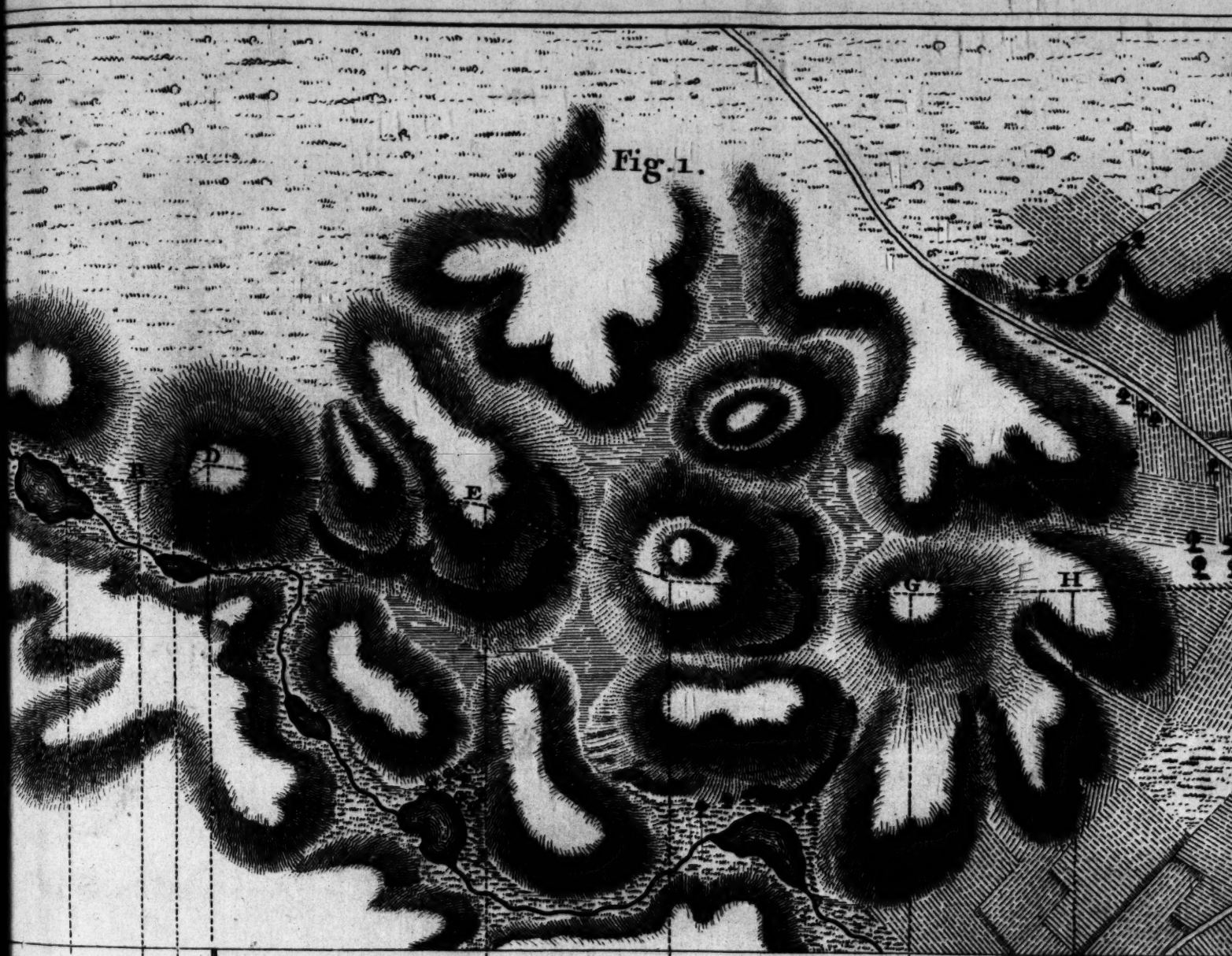


Fig. 1.

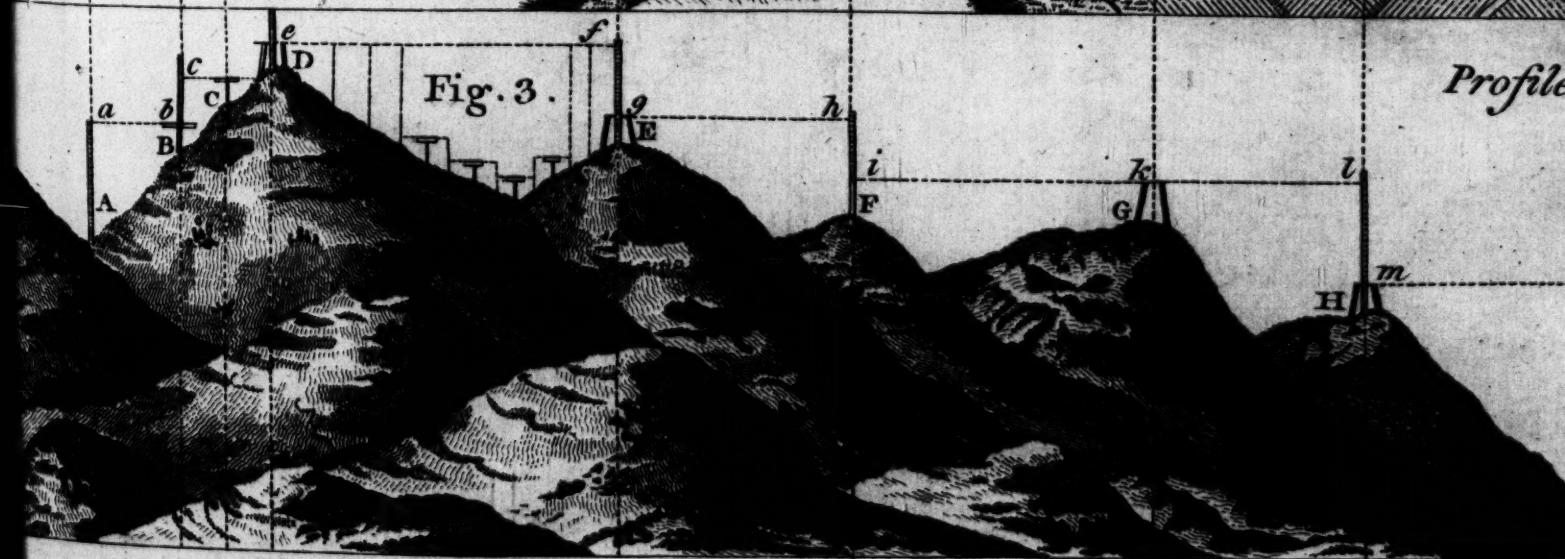
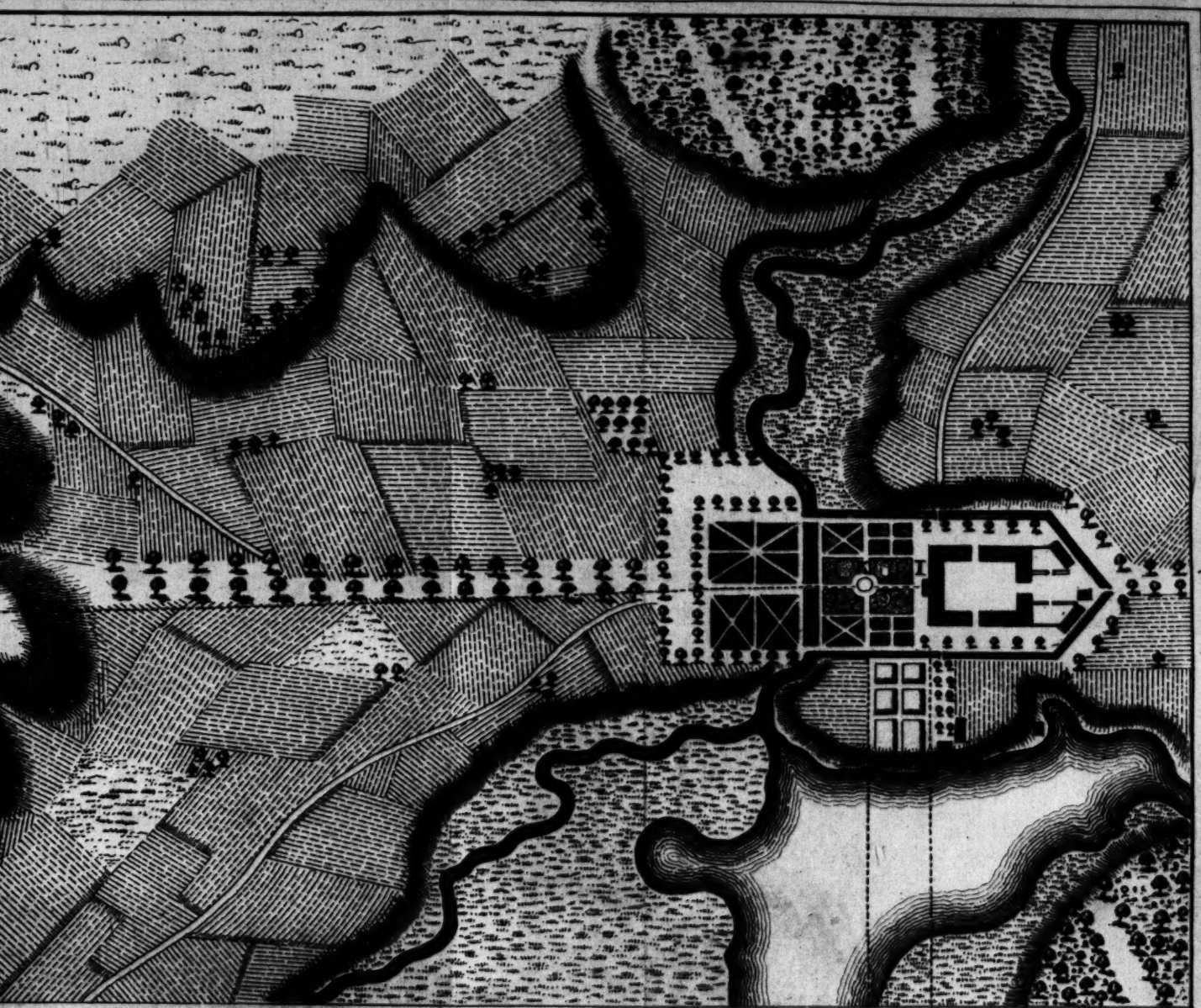


Fig. 3.

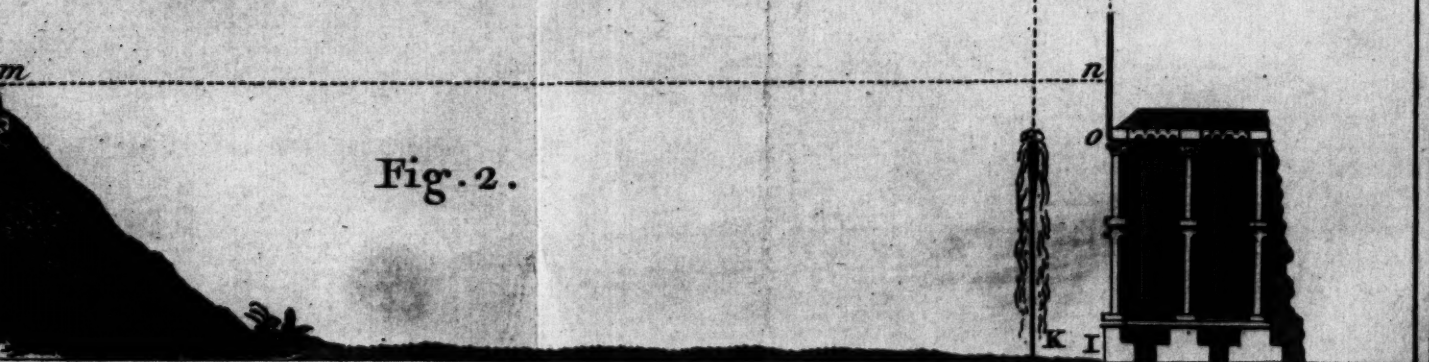
Profile

Line drawn?



Profile

Fig. 2.



ams. N° 60 Fleet Street. as the Act directs. June 1791.

J. Lodge sc.

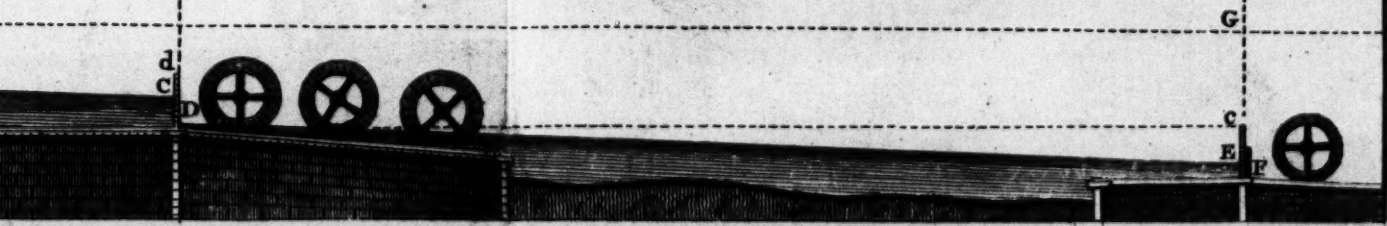


Fig. 2.

Fig. 1.



Profils



N° 60 Fleet Street. as the Act directs. June 11791.

In. Lodge sc.

Fig. 1.

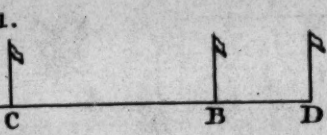
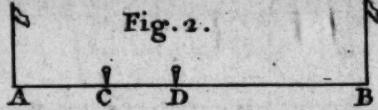


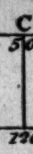
Fig. 2.



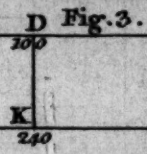
Aefgh E



C



D Fig. 3.



150



Fig. 6.

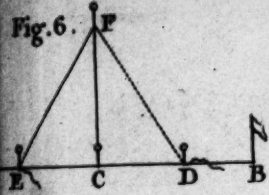


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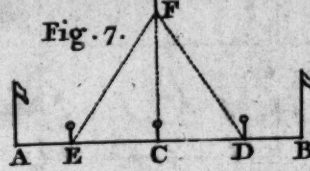


Fig. 8.

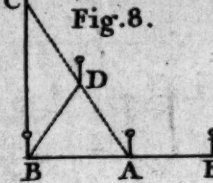


Fig. 9.

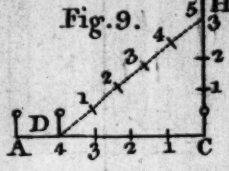


Fig. 10.

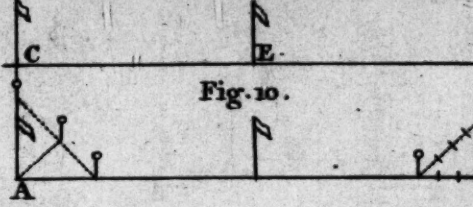


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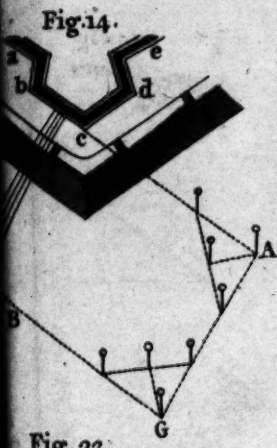


Fig. 15.

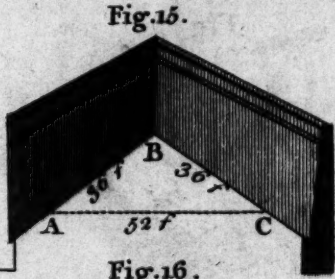


Fig. 16.

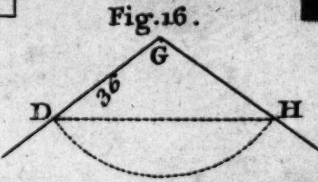


Fig. 17.

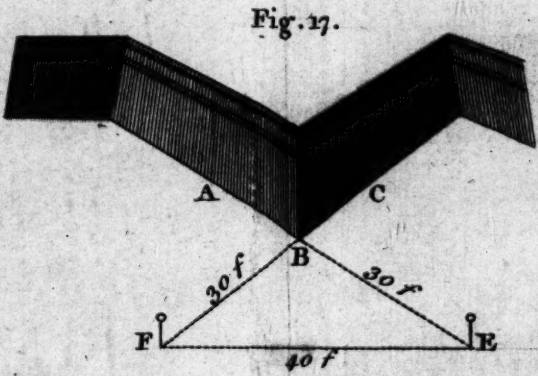


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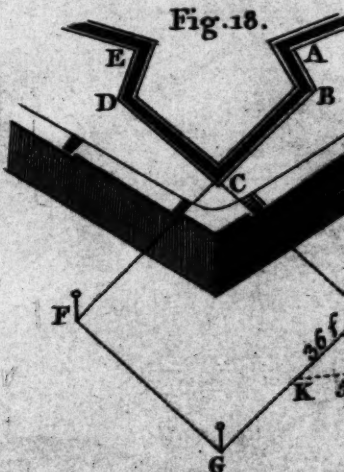


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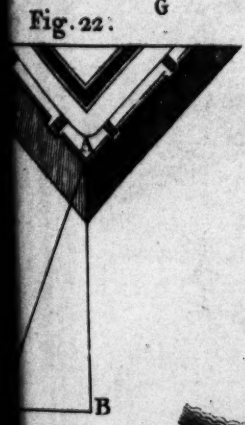


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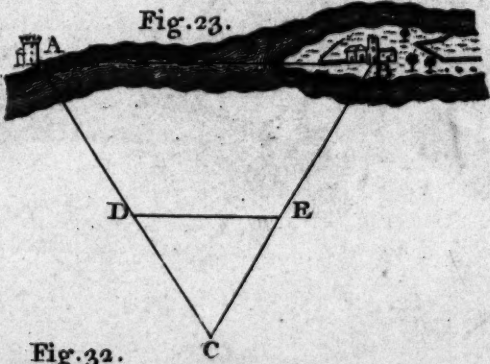


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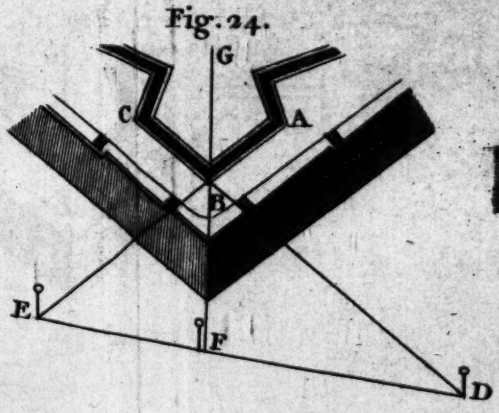


Fig. 32.

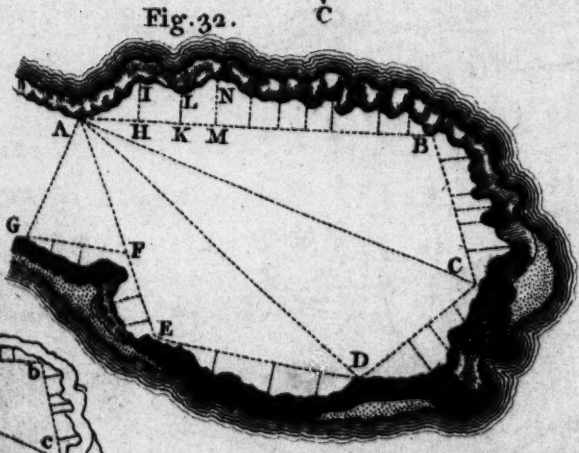


Fig. 33.



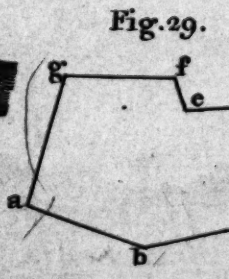
Fig. 30.

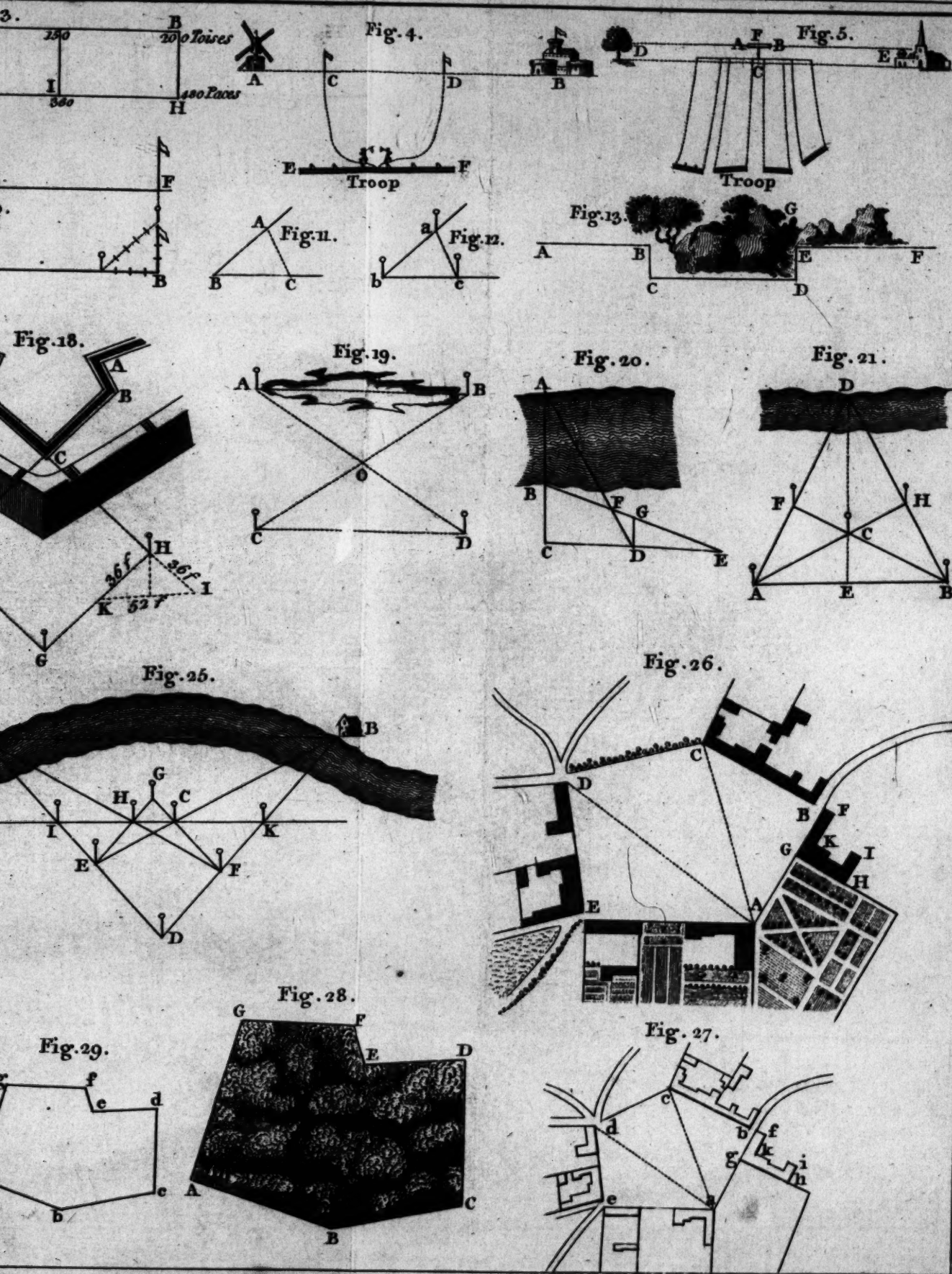


Fig. 31.



Fig. 29.





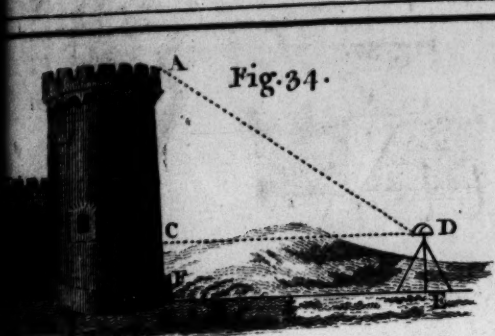


Fig. 34.

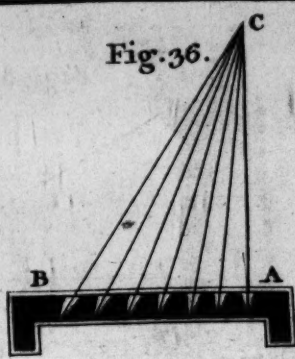


Fig. 36.

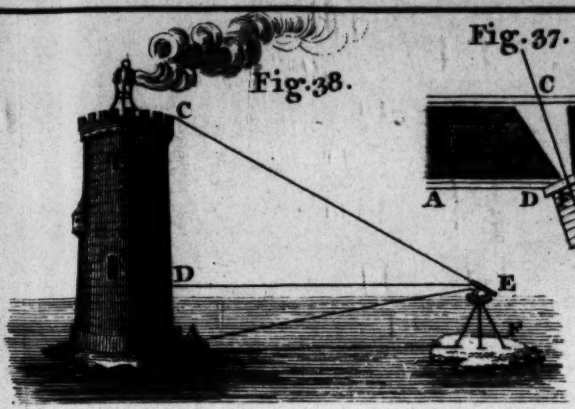


Fig. 38.

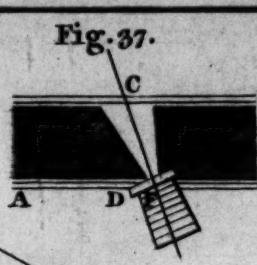


Fig. 37.



Fig. 35.

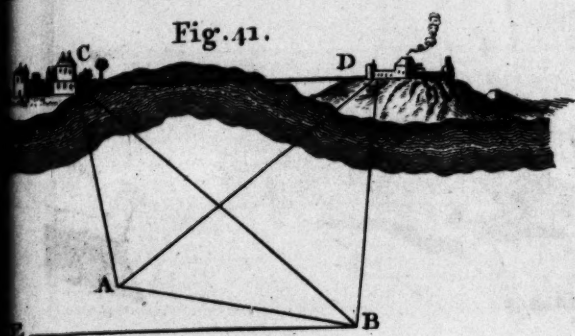


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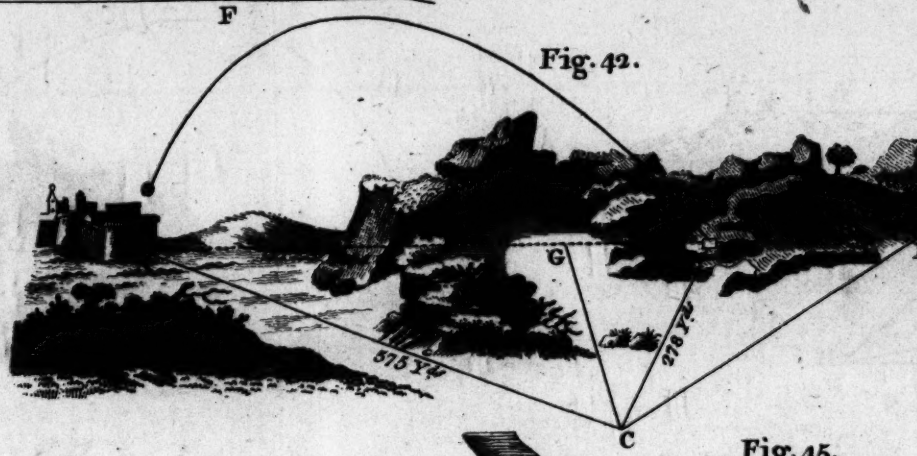


Fig. 42.

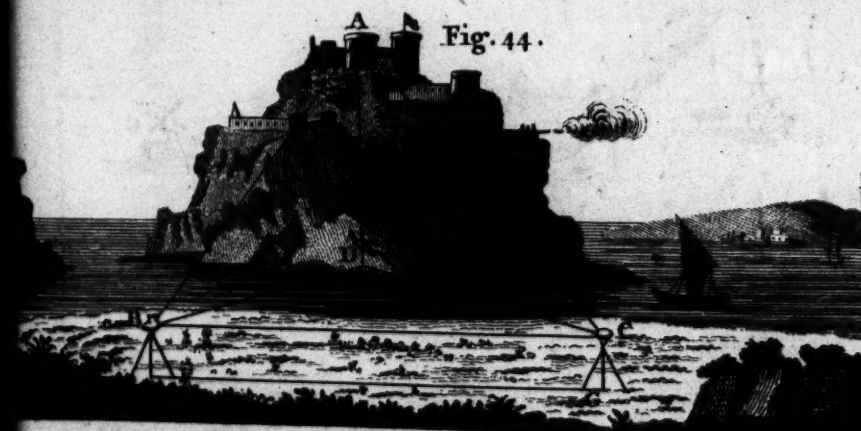


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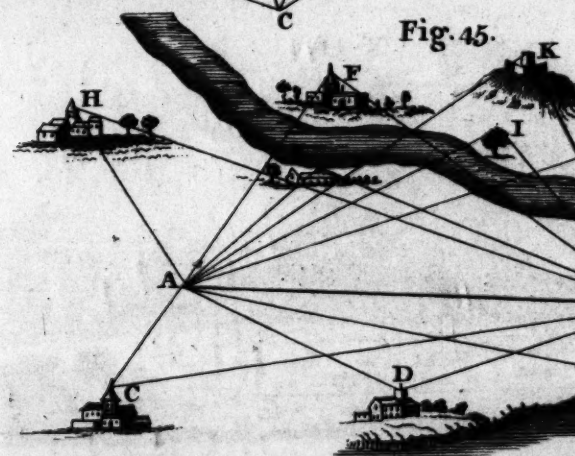


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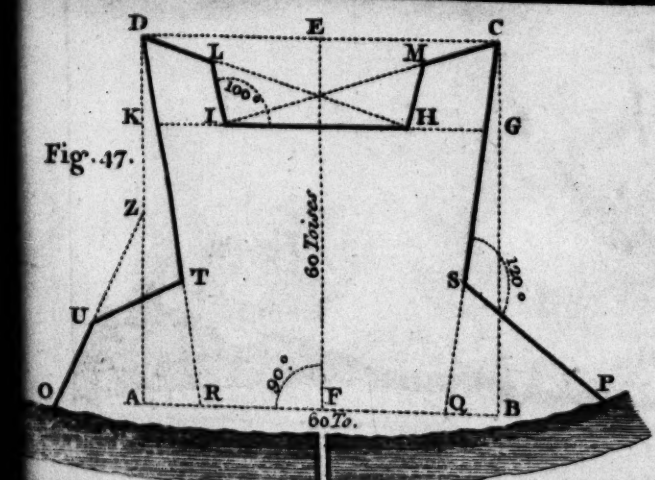


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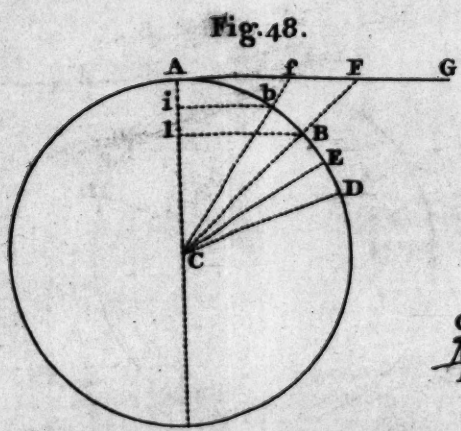
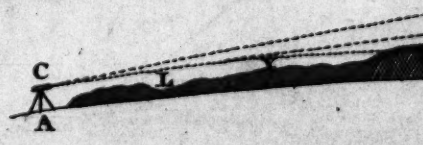


Fig. 48.



delin.

Fig. 37.

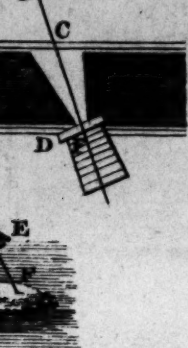


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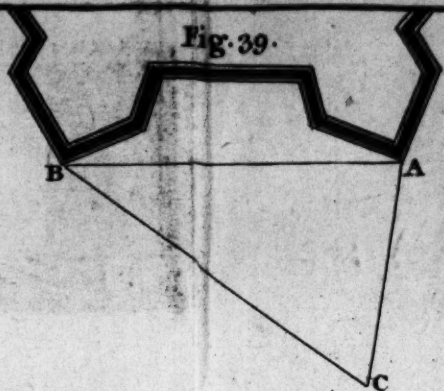


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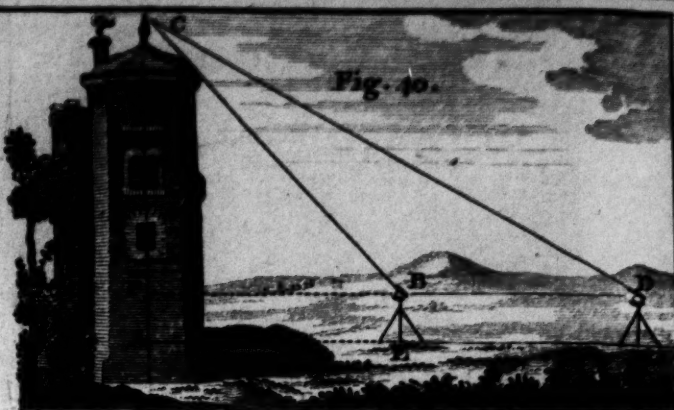


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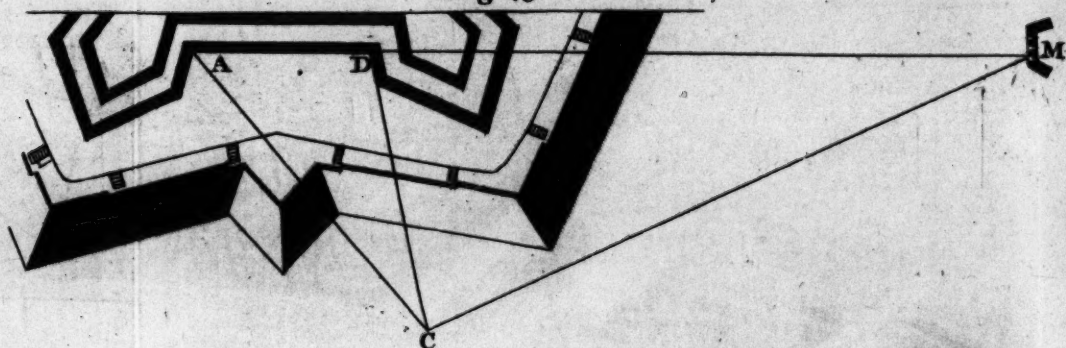


Fig. 45.

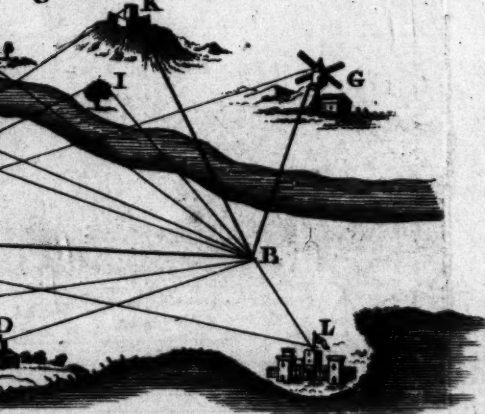


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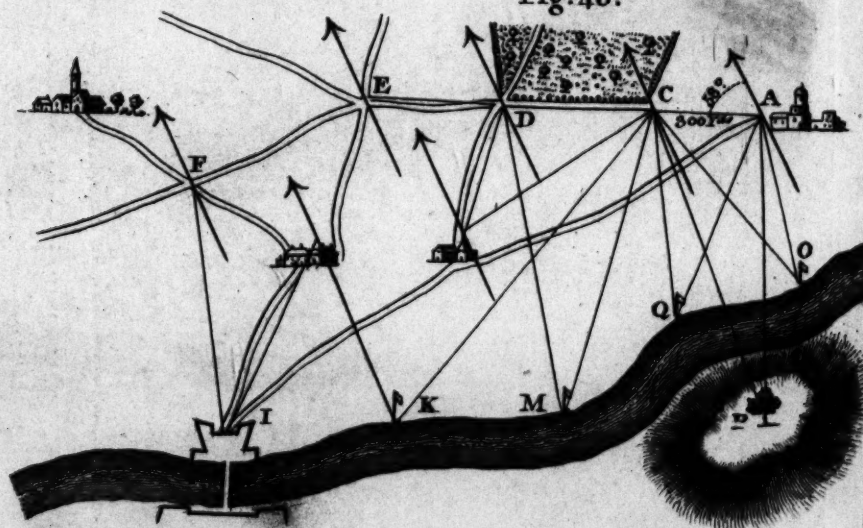


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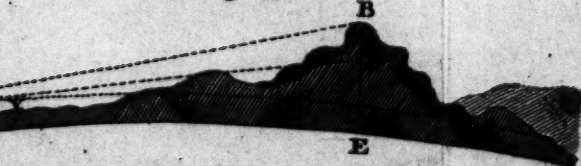


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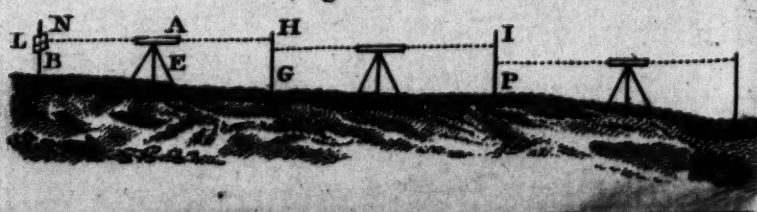
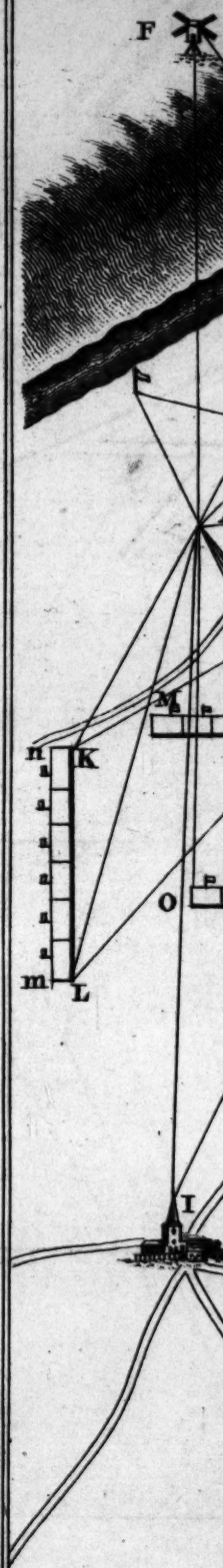
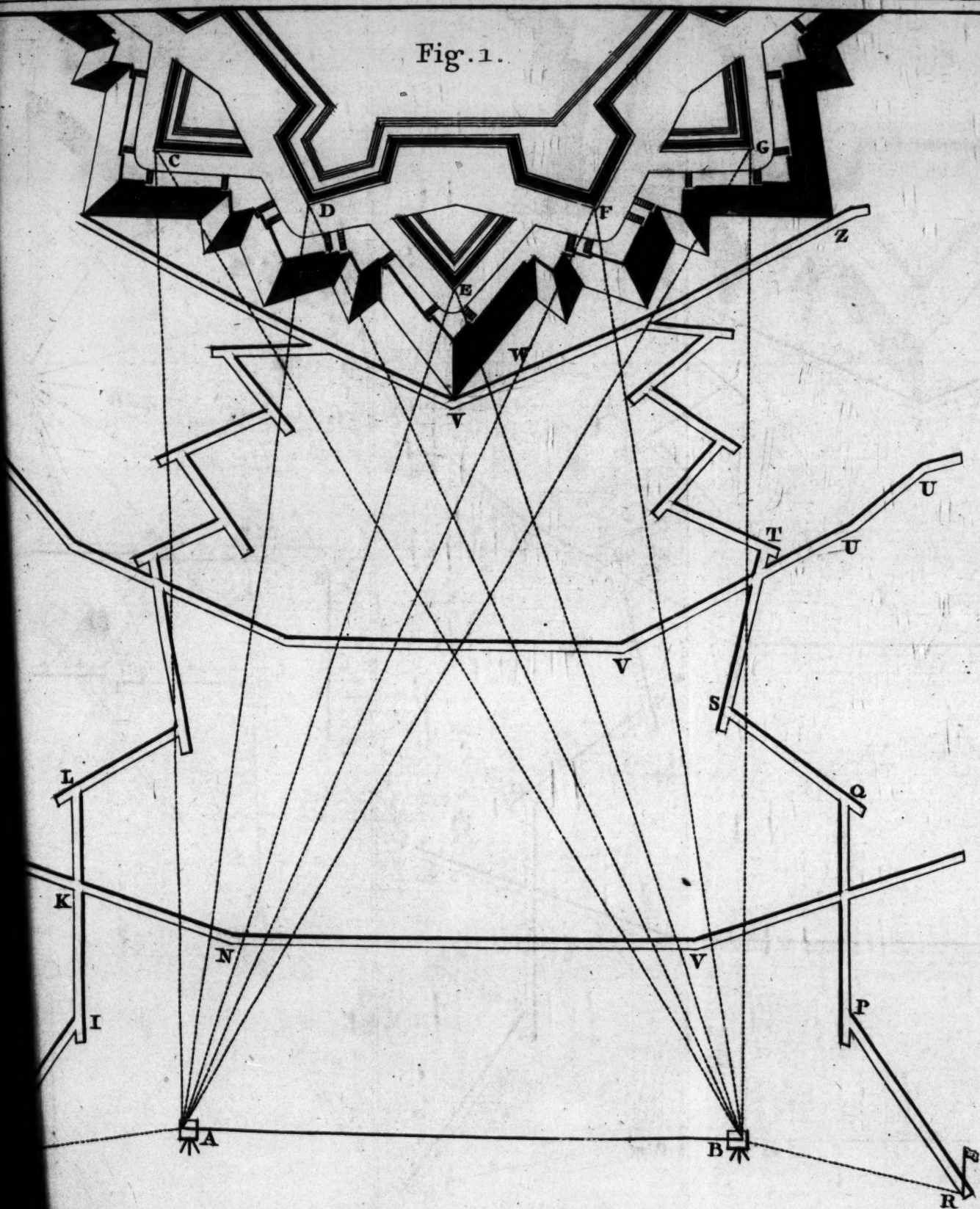
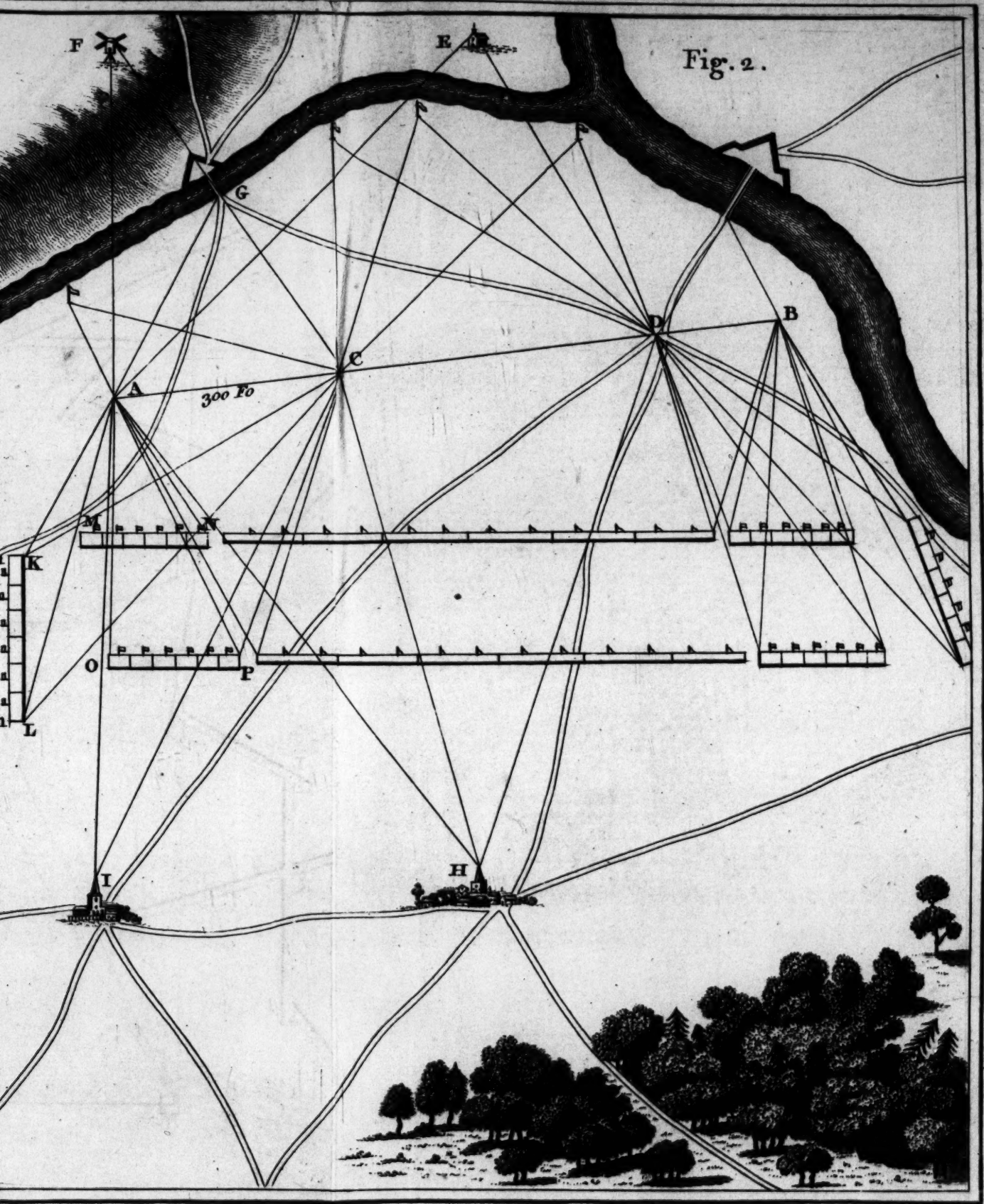


Fig. 1.





N^o 60, Fleet Street, as the Act directs, June 11 1790.

Jn^o Lodge Sc.

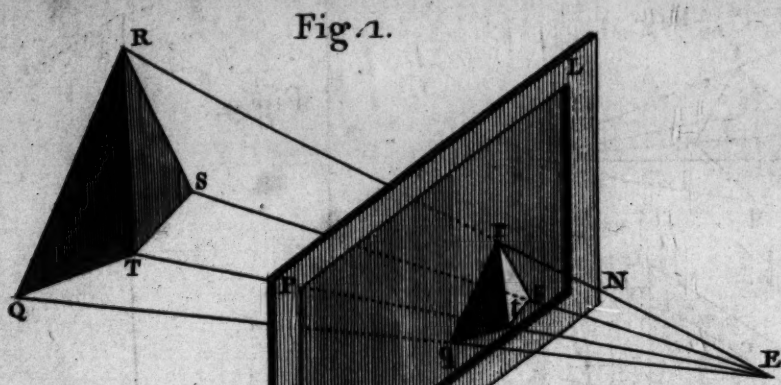


Fig. 1.

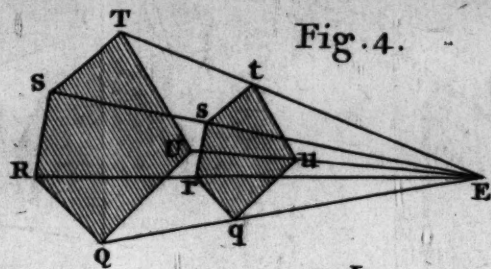


Fig. 4.

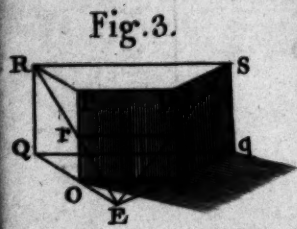


Fig. 3.

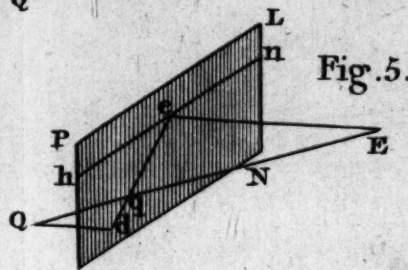


Fig. 5.

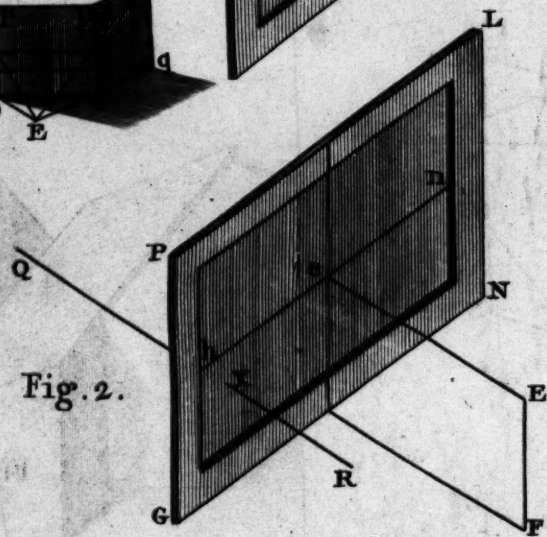


Fig. 2.

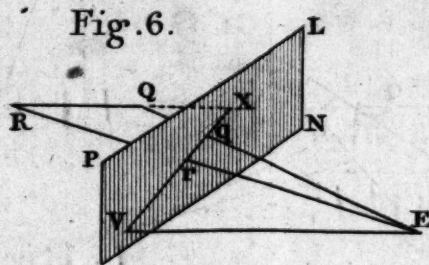


Fig. 6.

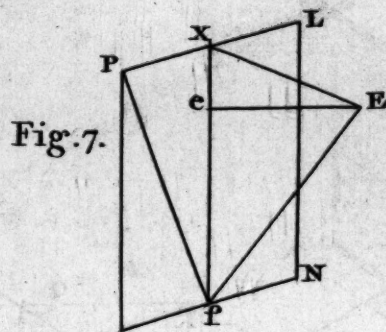


Fig. 7.

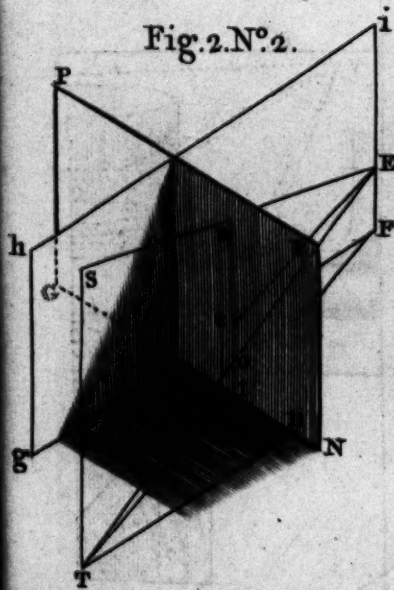


Fig. 2.N° 2.

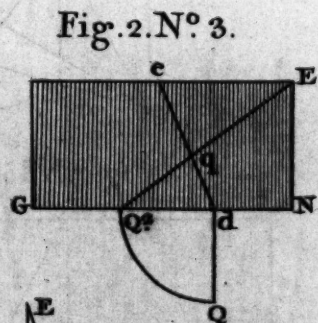


Fig. 2.N° 3.

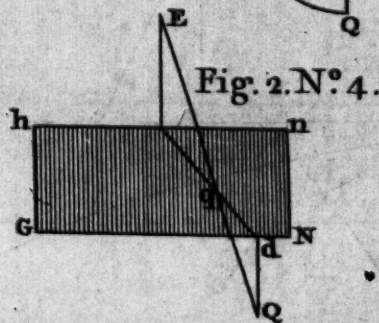


Fig. 2.N° 4.

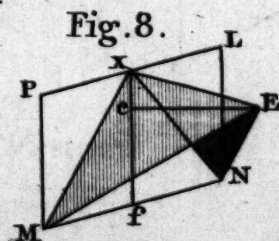


Fig. 8.

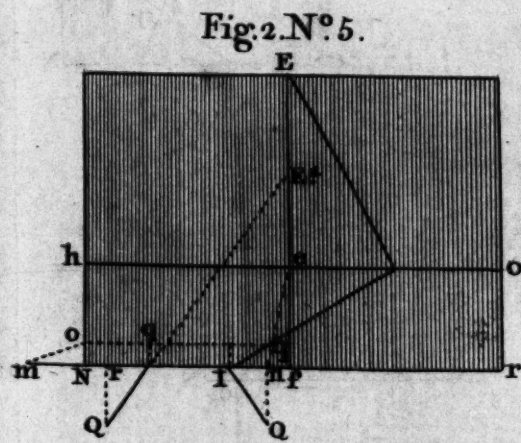


Fig. 2.N° 5.

Fig. 9.

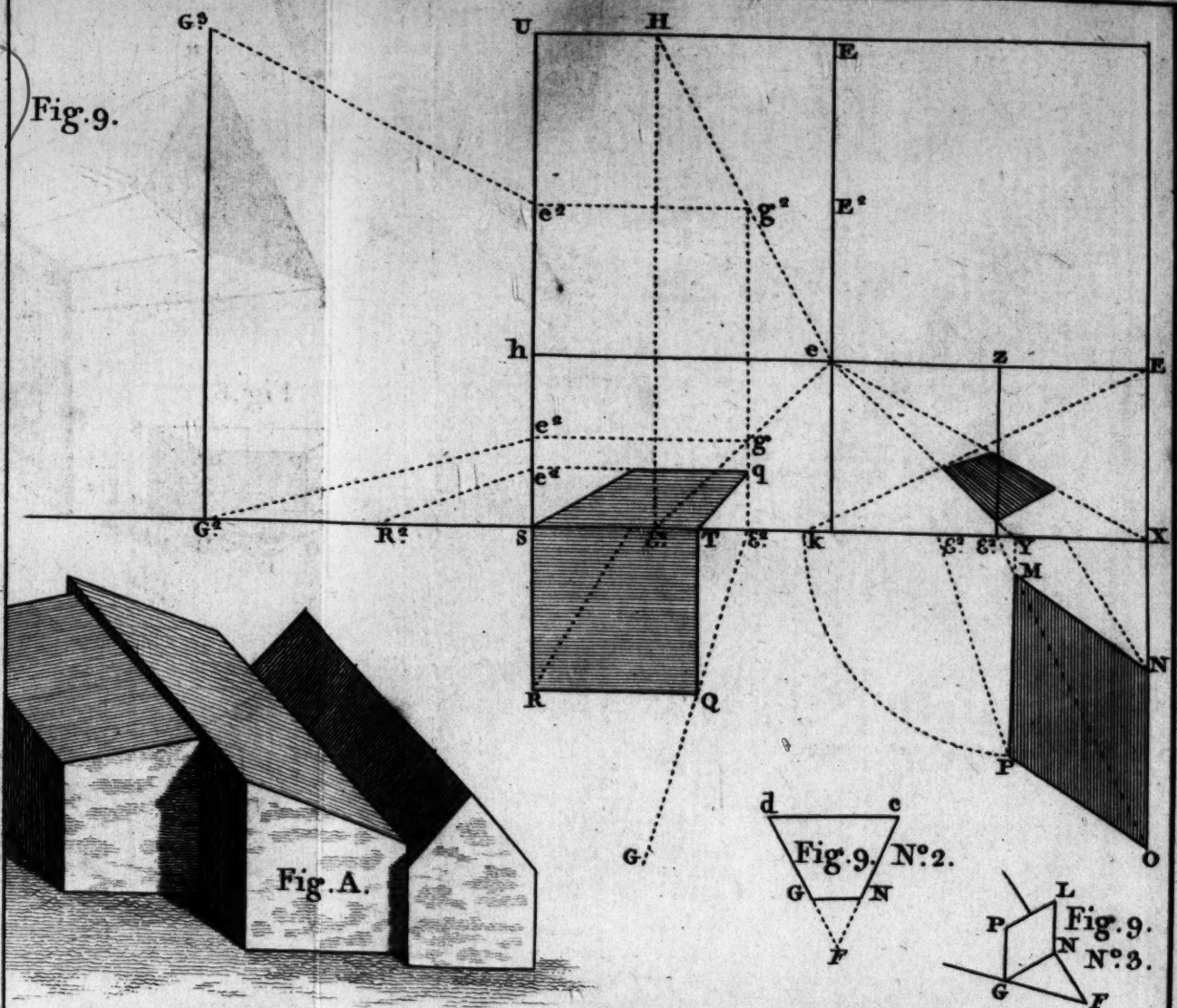


Fig. A.

Fig. 10.

In? Lodge sculp.

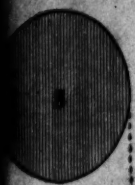


Fig. 11
No. 2.

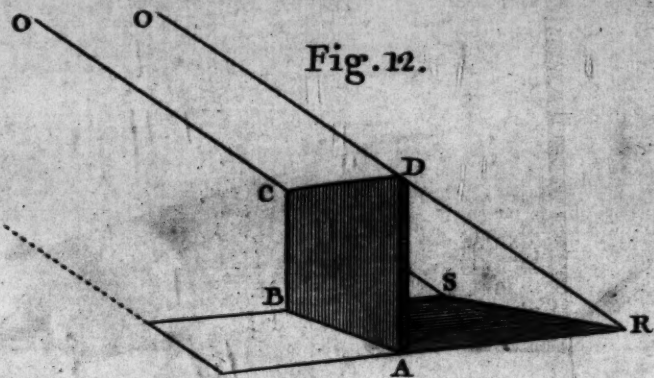


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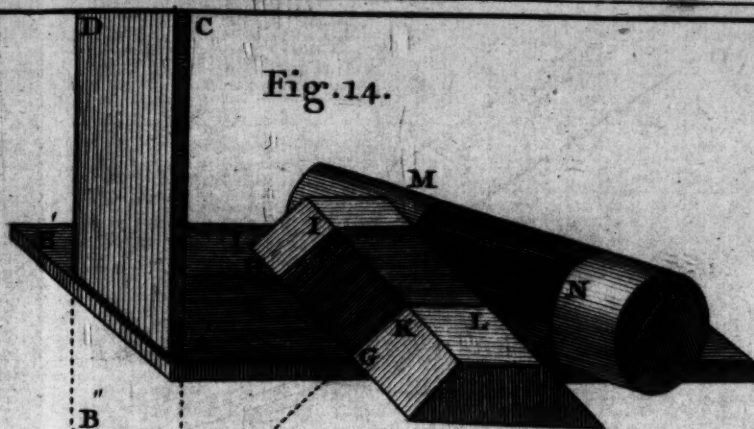


Fig. 14.

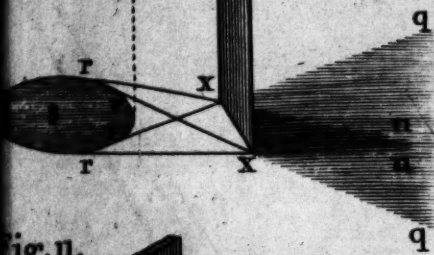


Fig. 11.

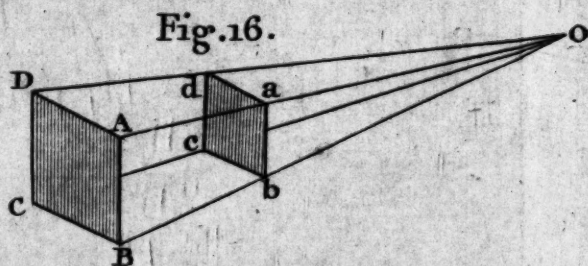


Fig. 16.

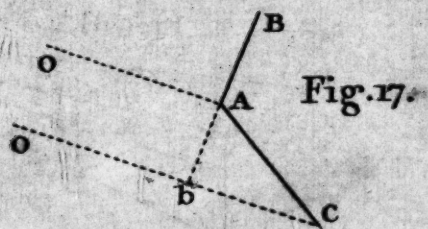


Fig. 17.

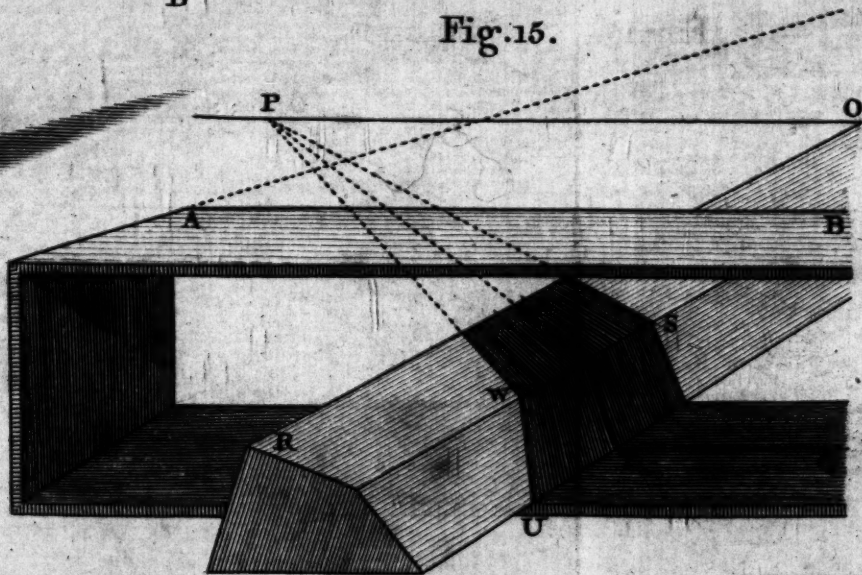


Fig. 15.

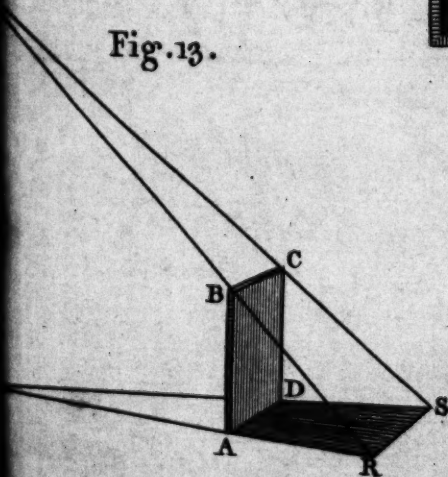


Fig. 13.



Fig. 18.

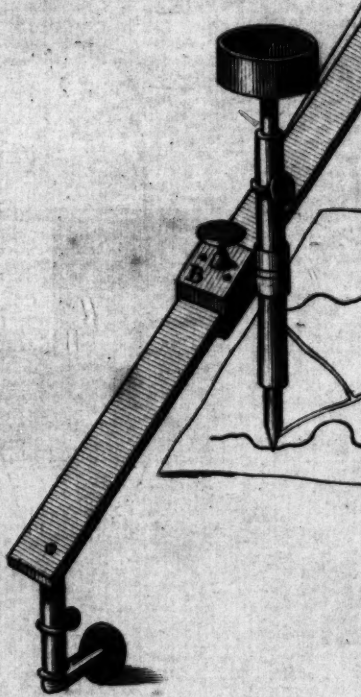


Fig. 19.

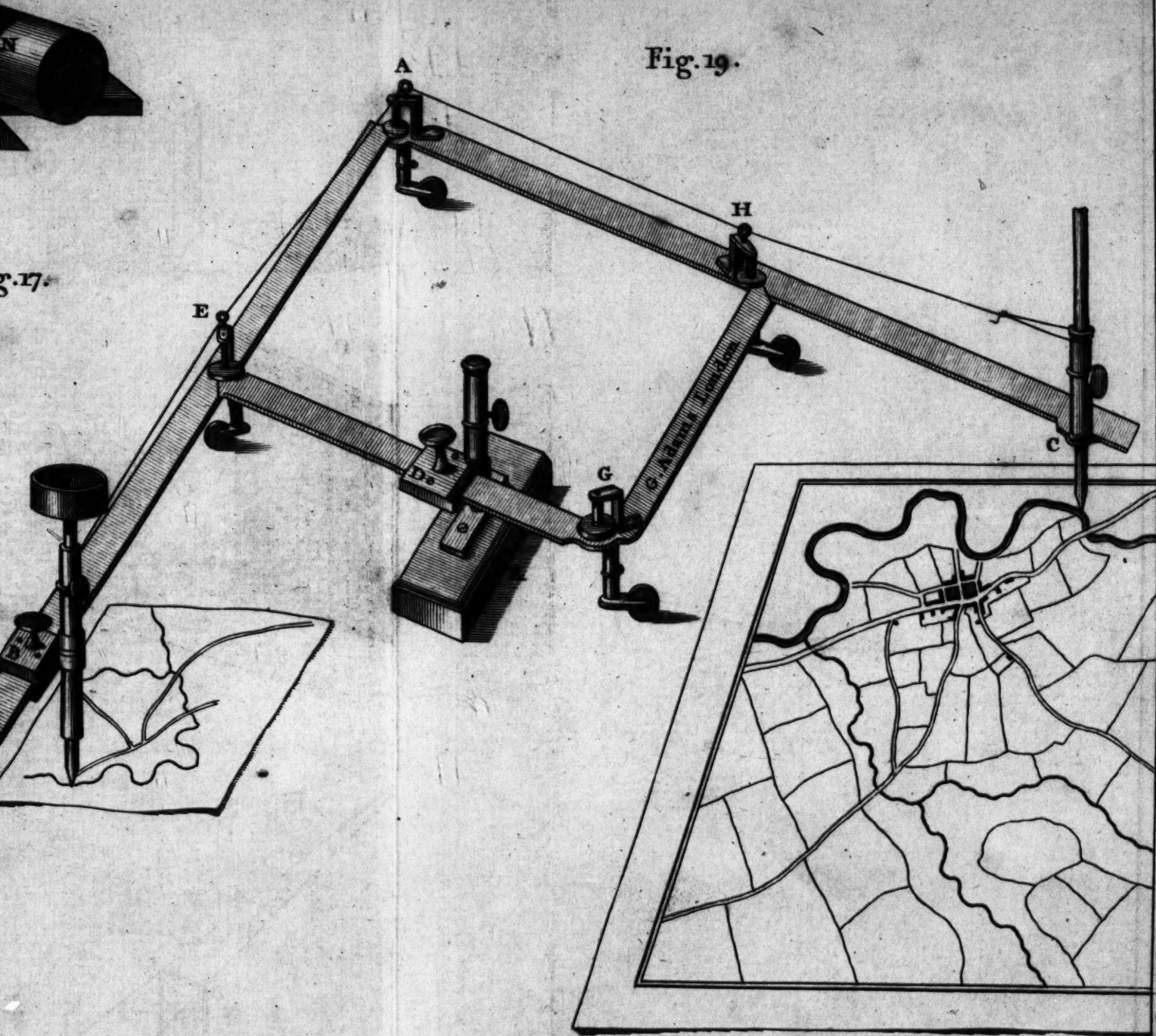
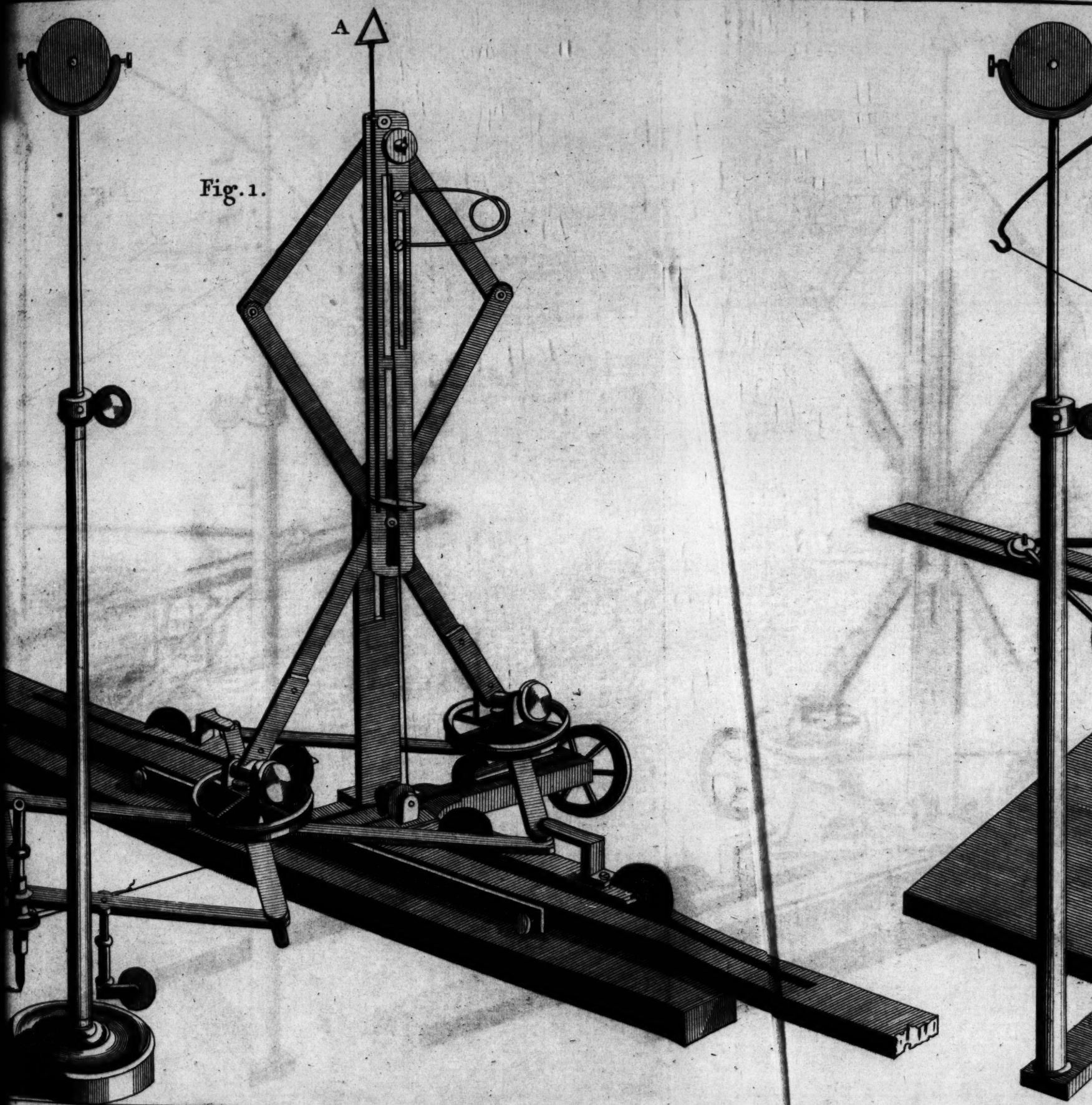


Fig. 1.



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